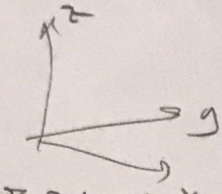
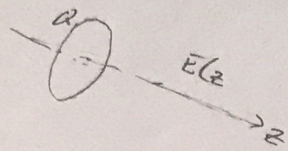
KOORDINATSYSTEMER

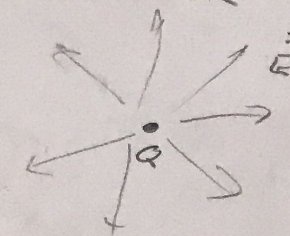
TFE4120-2018-I

8/1-18

KARTESISK | SYLINDRISK | SFÆRISK







INTEGRALER

Linje $\int_L \vec{A}\cdot d\vec{l}$ $\oint_C \vec{A}\cdot d\vec{l}$ | Flate $\int_S \vec{A}\cdot d\vec{S}$ $\oint_S \vec{A}\cdot d\vec{S}$ | Volum $\int_V \rho \, dv$

SKALARE FUNKSJONER

høyde: $h(x,y)$

temperatur: $T(x,y,z)$

$T(x,y,z,t)$

trykk: $P(x,y,z,t)$

Elektrisk potensial: $V(x,y,z)$

Ladningstetthet: $\rho(x,y,z)$

VEKTORFELT

elektrisk felt $\vec{E}(x,y,z \dots t)$

magnetisk felt $\vec{B}(x,y,z)$

strøm $\vec{J}(x,y,z)$

[vann, elektrisk] $\vec{I}(x,y,z)$

- Beskrive
- Integrere / derivere dem
- "Noen morsome sammenhenger mellom disse integralene og derivertene"

nyttige

GRADIENTEN

$$\nabla V = \left[\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right] \text{ Vektor :)}$$

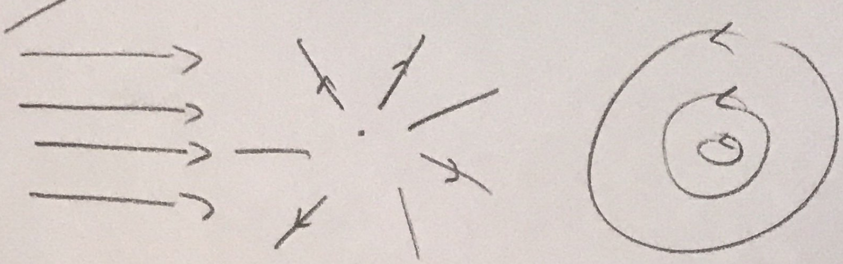
9/1-18

NABLAOPERATOREN: $\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$ VEKTOR

LAPLACEOPERATOREN: $\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ SKALAR

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

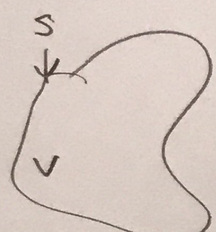
VEKTORFELT - Karakteristikker:



DIVERGENS:

$$\text{div}\,\vec{A} = \lim_{\Delta v \to 0} \frac{\oint \vec{A}\cdot d\vec{S}}{\Delta v} = \nabla\cdot\vec{A}$$ skalar

DIVERGENSTEOREMET:

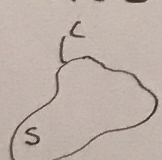


$$\oint_S \vec{A}\cdot d\vec{S} = \int_V \nabla\cdot\vec{A}\, dv$$

SIRKULASJON (CURL):

$$\text{curl}\,\vec{A} = \lim_{\Delta S \to 0} \frac{\oint \vec{A}\cdot d\vec{l}}{\Delta S} = \nabla\times\vec{A}$$ vektor

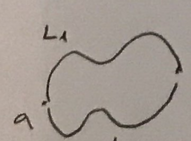
STOKES TEOREM:



$$\oint_C \vec{A}\cdot d\vec{l} = \int_S \nabla\times\vec{A}\cdot d\vec{S}$$

KONSERVATIVE FELT

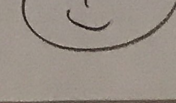
$$\vec{A} = -\nabla V$$



$$\int_{a,\,L_1}^{b} \vec{A}\cdot d\vec{l} = \int_{a,\,L_2}^{b} \vec{A}\cdot d\vec{l}$$

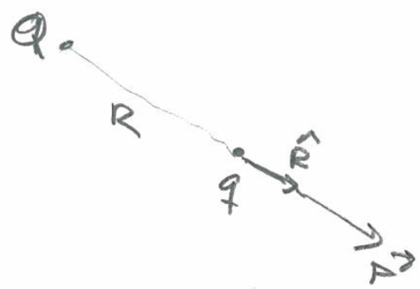
SISTE MORSOMME OPERATORRELASJON:

$$\nabla^2\vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla\times(\nabla\times\vec{A})$$



ELEKTROSTATIKK

Coulombs lov



$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{R^2} \hat{R}$$

ε_0 : Elektrisk permitivitet i vakuum

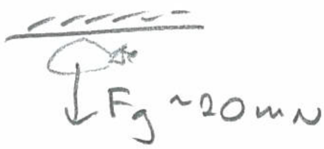
$$\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

Lading til et elektron q_e / proton : q_P

$$q_p = -q_e = 1{,}6 \cdot 10^{-19}\, C$$

15/1 2018

STØRRELSE



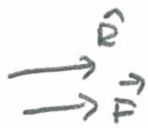
$R = 10^{-13}m \Rightarrow |F_{ep}| \sim 20mN$

RETNING

$$\hat{R} = \frac{\vec{R}}{R}$$

Like ladinger:

$\vec{F} \propto |Qq| \hat{R}$



Ulike ladinger

$\vec{F} \propto -|Qq| \hat{R}$

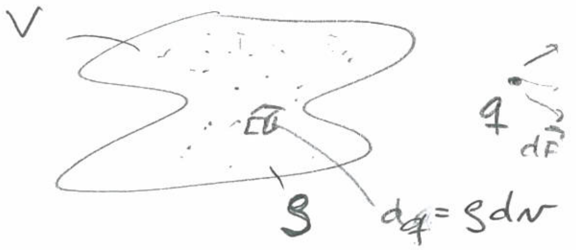
SUPERPOSISJON (Kraft fra FLERE punktladinger)

$Q_i \quad i = 1, n$

$$\vec{F} = \sum_i \vec{F}_i = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{Q_i}{R_i^2} \hat{R}_i$$

Regneeksempel 2.1 ⟷ typiske ladingsnh: batteri...

Coulombs lov for et utstrakt legeme



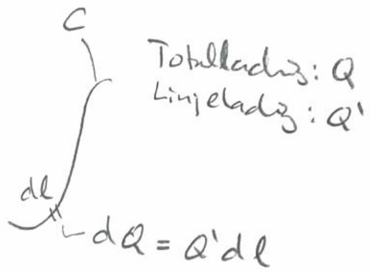
$$\vec{F} = \frac{q}{4\pi\varepsilon_0}\int_V \frac{\rho\, dv}{R^2}\hat{R}$$

ELEKTRISK FELT

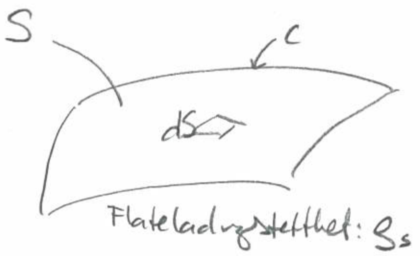
$$\boxed{\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}}$$

For punktladning Q: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{R}$

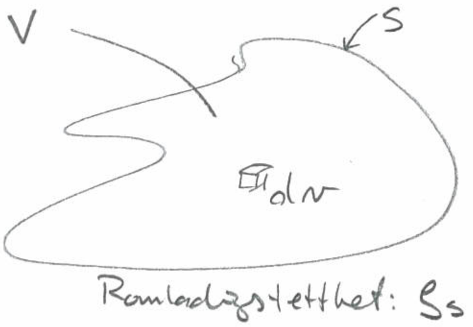
SUPERPOSISJON GJELDER OGSÅ FOR E-FELT



$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\int_C \frac{Q'\hat{R}}{R^2}\,dl$$



$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_s\hat{R}}{R^2}\,dS$$



$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\hat{R}}{R^2}\,dv$$

PRAKTISKE REGNEDVERSOR

→ Let etter symmetrier
(prøv å forstå feltet kvalitativt)

→ Velg et hensiktsmessig koordinatsystem

→ Sjekk dimensjoner og oppførsel f.eks i symmetripunkter / symmetriplan og/eller langt unna

16/1-2018

SKALARPOTENSIALET

$$V_A = \int_A^{ref} \vec{E} \cdot d\vec{l}$$

Enhet:

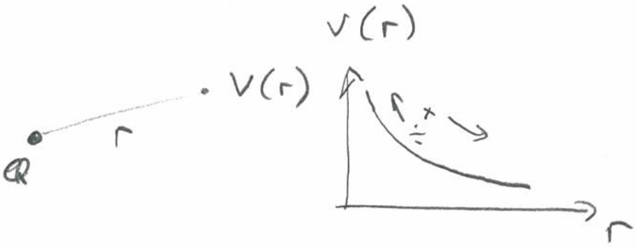
$$[V] = \frac{J}{C} \equiv V$$

$$\Rightarrow [\vec{E}] = V/m$$

⟹ Uavhengig av integrasjonsvei fra A til ref ☺

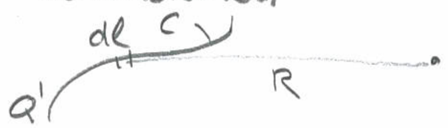
POTENSIAL FRA EN PUNKTLADNING, Q relativt til ∞

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$



POTENSIAL FRA EN....

- LINJELADNING



$$V = \frac{1}{4\pi\varepsilon_0} \int_C \frac{Q' dl}{R}$$

! ingen vektorer ☺

- FLATELADNING

$$V = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\rho_s ds}{R}$$

- ROMLADNING

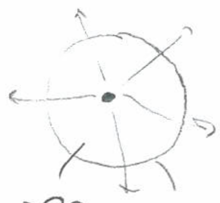
$$V = \frac{1}{4\pi\varepsilon_0} \int_v \frac{\rho\, dv}{R}$$

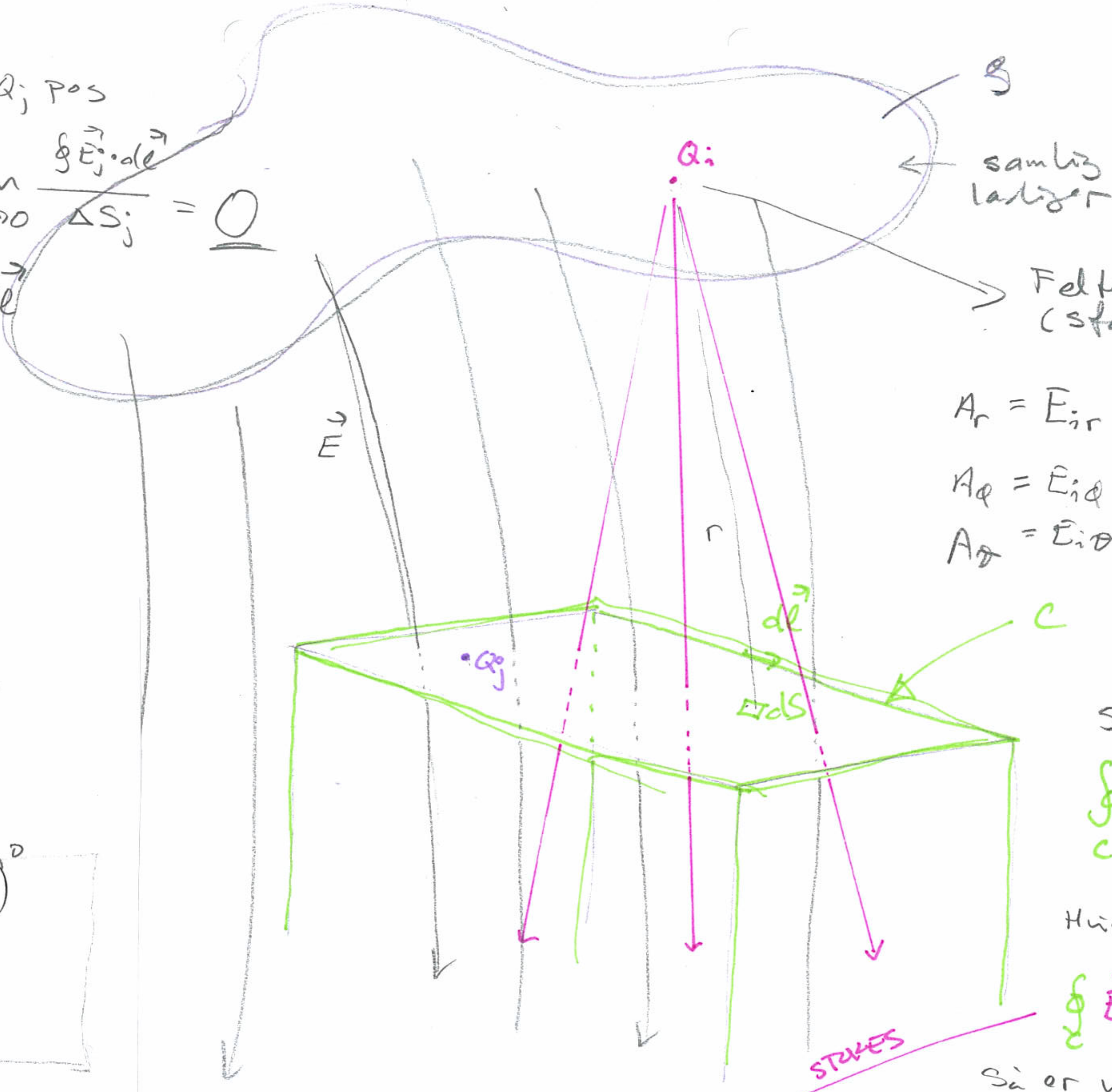
relativt til ∞

Curl til $\vec{E}_j$ i Q_j pos

$$(\nabla \times \vec{E}_j)_{kom} = \lim_{\Delta S_j \to 0} \frac{\oint \vec{E}_j \cdot d\vec{l}}{\Delta S_j} = \underline{0}$$

$\vec{E}_j \perp d\vec{l}$





Feltet rundt denne (sfæriske koordin.) Q_i i origo.

$A_r = E_{ir} = \frac{1}{4\pi\varepsilon_0}\frac{Q_i}{r^2}$

$A_\phi = E_{i\phi} = 0$

$A_\theta = E_{i\theta} = 0$

Sfærisk koordinatsystem:

$$\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\boxed{\begin{aligned}\nabla \times \mathbf{A} = {} & \frac{\hat{\mathbf{r}}}{r\sin\theta}\left(\frac{\partial(\sin\theta \cancel{A_\phi})}{\partial\theta}^{\to 0} - \frac{\partial \cancel{A_\theta}}{\partial\phi}^{\to 0}\right) \\ & + \frac{\hat{\boldsymbol{\theta}}}{r}\left(\frac{1}{\sin\theta}\frac{\partial \cancel{A_r}}{\partial\phi}^{\to 0} - \frac{\partial(r\cancel{A_\phi})}{\partial r}^{\to 0}\right) \\ & + \frac{\hat{\boldsymbol{\phi}}}{r}\left(\frac{\partial(r\cancel{A_\theta})}{\partial r}^{\to 0} - \frac{\partial \cancel{A_r}}{\partial\theta}^{\to 0}\right)\end{aligned}}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

Skal vise

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

Hvis vi viser

$$\oint_C \vec{E}_i \cdot d\vec{l} = 0$$

STOKES

$$\int_S \underbrace{\nabla \times \vec{E}_i}_{0}\, dS = ?$$

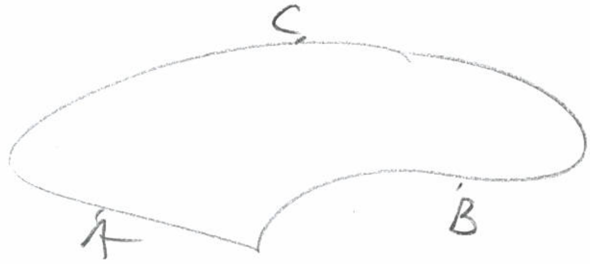
Så er vi i MÅL

$$\vec{E} = \sum \vec{E}_i$$

SPENNING (uavhengig av referanse for V_A, V_B)

$$V_{AB} = V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

$\Rightarrow$ KIRCHHOFFS SPENNINGSLOV



$V_{AB} + V_{BC} + V_{CA} = 0$

$$\boxed{\vec{E} = -\nabla V}$$

Nyttig sak

EKVIPOTENSIALFLATER:

$V = \text{konst}$

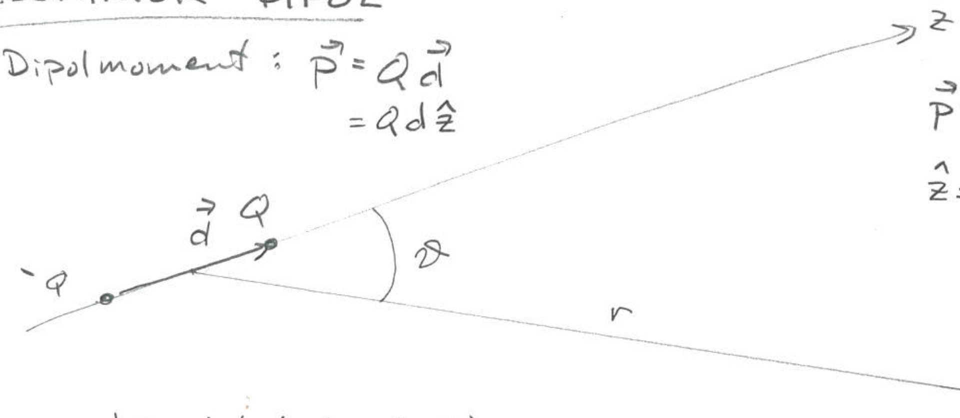
22/1 2018

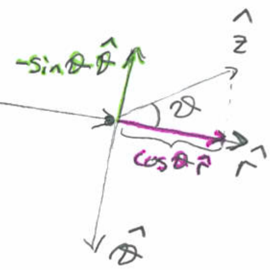
ELEKTRISK DIPOL

Dipolmoment: $\vec{p} = Q\vec{d}$
$= Qd\hat{z}$

$\vec{p} \cdot \hat{r} = Qd\cos\theta$

$\hat{z} = \underbrace{\cos\theta \cdot \hat{r}} \underbrace{- \sin\theta \cdot \hat{\theta}}$





I punktet (r, ϕ, θ):

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0} \frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}}$$

$$\boxed{\vec{E} = \frac{Qd\,(3\cos\theta \cdot \hat{r} - \hat{z})}{4\pi\varepsilon_0\, r^3} = \frac{1}{4\pi\varepsilon_0} \frac{3(\vec{p} \cdot \hat{r}) \cdot \hat{r} - \vec{p}}{r^3}}$$

enhetsvektorer: samme plan som arket