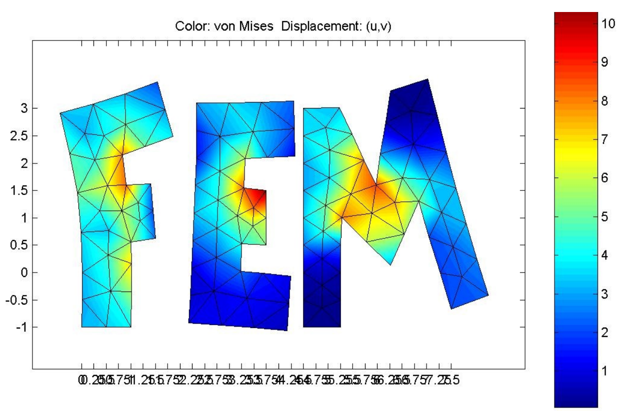
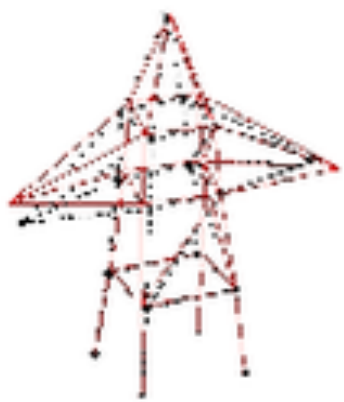
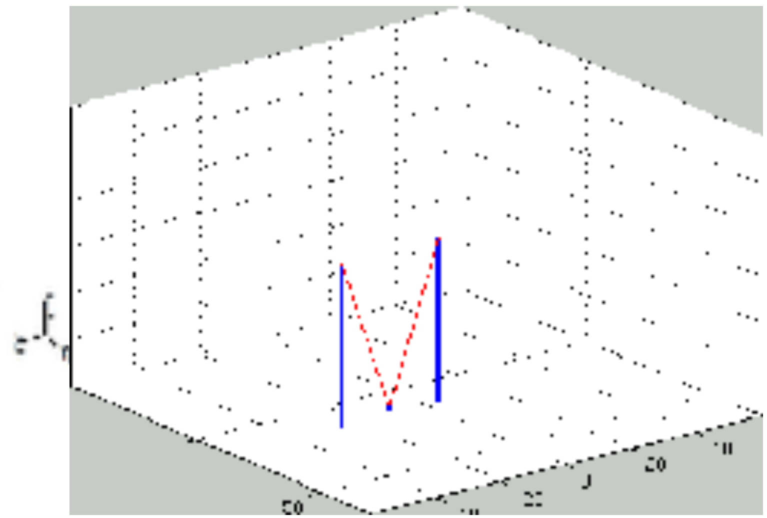
Mathematics in mechanical engineering education: modelling, simulation, computations and interactive learning







[TOUT,YOUT] = ODE23(ODEFUN,TSPAN,Y0)

Mikael Enelund,

Head of Mechanical Engineering program, Chalmers

OUTLINE

- Integration of simulation based mathematics education
- Background
- Reformed Education
- Program learning outcomes and program design
- Integrated curriculum
- Case studies
- Evaluation and results
- Current focus

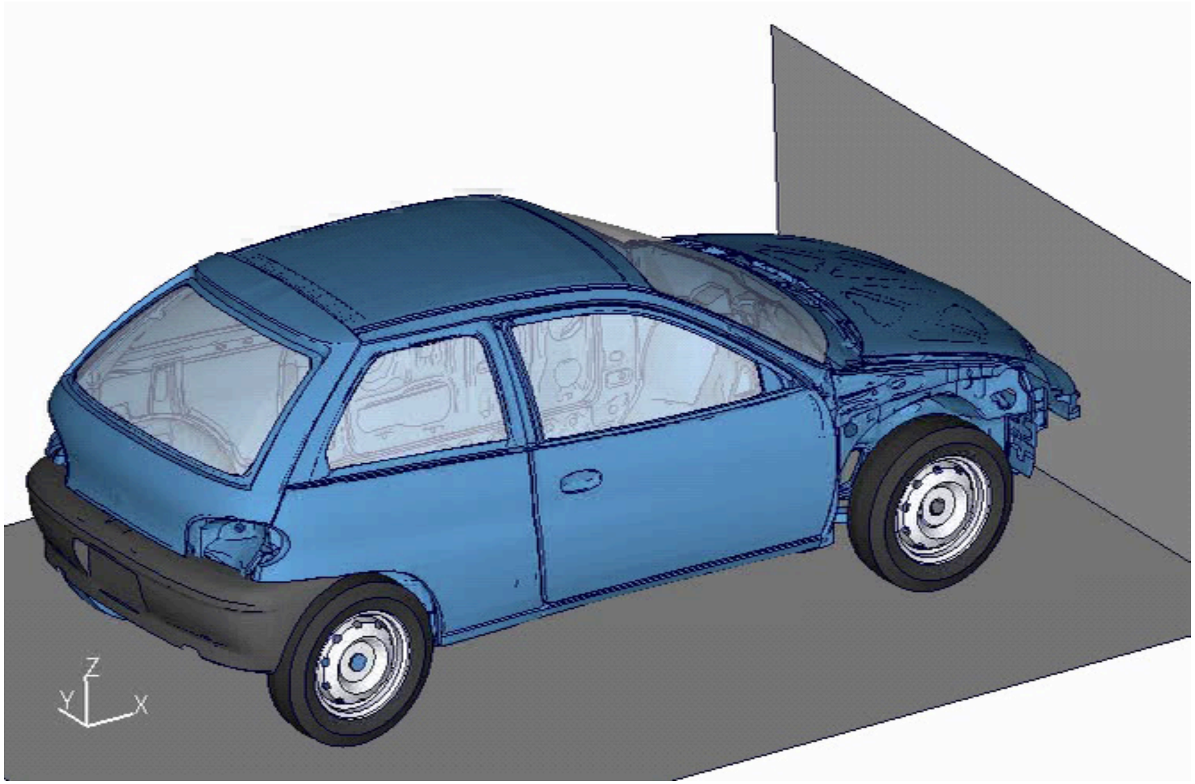
CO DEVELOPPERS

Stig Larsson, Professor in Applied Mathematics
Examiner Mathematical Analysis in Several variables
stig@chalmers.se

Anders Logg, Professor in Computational Mathematics
Examiner Introductory Mathematics
anders.logg@chalmers.se

Mikaael Enelund, Professor in Structural Dynamics
Head of ME program
Examiner Solid Mechanjcs
mikael.enelund@chalmers.se

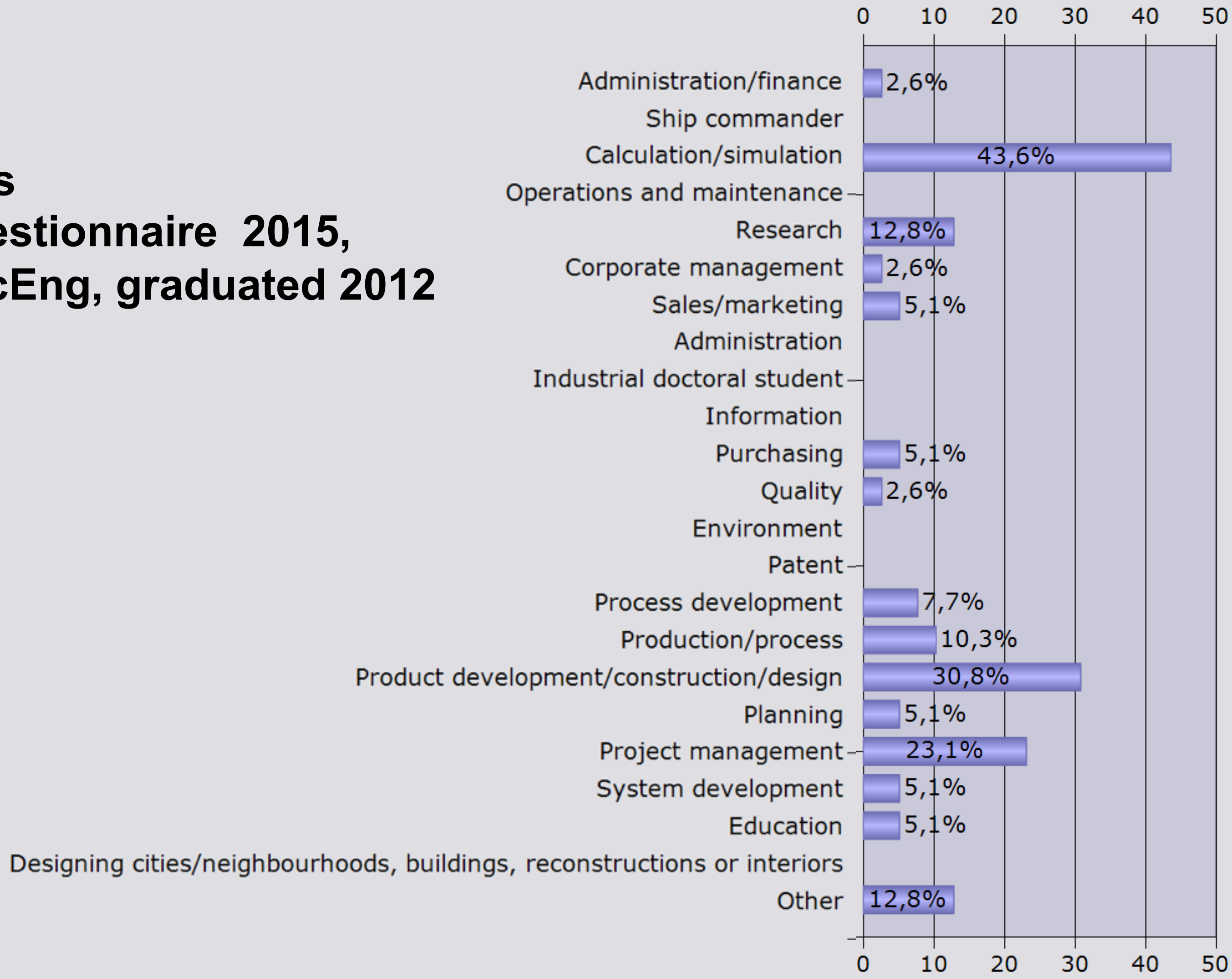
ME ENGINEERS USE A LOT OF ADVANCED MATH



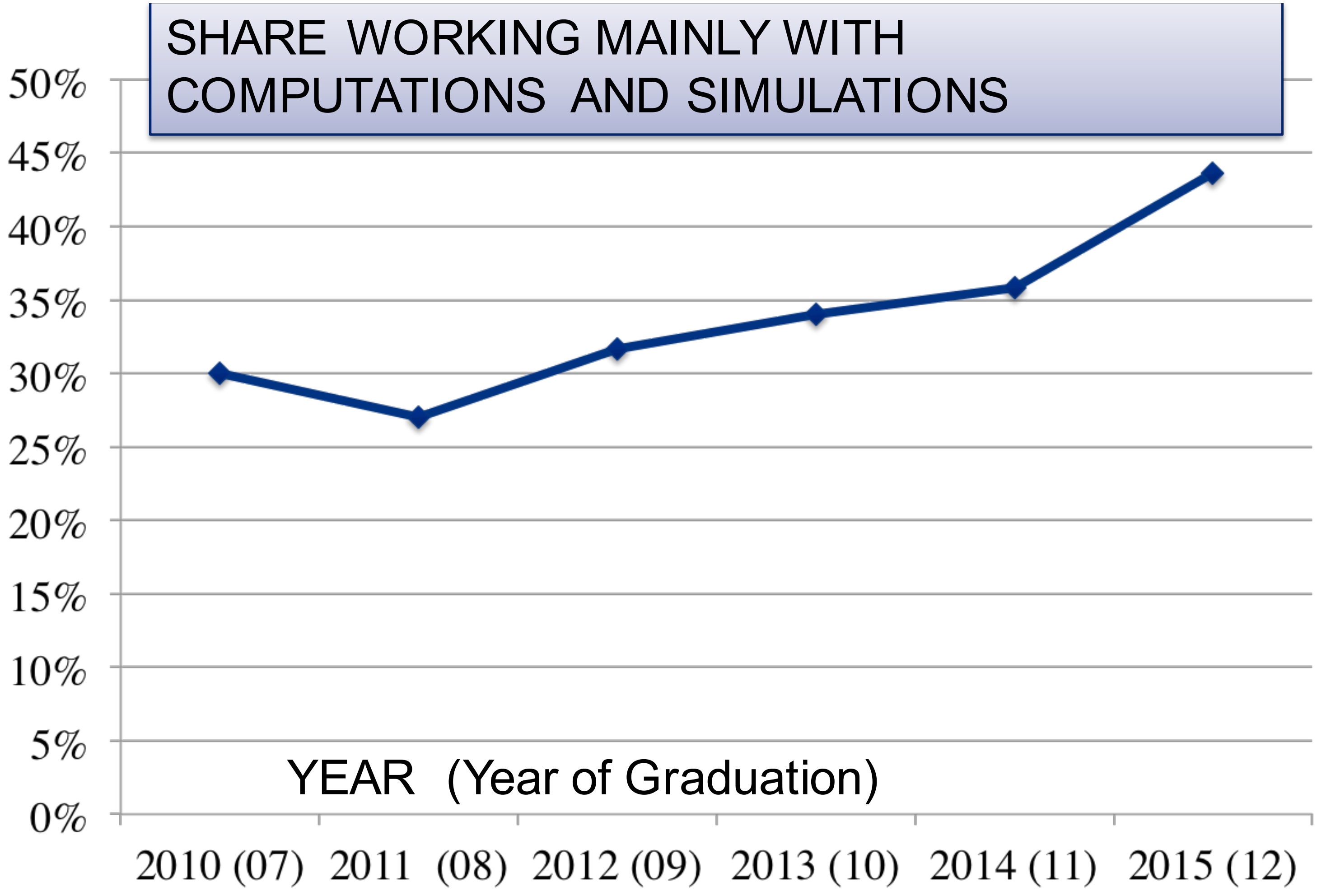
- Simulation driven design
- CAE
- Optimization
- Control
- Industry 4.0
- Internet of things
- Computerization and digitalization
- …..

What do mechanical engineers three years after graduation?

Main Duties
Alumni questionnaire 2015,
MSc in MecEng, graduated 2012



What do mechanical engineers three years after graduation?



CALCULUS: A complete course, Adams

EXERCISES 5.6

Evaluate the integrals in Exercises 1–44. Remember to include a constant of integration with the indefinite integrals. Your answers may appear different from those in the Answers section but may still be correct. For example, evaluating $I = \int \sin x \cos x \, dx$ using the substitution $u = \sin x$ leads to the answer $I = \frac{1}{2}\sin^2 x + C$; using $u = \cos x$ leads to $I = -\frac{1}{2}\cos^2 x + C$; and rewriting $I = \frac{1}{2}\int \sin(2x)\,dx$ leads to $I = -\frac{1}{4}\cos(2x) + C$. These answers are all equal except for different choices for the constant of integration C:
$\frac{1}{2}\sin^2 x = -\frac{1}{2}\cos^2 + \frac{1}{2} = -\frac{1}{4}\cos(2x) + \frac{1}{4}$.

You can always check your own answer to an indefinite integral by differentiating it to get back to the integrand. This is often easier than comparing your answer with the answer in the back of the book. You may find integrals that you can't do, but you should not make mistakes in those you can do because the answer is so easily checked. (This is a good thing to remember during tests and exams.)

1. $\int e^{5-2x}\,dx$
2. $\int \cos(ax+b)\,dx$
3. $\int \sqrt{3x+4}\,dx$
4. $\int e^{2x}\sin(e^{2x})\,dx$
5. $\int \frac{x\,dx}{(4x^2+1)^5}$
6. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
7. $\int x e^{x^2}\,dx$
8. $\int x^2 2^{x^3+1}\,dx$
9. $\int \frac{\cos x}{4+\sin^2 x}\,dx$
10. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$
11. $\int \frac{e^x+1}{e^x-1}\,dx$
12. $\int \frac{\ln t}{t}\,dt$
13. $\int \frac{ds}{\sqrt{4-5s}}$
14. $\int \frac{x+1}{\sqrt{x^2+2x+3}}\,dx$
15. $\int \frac{t\,dt}{\sqrt{4-t^4}}$
16. $\int \frac{x^2\,dx}{2+x^6}$
17. $\int \frac{dx}{e^x+1}$
18. $\int \frac{dx}{e^x+e^{-x}}$
19. $\int \tan x \ln\cos x\,dx$
20. $\int \frac{x+1}{\sqrt{1-x^2}}\,dx$
21. $\int \frac{dx}{x^2+6x+13}$
22. $\int \frac{dx}{\sqrt{4+2x-x^2}}$
23. $\int \sin^3 x\cos^5 x\,dx$
24. $\int \sin^4 t\cos^5 t\,dt$
25. $\int \sin ax\cos^2 ax\,dx$
26. $\int \sin^2 x\cos^2 x\,dx$
27. $\int \sin^6 x\,dx$
28. $\int \cos^4 x\,dx$
29. $\int \sec^5 x\tan x\,dx$
30. $\int \sec^6 x\tan^2 x\,dx$
31. $\int \sqrt{\tan x}\sec^4 x\,dx$
32. $\int \sin^{-2/3} x\cos^3 x\,dx$
33. $\int \cos x\sin^4(\sin x)\,dx$
34. $\int \frac{\sin^3 \ln x\cos^3 \ln x}{x}\,dx$
35. $\int \frac{\sin^2 x}{\cos^4 x}\,dx$
36. $\int \frac{\sin^3 x}{\cos^4 x}\,dx$
37. $\int \csc^5 x\cot^5 x\,dx$
38. $\int \frac{\cos^4 x}{\sin^8 x}\,dx$
39. $\int_0^4 x^3(x^2+1)^{-\frac{1}{2}}\,dx$
40. $\int_1^{\sqrt{e}} \frac{\sin(\pi \ln x)}{x}\,dx$
41. $\int_0^{\pi/2} \sin^4 x\,dx$
42. $\int_{\pi/4}^{\pi} \sin^5 x\,dx$
43. $\int_e^{e^2} \frac{dt}{t\ln t}$
44. $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{9}} \frac{2^{\sin\sqrt{x}}\cos\sqrt{x}}{\sqrt{x}}\,dx$

45. Use the identities $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ and $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ to help you evaluate the following:

$$\int_0^{\pi/2} \sqrt{1+\cos x}\,dx \quad \text{and} \quad \int_0^{\pi/2} \sqrt{1-\sin x}\,dx$$

46. Find the area of the region bounded by $y = x/(x^2+16)$, $y = 0$, $x = 0$, and $x = 2$.

47. Find the area of the region bounded by $y = x/(x^4+16)$, $y = 0$, $x = 0$, and $x = 2$.

48. Express the area bounded by the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ as a definite integral. Make a substitution that converts this integral into one representing the area of a circle, and hence evaluate it.

49. Use the addition formulas for $\sin(x \pm y)$ and $\cos(x \pm y)$ from Section P.6 to establish the following identities:

$$\cos x\,\cos y = \frac{1}{2}\Big(\cos(x-y) + \cos(x+y)\Big),$$
$$\sin x\,\sin y = \frac{1}{2}\Big(\cos(x-y) - \cos(x+y)\Big),$$
$$\sin x\,\cos y = \frac{1}{2}\Big(\sin(x+y) + \sin(x-y)\Big).$$

50. Use the identities established in Exercise 49 to calculate the following integrals:
$\int \cos ax\,\cos bx\,dx$, $\int \sin ax\,\sin bx\,dx$, and $\int \sin ax\,\cos bx\,dx$.

51. If m and n are integers, show that:
(i) $\int_{-\pi}^{\pi} \cos mx\,\cos nx\,dx = 0$ if $m \ne n$,
(ii) $\int_{-\pi}^{\pi} \sin mx\,\sin nx\,dx = 0$ if $m \ne n$,
(iii) $\int_{-\pi}^{\pi} \sin mx\,\cos nx\,dx = 0$.

52. **(Fourier coefficients)** Suppose that for some positive integer k,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{k}(a_n\cos nx + b_n\sin nx)$$

BETA Mathematics handbook

31. $\int \frac{\sqrt{ax+b}}{x^n}\,dx = -\frac{1}{b(n-1)}\left(\frac{(ax+b)^{3/2}}{x^{n-1}} + \frac{(2n-5)a}{2}\int\frac{\sqrt{ax+b}}{x^{n-1}}\,dx\right)$

32. $\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$

33. $\int \frac{x}{\sqrt{ax+b}}\,dx = -\frac{2b\sqrt{ax+b}}{a^2} + \frac{2(ax+b)^{3/2}}{3a^2}$

34. $\int \frac{x^2}{\sqrt{ax+b}}\,dx = \frac{2b^2\sqrt{ax+b}}{a^3} - \frac{4b(ax+b)^{3/2}}{3a^3} + \frac{2(ax+b)^{5/2}}{5a^3}$

35. $\int \frac{x^n}{\sqrt{ax+b}}\,dx = \frac{2}{a(2n+1)}\left(x^n\sqrt{ax+b} - bn\int\frac{x^{n-1}}{\sqrt{ax+b}}\,dx\right)$

36. $\int \frac{dx}{x^n\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)\,bx^{n-1}} - \frac{(2n-3)\,a}{(2n-2)\,b}\int\frac{dx}{x^{n-1}\sqrt{ax+b}} \quad (n \ne 1)$

37. $\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| & (b>0) \\ \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}} & (b<0) \end{cases}$

38. $\int \frac{dx}{c+\sqrt{ax+b}} = \frac{2}{a}\left(\sqrt{ax+b} - c\ln|c+\sqrt{ax+b}|\right)$

39. $\int \frac{\sqrt{ax+b}}{c+\sqrt{ax+b}}\,dx = \frac{1}{a}\left(ax+b-2c\sqrt{ax+b}+2c^2\ln|c+\sqrt{ax+b}|\right)$

40. $\int \frac{x}{c+\sqrt{ax+b}}\,dx =$
$= \frac{1}{a^2}\left(2(c^2-b)\sqrt{ax+b} - c\,(ax+b) + \frac{2}{3}(ax+b)^{3/2} - 2c(c^2-b)\ln|c+\sqrt{ax+b}|\right)$

41. $\int \frac{dx}{\sqrt{ax+b}\,(c+\sqrt{ax+b})} = \frac{2}{a}\ln|c+\sqrt{ax+b}|$

42. $\int \frac{dx}{(ax+b)(c+\sqrt{ax+b})} = \frac{2}{ac}\ln\left|\frac{\sqrt{ax+b}}{c+\sqrt{ax+b}}\right|$

43. $\int \frac{dx}{(c+\sqrt{ax+b})^2} = \frac{2c}{a(c+\sqrt{ax+b})} + \frac{2}{a}\ln|c+\sqrt{ax+b}|$

44. $\int \frac{\sqrt{ax+b}}{(c+\sqrt{ax+b})^2}\,dx = \frac{2\sqrt{ax+b}}{a} - \frac{2c^2}{a(c+\sqrt{ax+b})} - \frac{4c}{a}\ln|c+\sqrt{ax+b}|$

Instead, write a computer program that solves all problems(integrals)


```
def f(x):
    return cos(x)**2

a = -pi
b = pi
n = 100
dx = (b - a)/n
F = 0
for i in range(n):
    F += f((i+0.5)*dx)*dx

print(F)
```

$$\int_{-\pi}^{\pi} (\cos(x))^2 \mathrm{d}x$$

```
3.141592653589793
```

Mathematics + programming = true

- A computer program solves the "general" problem
- Reduce repetitive exercising to practice more on understanding, problem definition and computations
- Opportunities to practice math and problem solving at a higher level
- Logical and algorithmic thinking, creativity and problem solving
- Requires knowledge of mathematics and programming
- Programming creates an understanding of the digital world and opportunities to solve new problems, create new systems, processes and products

BACKGROUND – REFORMED MATH

- Rapid development of computers and the internet
  - Solving most problems faced in modern engineering includes high precision digital models and simulations.
  - Preparing students for a modern approach based on modelling, simulation and analysis
- CDIO raises the need for a reform of math education
  - Need for a toolbox to handle real (complex) problems
- Young persons learn much differently than they used to a few decades ago. “the Nintendo Syndrome”
  - Do not read manuals but go and try. If “killed” try something else until you get to the next level. If no success go to the internet to get a hint.
  - This comes through in studying: it is becoming increasingly hard to make students read books. At the same time providing them with ways to try before reading proves educationally rewarding.
- Need for reformed math education, not less math

REFORMED SIMULATION BASED MATHEMATICAL EDUCATION

- Launched 2006/2007 and continuously improved,
- New math courses including a basic course in Matlab programming,
- Focus transferred from solving oversimplified special problems with known solutions to more open general problems,
- Interactive/virtual learning learning environments,
- Teaching and learning in computer lab,
- Textbook book in Computational math. Programming, numerics and simulations integrated and
- Integration of mathematics in other fundamental engineering courses.

CORNERSTONES

- To highlight and clarify modelling, computations, analyses and simulations,
- Full integration of computational aspects (including programming) and symbolic aspects of mathematics,
- Construction of algorithms and writing own programs (programming skills and understanding of mathematics and algorithm construction)
- General equations instead of the simplified special equations whose solutions can be written in elementary functions
- The finite element taught in first year math course Calculus in Several Variables and used in Solid Mechanics course
- Computer-oriented exercises, assignments and team projects that are used simultaneously in the mathematics courses and in courses of mechanics and solid mechanics

PROGRAM LEARNING OUTCOMES: MATH

The Master of Science in Mechanical Engineering graduate shall:

1 Be able to put into practice (apply) mathematics with focus on being able to

- *1.1 solve linear and nonlinear systems of algebraic equations by numerical methods,*
- *1.2 solve ordinary differential equations of the following types; separable, inhomogeneous with constant coefficients and Euler's,*
- *1.3 solve by numerical methods linear and nonlinear ordinary differential equations inclusive reformulating to a first order system,*
- *1.4 solve the eigenvalue problem for continuous and discretized systems*
- *1.5 use the Finite element method to solve partial differential equations,*
- *1.6 explain the fundamentals of probability theory and statistics and being able to plan experiments with respect to statistical variations*
- *1.7 program solutions, including graphic presentations of engineering problems in Matlab and/or Python.*

PROGRAM LEARNING OUTCOMES – MATH, CONT'D

The Master of Science in Mechanical Engineering graduate shall:

2 *Be able to formulate theoretical models and set up equations to describe the models. Solve equations in order to simulate reality and assess the reasonableness of the choice of model and the solution's level of accuracy.*

3 *Be able to analyze, solve and simulate advanced mechanical engineering problems within the selected specialization area/master's program by using modern, computer-based tools and from these, selecting the most appropriate ones*

The reformed courses are developed to meet these goals.

ME PROGRAMME – INTERGRATED CURRICULUM

YEAR 1

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Programming in Matlab (4.0)	Computer aided engineering (4,0)	**Linear algebra** (7,5)	**Mathematical analysis in several variables** (7,5)
Introductory mathematics (7,5)	**Mathematical analysis in a single variable** (7.5)	Statics & strength of materials (7.5)	Solid Mechanics (7,5)
Introduction to Mechanical Engineering (7,0)			

- Joint exercises/assignments/projects
- Matlab programming, numerical solutions and simulations
- Simulation using industrial software (CATIA, ANSYS, ADAMS, FLUENT…)

YEAR 2

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Mechanics - Dynamics (7,5p)	Machine element (7,5)	Thermodynamics and energy technology (7,5)	Industrial production and organisation (6)
Material technology (7,5)	Material and manufacturing technology (7,5)	Integrated design and manufacturing (7,5)	
		Sustainable product development (4,5)	Engineering economics (4,5)

- Joint exercises/assignments
- Matlab programming, numerical solutions and simulations
- Simulation using industrial softwares (CATIA, ANSYS, ADAM, FLUENT…)

YEAR 3

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Mechatronics (7,5)	Automatic control (7,5)	Bacheleor diploma project (15)	
Fluid mechanics (7,5)	Elective 1 (7,5)	Elective 2 (7,5)	Mathematical statistics (7,5)

Elective 1

- Energy conversion
- Finite element method
- Machine design
- Simulation of production

Elective 2

- Logistics
- Sound and vibration
- Material and process selection
- Objectoriented programming
- Transforms and differential equations
- Heat transfer

Matlab programming, numerical solutions and simulations

Simulation using industrial softwares (CATIA, ANSYS, ADAMS, FLUENT…)

YEARS 4 and 5

Second cycle, 2 years international master programme. 8 master programmes belong to Mechanical Engineering

- **MSc PROGRAM IN APPLIED MECHANICS**
- **MSc PROGRAM IN AUTOMOTIVE ENGINEERING**
- MSc PROGRAM IN MATERIALS ENGINEERING
- **MSc PROGRAM IN NAVAL ARCHITECTUREAND OCEANS ENGINEERING**
- **MSc PROGRAM IN PRODUCT DEVELOPMENT**
- MSc PROGRAM IN PRODUCTION ENGINEERING
- **MSc PROGRAM IN SUSTAINABLE ENERGY SYSTEMS**
- MSc PROGRAM IN TECHNOLOGY, SOCIETY AND THE ENVIRONMENT

Goals MATH Courses (mandatory)	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2	3
Introduction to Mechanical Eng.									
Introductory course in Mathematics	T	I						T	
Programming in Matlab	T							T	I
Calculus in a Single Variable	T	T	T						
Computer Aided Design								T	T
Linear Algebra	T		T	T				T	
Statics and Solid Mechanics	U	U	U					T	
Calculus in Several Variables	U	T	T	T	T			T	
Solid Mechanics	U	U	U		T			T	T
Mechanics – Dynamics	U	U	U	U				U	
Engineering Materials	U	U		U				U	T
Machine Elements	U	U	U	U	U			U	U
Material and Manufact. Techn.	U	U	U	U	U			T	U
Thermodynamics and Energy Technology	U	U					T		
Sustainable Product Develop.								U	
Integrated Design and Manufact.	U	U							T
Industrial Production and Org.									
Engineering Economics									
Mechatronics	U	U	U					T	
Fluid Mechanics	U	U	U	U	U		U		T
Automatic Control	U	U	U	U					
Mathematical Statistics	U	U	U			T			T
Bacheleor Diploma Project									U

Part of program design matrix – teaching math

I = Introduce
T = Teach
U = Utilize

Systematic approach to design an integrated curriculum

Mechanical Engineering

Goals MATH Courses (mandatory)	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2	3
Introductory course in Mathematics	W, CE						CE		
Programming in Matlab							W, CE		
Calculus in a Single Variable	W	W, CE	W, CE				CE		
Computer Aided Design									W, P
Linear Algebra	W, CE			W, CE			CE		
Statics and Solid Mechanics	W CE	W					CE	CE	
Calculus in Several Variables	W, CE	W, CE		W, CE	W, CE		CE		
Solid Mechanics	W, CE	W, CE			W, CE		CE	CE	CE
Mechanics – Dynamics	W	W, CE	W, CE				CE	CE	CE
Engineering Materials									
Machine Elements	W	W					P	P	
Material and Manufact. Techn.					P				
Thermodynamics and Energy Technology							P	P	
Sustainable Product Develop.									
Integrated Design and Manufact.								P	P
Mechatronics							CE	CE	
Fluid Mechanics	W, CE	CE	CE				CE	CE	L
Automatic Control	W, CE	W, CE	W, CE				CE	CE	CE
Mathematical Statistics						W, P	P	P	P

Part of program design matrix – math assessment

W = Written exam/quiz
CE = Computer exercise
P = Project
L=Lab

Systematic approach to evaluate the program learning outcomes

FIRST YEAR MATH COURSES

Quarter 1: Introductory Mathematics

Function, continuity, derivative in one variable. Series.

Computer exercises:

1. Function gallery
2. Bisection algorithm
3. Fixed point iteration
4. Numerical derivative
5. Newton’s method

Quarter 2: Mathematical Analysis in One Variable

Integral, ODE, transforms Geometry and vector algebra.

Computer exercises:

1. ODE1: primitive function (integral)
2. ODE2: Euler’s method for systems of ODE
3. ODE3: implicit methods
4. ODE4: boundary value problems (shooting methods with Euler Solver)

FIRST YEAR MATH COURSES, CONT'D

Quarter 3: Linear Algebra (taught in parallel with Statics and Strength of Materials)

Gauss elimination, matric algebra, determinant, inverse matrix. Orthogonality, eigenvalue problem. Least squares method.

Computer exercises:

1. Matrix algebra
2. Geometry
3. Systems of linear equations, error analysis, condition number (Elastic Truss)
4. Least squares (calibration of Norton's law for creeping)

FIRST YEAR MATH COURSES, CONT'D

Quarter 4: Mathematical Analysis in Several Variables (taught parallel with Solid Mechanics)

Partial derivative. Linearization, Jacobi matrix, Newton's metod. Taylor's formula. Optimization. Curves and surfaces. Double and triple integral. Curve integral, surface integral. Boundary value problems and the finite element method.

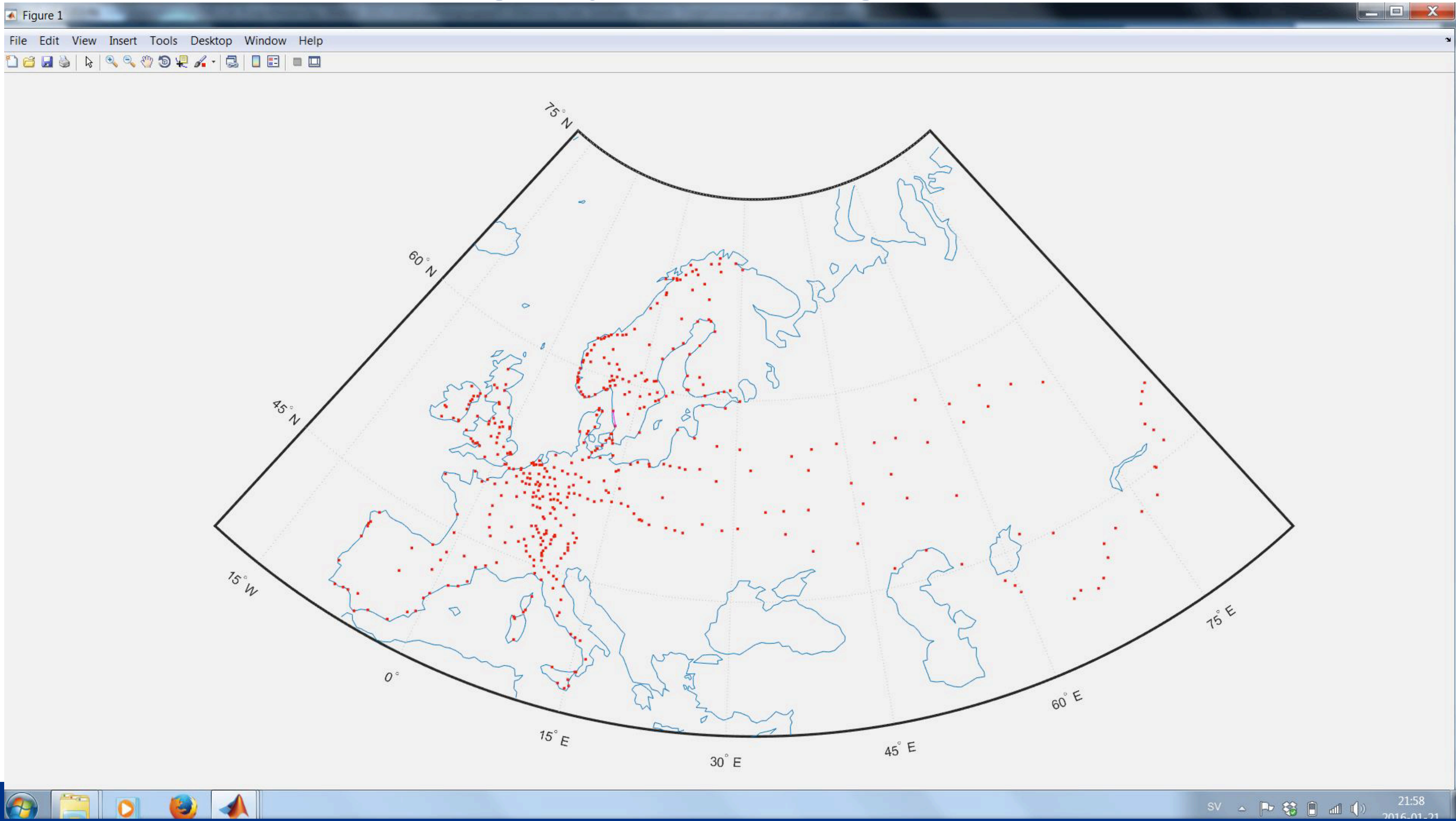
Computer exercises:

1. Visualization of multivariable functions
2. Jacobi matrix and Newton's method
3. Optimization
4. The finite element method in 1–D (own code, bar structure, rotating disc)
5. The finite element method in 2–D (Matlab's PDE Toolbox, ANSYS, heat transfer, stress concentration)

PROGRAMMING IN MATLAB

- General methodology requires programming
- Experience shows the need for a separate programming course
- Aim: Develop own programs from problem description to working code
- 4 programming assignments and final exam in computer lab
- Why Matalb?
  - Easy to use and suitable as a first programming environment
  - Used in all applied courses and in applied research
  - Toolboxes and built-in function
- Third year course in Object-oriented programing in Python

Example: Assignment "Least cost path using Dijkstra's algorithm"



Example: Introductory mathematics (taught in parallel with Programming in Matlab)

Compute square $\sqrt{2}$ by bisection algorithm $x_n = \dfrac{x_{n-1} + 2/x_{n-1}}{2}$

$$
\begin{aligned}
x_0 &= 1 \\
x_1 &= (x_0 + 2/x_0)/2 &\approx 1.5 \\
x_2 &= (x_1 + 2/x_1)/2 &\approx 1.41666666666667 \\
x_3 &= (x_2 + 2/x_2)/2 &\approx 1.41421568627745 \\
x_4 &= (x_3 + 2/x_3)/2 &\approx 1.41421356237747 \\
x_5 &= (x_4 + 2/x_4)/2 &\approx 1.41421356237731 \\
x_6 &= (x_5 + 2/x_5)/2 &\approx 1.41421356237731 \\
x_7 &= (x_6 + 2/x_6)/2 &\approx 1.41421356237731 \\
x_8 &= (x_7 + 2/x_7)/2 &\approx 1.41421356237731 \\
x_9 &= (x_8 + 2/x_8)/2 &\approx 1.41421356237731 \\
x_{10} &= (x_9 + 2/x_9)/2 &\approx 1.41421356237731
\end{aligned}
$$

MATLAB

```
x   = 1;
tol = 1e-10;
dx = 2*tol;

while abs(dx) > tol
    dx = (x + 2/x) / 2 - x;
    x += dx;
end
```

C++

```
int main()
{
  double x = 1.0;
  double tol = 1e-10;
  double dx = 2*tol;

  while (abs(dx) > tol)
  {
    dx = (x + 2/x) / 2 - x;
    x += dx;
  }

  return 0;
}
```

Example: Introductory mathematics

Write a program that solves the equation *f (x)* = 0 with arbitrary accuracy.

Solve $5x + 5 = 10, \quad x^2 = 2, \quad x^5 - x + 1 = 0, \quad x = \cos x$

Example: Mathematical analysis in several variables

Write an FE-program that solves the boundary value problem

$$-\frac{d^2u}{dr^2} - \frac{1}{r}\frac{du}{dr} + \frac{1}{r^2}u = \frac{1-\nu^2}{E}K_r \quad \text{for } r \in I = (a,b)$$

$$u(a) = 0, \; u'(b) = 0.$$

with $K_r = \rho\omega^2 r$.

Governing equation for a rotating elastic disc

ASSESMENT

- Mandatory computer assignments to hand-in or present in computer lab
- Web based quizzes that can give credits to be included in the final exam
- Final exam with theory, explanatory and deductive problems, coding problems and traditional problems

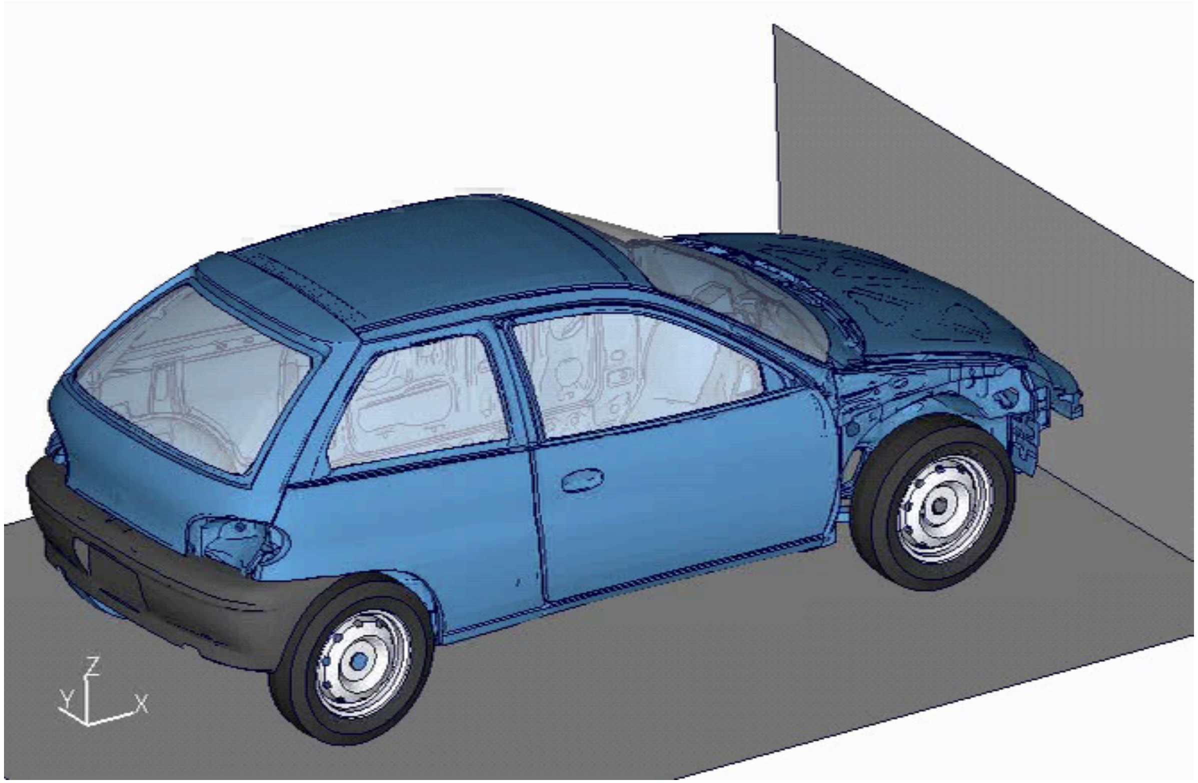
Some Remarks

- Mathematics as a general tool for modeling
- Mathematics as a general tool for solving equations
- General methodology requires numerics + programming
- General methodology requires mathematical theory
- General methodology same for $x^2 = 2$ as for systems of coupled nonlinear partial differential equations

The same method solves the Einstein's field equations

$$R_{ab} - \frac{1}{2} R g_{ab} + g_{ab}\Lambda = 8\pi T_{ab}$$

And Newton’s equations of motion

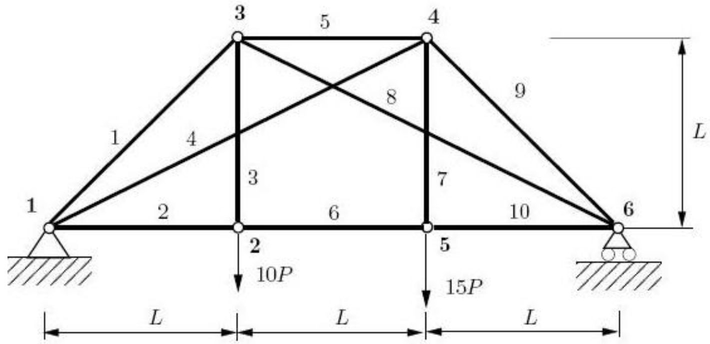


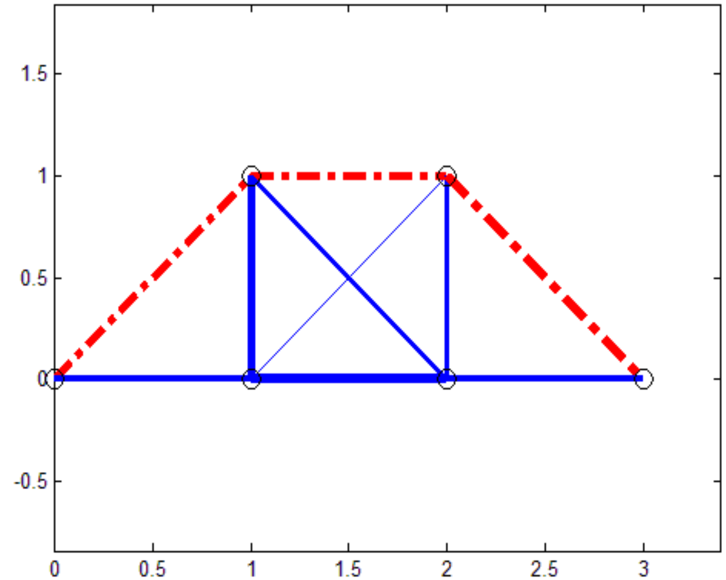
JOINT COMPUTER ASSIGNMENTS, 2 EXAMPLES

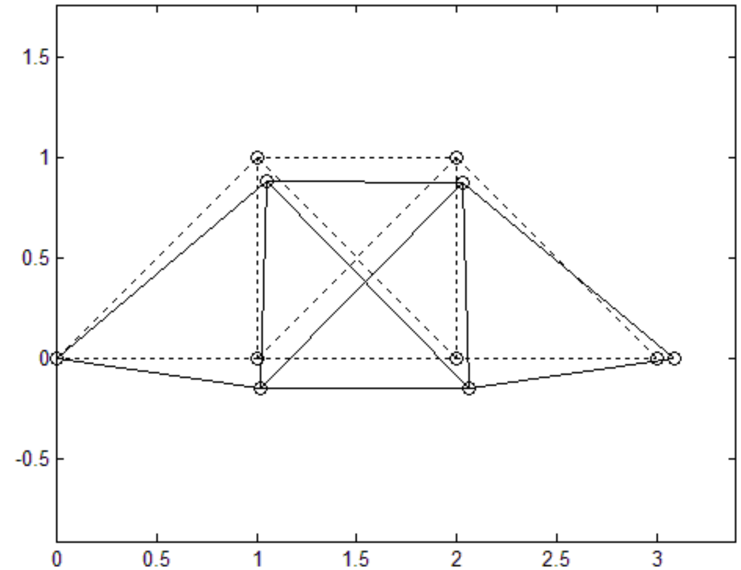
Courses Statics and strength of materials/Linear algebra:

Analysis of elastic truss frame

- Programming: from problem definition to code
- Manage large systems of equations,
- Visualize the stress distribution and deformations and optimization
- Introduction to FEM and Structural Mechanics



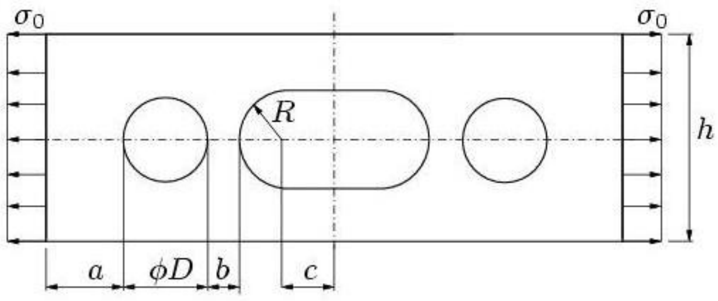


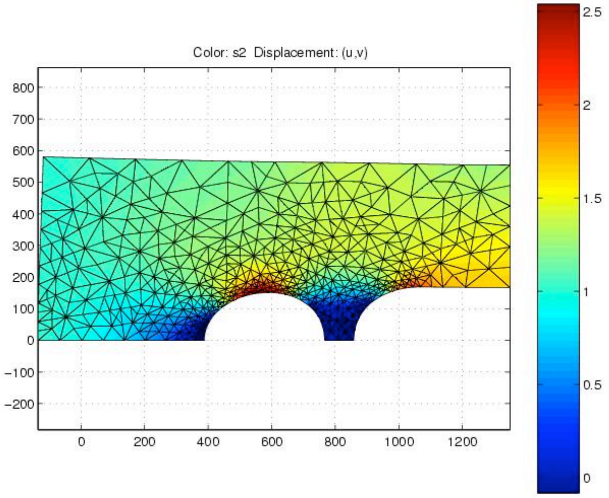


Courses: Mathematical Analysis in Several Variables and Solid Mechanics

Stress analysis of plane elastic plate with 3 holes

- Develop knowledge about stress distribution and how the stress is increased due to abrupt changes in geometry
- Skills to use the finite element method and introduction to error estimation and adaptive mesh refinement



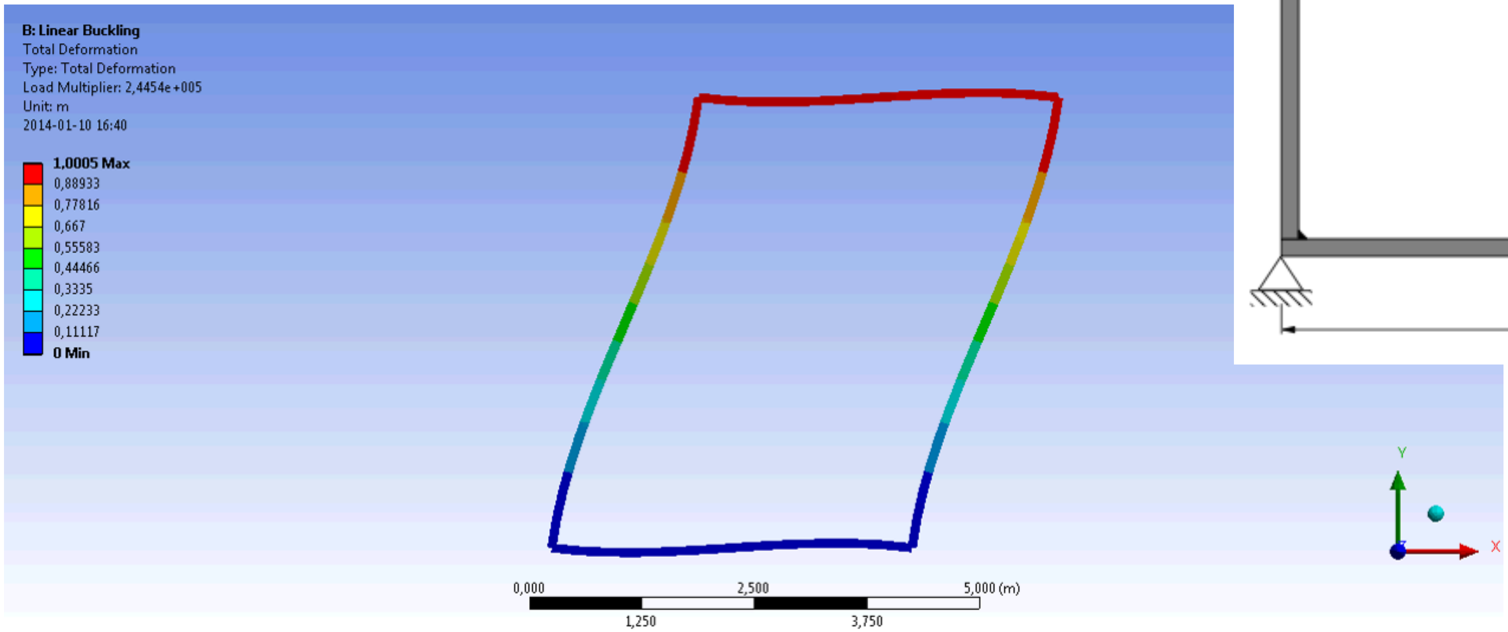


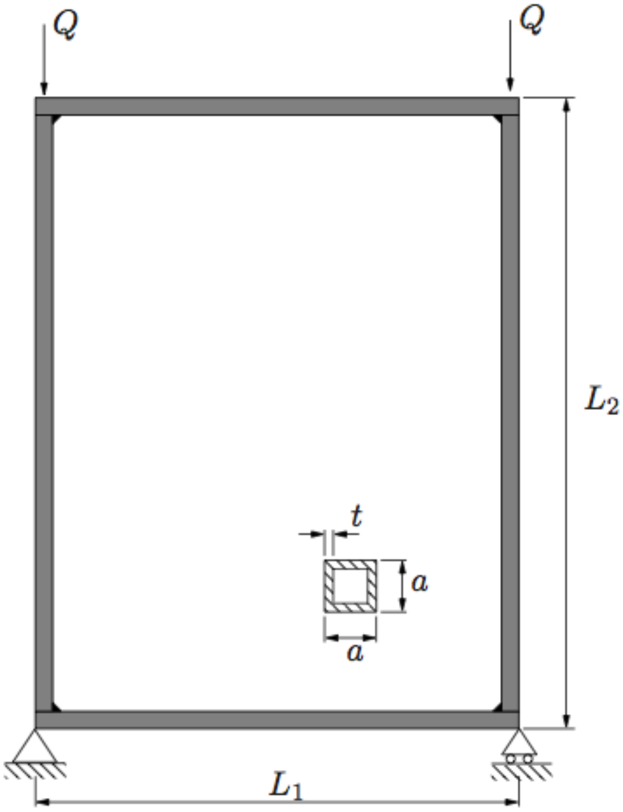
Case Study: Solid Mechanics

- Bachelor 1st year course in Q4, 7.5 ECTS, about 170 students
- Taught in parallel and in cooperation with Mathematics in Several Variables
    - Mathematical background to FEM in math course
    - Applications and techniques in Solid Mechanics
- FEM – multiple purposes
    - To prepare students for future careers
    - To equip the students with a design/simulation tool for upcoming product development projects (Design-Build-Test)
    - As pedagogical tool to illustrate and simulate theory, principles and phenomena
- Five projects: beam structure, stability of frame, stress analysis of plate with hole, shrink fit assembly and stress analysis and life length prediction of 3D bracket

Stability analysis of plane frame

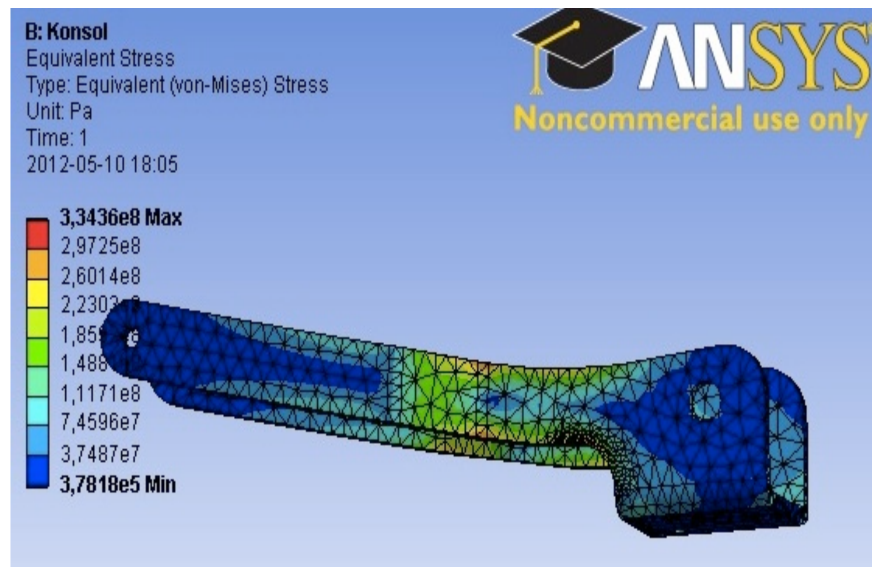
- Solved by use of ANSYS
- Use Finite element method for stability analysis
- Antisymmetric buckling mode





Stress analysis and life length prediction of 3D bracket

- Industrial problem
- Solved by use of CATIA &ANSYS
- Illustrate stress concentration
- Life length prediction and crack propagation by FEM
- Mesh refinement, ionvergence, error estimation



EVALUATION AND RESULTS

- The computer is considered a valuable tool for match calculations and learning of mathematics
- Motivation to study, number of passed and students' general impression of math courses have increased
- Programming skills have increased significantly
- The ability to perform traditional analyses (derive and solve the special differential equations) in the mechanics has not decreased
- Decision making is brought forward in the sense that students consider real systems and structures and solve real problem (reasoning and decision making at a higher level),
- Active learning is emphasized in simulations, open-ended problems and in the virtual/interactive learning environments that are used,

EVALUATION AND RESULTS

- The main goal that each student should gain knowledge, skills and ability to effectively use computational mathematical modeling and simulations in applications has been reached to a large extent
- Employers claim that the mechanical engineering students have became significantly better prepared for the managing and solving of open-ended problem, carrying out numerical simulations, programming and using modern industrial software.
- Teachers of advanced level courses verify that the students' ability to solve large complex problems has improved and that the computational skills in general are much better.
  - "Mechanical engineering students in average are much better prepared for the courses and can handle computations and projects more efficiently and at higher level compared to engineering students from other disciplines"

CURRENT FOCUS

- Interactive (Virtual) Learning Environments T, L & A
- New textbook (e-book) for all math courses in the ME program including new programming and simulations assignments
- Preparatory web based course in programming
- Integrating concepts and techniques of modern optimization theory and practice with traditional design methods'
- New course in programming (Python) on intermediate level.

References

- Match course descriptions (learning outcomes, computer assignments, old exams etc) http://www.chalmers.se/sv/institutioner/math/utbildning/grundutbildning-chalmers/arkitekt-och-civilingenjor/maskinteknik/Sidor/default.aspx

- Integration of Computational Mathematics Education in the Mechanical Engineering Curriculum, *Proceedings of 7th International CDIO Conference, Copenhagen, Denmark,* 2011

- A computational mathematics education for students of mechanical engineering, *World Transactions on Engineering and Technology Education*, vol 5, 2006.

Questions?