Chapter 2

GÖDEL: WHAT IS DECIDABLE?

- **PHILOSOPHICAL AND MATHEMATICAL LOGIC**
  - If all humans (B's) are mortal (A), and all Greeks (C's) are humans (B's), then all Greeks (C's) are mortal (A).
  - Leibniz: Logic and reasoning as a series of mechanical and symbolic tasks.

- **Deductive and Inductive Reasoning**
  - Deductive: Spesific conclution from a generalization.
  - Inductive: Broad generalization from specific observation.

- **Paradox**
  - Contradictions
  - Led to significant advances in science, philosophy and mathematics

- **Formal and Informal Logic**
  - Formal: Set of descrete symbols, syntax and semantics.
  - Hoare triple: {P} C {Q}
    - “Whenever P holds for the state before the execution of C, then Q will hold afterward.”
  - Informal: Natural language arguments.
    - Complicated
- **Mathematical Logic**
  - Apply logic to mathematics: reduce mathematics to tetologies.
    - Russell’s paradox: https://www.youtube.com/watch?v=GpVRePLMLbU
  - Logic programming: Set of axioms and rules.
    - Proofs by humans can be computer assisted if too lengthy for humans.
- **Decidability**
  - Kurt Gödel incompleteness theorem: In certain mathematical domains, there are ploblems that can not be solved or propositions that can not be proven.

- **Propositional Logic (or calculus)**

Symbols	Statement	Connectives
$p \vee q$	“either p is true, or q is true, or both”	Disjunction
$p \cdot q$	“both p and q are true”	Conjunction
$p \supset q$	“if p is true, then q is true”	Implication
$p \equiv q$	“p and q are either both true or both false”	Equivalence

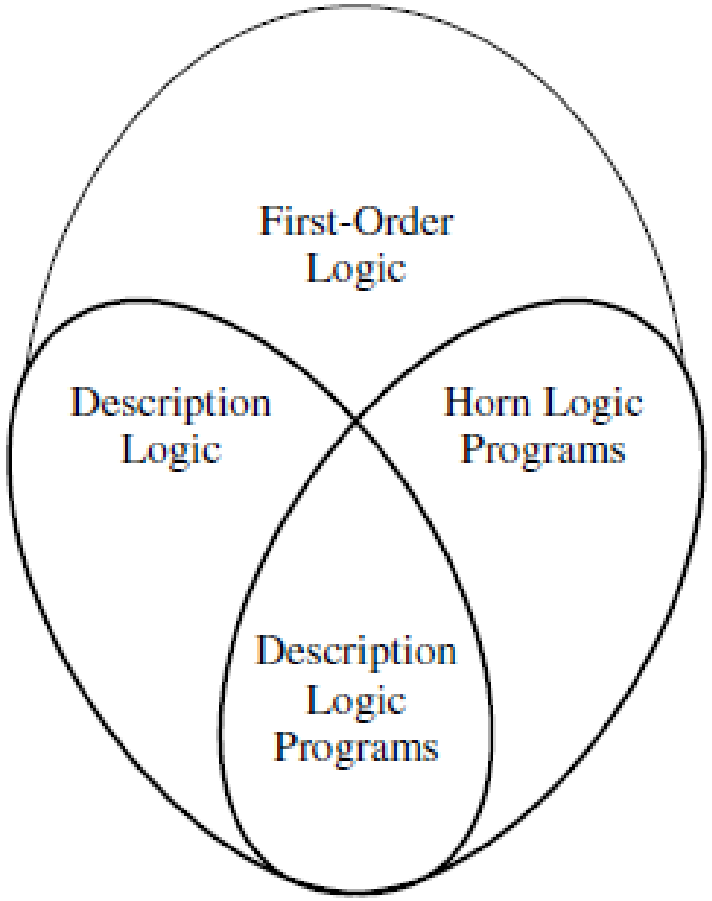
- **First-Order Logic**
  - “there exists an *x* such that . . .”
  - “for any *x*, it is the case that . . .”
  - “x” is values not properties
  - Can formulize all of set theory and most of mathematics.
  - Model Theory: connections between FO-properties and FO-structures
    - Restrictive thus questions cannot be discussed, but FOL have precise grammer

- **Automated Inference for FOL**
  - Gödel's Completeness Theorem says that FOL is only semidecidable.
  - If sentence is true given a set of axioms, there is a procedure that will determine this. But is it is false, there is no guarantee that a procedure will determine this.
    - Thus the Truth Table is not complete for FOL, as the size may be infinite.
  - Natural deduction is complete in FOL, but not practical.
    - "Branching factor"
  - Modus Ponens: Given the statements p and *if p then q,* infer q.
    - Generalized MP is not complete for FOL
    - GMP is complete for Knowledge Bases containing Horn clauses (chapter 8)
      - $(\forall x)(P1(x) \wedge P2(x) \wedge \ldots \wedge Pn(x)) \Rightarrow Q(x)$

Example from page 30

- KB = All cats like fish, cats eat everything they like, and Molly is a cat. In FOL then,
  - KB = $(\forall x)\ \text{cat}(x) \Rightarrow \text{likes}(x, \text{Fish})$ (1)
  - $(\forall x)(\forall y)\ (\text{cat}(x) \wedge \text{likes}(x, y)) \Rightarrow \text{eats}(x, y)$ (2)
  - $\text{cat}(\text{Molly})$ (3)
- Query: Does Molly eat fish?
- Proof:
  - Use GMP with (1) and (3) to derive: (4) likes(Molly, Fish)
  - Use GMP with (3), (4) and (2) to derive: eats(Molly, Fish)
- Conclusion: Yes, Molly eats fish.

- **Description Logic**
  - allow specifying a terminological hierarchy using arestricted set of first-order formulas.
- **Relating Logic Fragments ->**
- **Higher Order Logic**
  - Variables can appear in places where predicate or functions sympols appear.
  - N order cant takr (N-1) order predicates as arguments.



- **Recursion Theory**
  - The Fibonacci numbers
    - 0, 1, 1, 2, 3, 5, 8, 13, . . .

- **KNOWLEDGE REPRESENTATION**
  - “Data,” “information,” “knowledge,” and “understanding.”
  - Logic-based approaches
    - Complex relationships, well defined syntax, semantics and proof theory.
  - Rule-based systems
    - IF-THEN
  - Frames and semantic networks.
    - Network capturing related object, creating a class hierarchy.
    - Matching and property inheritence

- **COMPUTATIONAL LOGIC**
  - Logic + Control = Algorithms

- **ARTIFICIAL INTELLIGENCE**
  - Strong and weak AI

- **WEB ARCHITECTURE AND BUSINESS LOGIC**
  - Front-end is stupid HTML, simple asci text.
  - Business logic is deligated to backend scripts.
  - Calculations on server is efficient and fast, but must communicate through inefficient XML in ascii.

- **THE SEMANTIC WEB**
  - HTML and XML made for humans to read.
  - New markup schemas for automatic agents.

- **Inference Engines for the Semantic Web**
  - Process the knowledge available in the Semantic Web by deducing new knowledge from already specified knowledge.

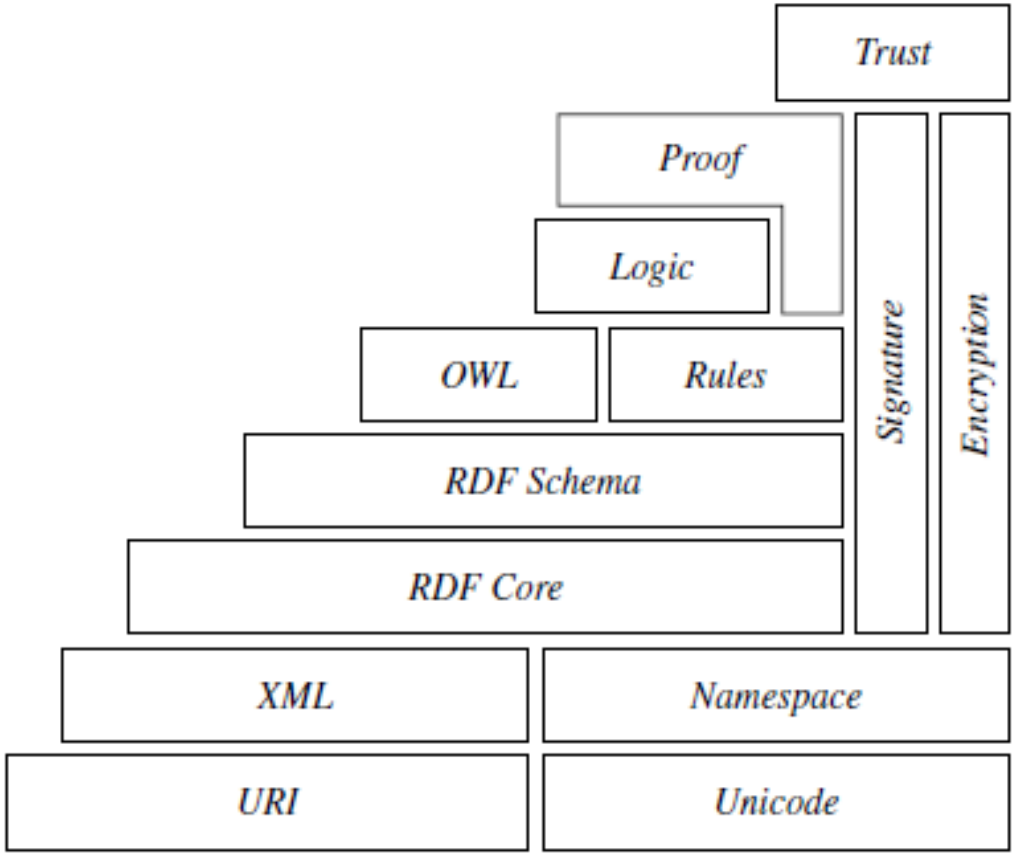


Figure 2-3. Markup language pyramid.