An Extended Cascading Failure Model for Loading Dependent Systems with Multi-state Components

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Title: An Extended Cascading Failure Model for Loading Dependent Systems with Multi-state Components

Motivation:

Cascading failures; overloading components; loading dependent systems

Work:

Extended multi-state CASCADE model; cascading process; probability distributions; stop scenarios; numerical examples

Significance:

Expected to provide a reference for reliability analysis of loading dependent systems whose operating efficiency or maintenance strategy is affected by overloading components

Key words: Multi-state CASCADE Model, loading dependent, multi-component system, cascading failure, Overloading.

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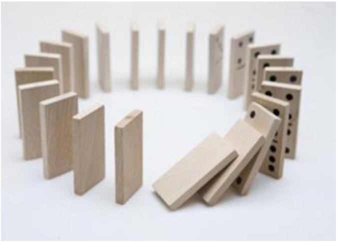
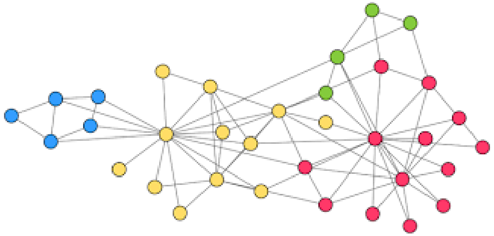
❑ Introduction

- **Research Motivation**



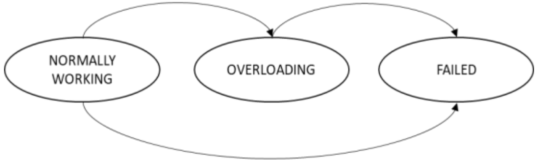
Cascading failures

- The chemical explosive accident in Mexico in 1984
- Blackout in American in 1996
- Blackout in Italy in 2003
- The Fukushima nuclear accident generated by a tsunami in 2011



Loading dependent system

- Wind farms
- Energy charging stations
- Piping networks
- Medical devices
- Road systems

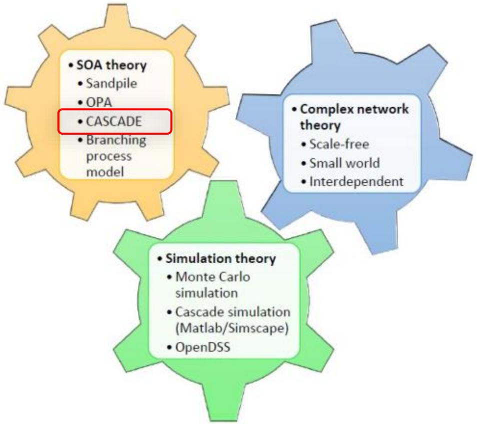


Overloading state

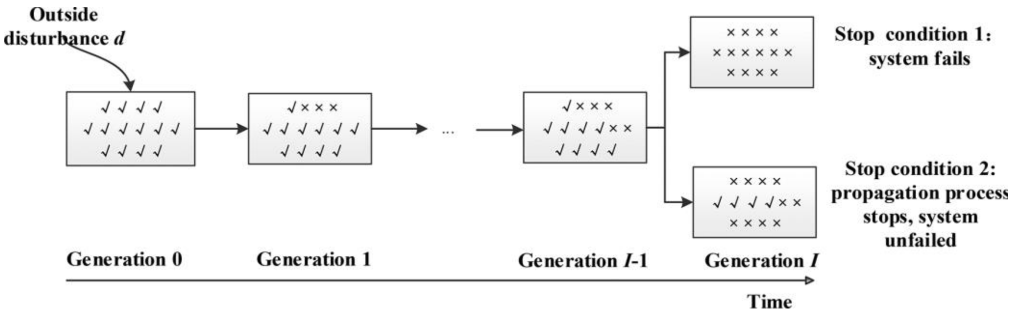
- Workload/Capacity
- Degradation

❑ Introduction

- **Previous contributions**



Representative models of cascading failures



CASCADE Model

The cascading process in a loading dependent system was first investigated by the CASCADE model.

CASCADE model focuses on instantaneous failure propagation but ignore the overloading phenomenon.

[1]L. D. Xing, "Cascading failures in Internet of Things review and perspectives on reliability and resilience," Ieee Internet Things, vol. 8, no. 1, pp. 44-64, Jan. 2021.
[2]H. Dong and L. R. Cui, "System reliability under cascading failure models," Ieee T Reliab, vol. 65, no. 2, pp. 929-940, Jun. 2016.

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- Problem description
- Quantitative analysis with multi-state CASCADE model
- Numerical examples
- Conclusion

Problem description

- **Overloading as a cascading mechanism**

TABLE DIFFERENCE AND SIMILARITIES BETWEEN DIRECT AND INDIRECT CAFS.

Category		Direct	Indirect
Difference	Driving force	Sudden shock and damage	Loading dependence
	Effects on components in sequence	Failures or degradation	Failures, degradation or overloading components
Similarities	Trigger	One failure or failures	
	Stop condition	There are no more new failures	

States including Normally Working, Overloading and Failed for each component.

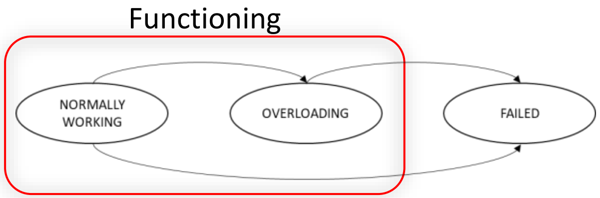
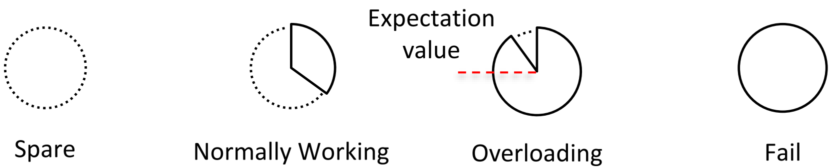


Fig. 1. State transition of components.



load-capacity ratio $r=l/c$

❑ Problem description

- **Model description and algorithm**

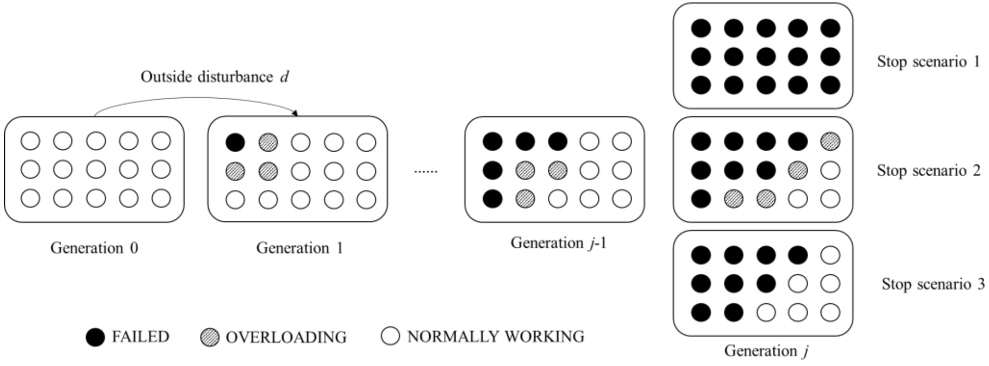
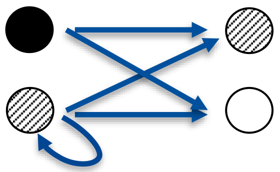


Fig. 2. Cascading process and stop scenarios of multi-state CASCADE Model.



This cascading process may stop when

a) all components fail (system fails); or
b) the performance level of the functioning component is less than the failure threshold (cascading process stops, system not failed).

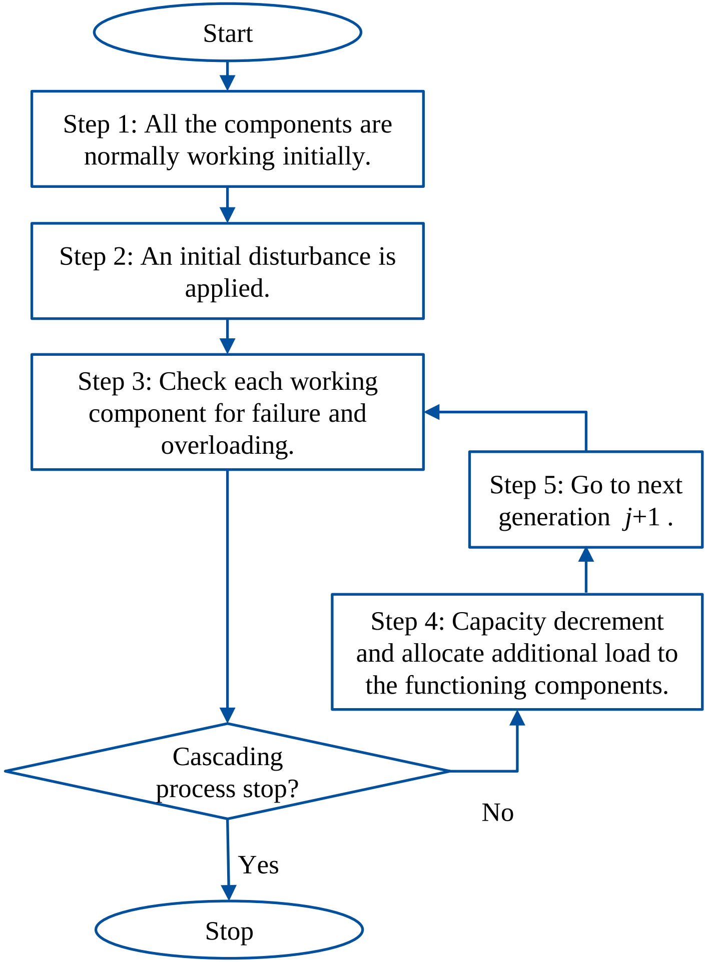
Assumptions:

1) The total number of components n in the system is finite.
2) All components in the system are identical, exchangeable and nonrepairable.
3) The capacity of every functioning component degrades naturally as the cascading failure propagate. The value of capacity decrement in every generation is c_d.

❑ System description and assumptions

• Model description and algorithm

1. All components are normally working initially with capacity $c_0 = 1$ and random loads l_i uniformly distributed in [0, 1].
2. An initial outside disturbance *d* to all components triggers the initial event followed by failure propagation. The initial failure is set as a trigger in generation 0 of a CAF.
3. Check states for each component. The performance level is represented by ratio of workload to capacity *l*/*c*. If the ratio of component *i* $r_i < r^*$, then component *i* is working well. When the ratio r_i of component *i* exceeds 1, the workload of the component will be more than its capacity could endure, so the component fails. Otherwise, the component is overloading. Suppose that there are n_{fj} components failed and n_{oj} components overloading in generation *j*. If $m_{fj} = 0$, the cascade model stops.
4. The additional load on the functioning components due to each failure in this generation in next generation is l_f. The additional load on the functioning components in next generation due to each overloading component in this generation is l_o. It is natural that l_o is considered smaller than l_f. Additional loads $n_{fj}l_f + n_{oj}l_o$ are allocated according to the number of failed and overloading components and added to each functioning component. The capacity of every component decreases due to degradation, so we have the capacity of the component in the *j*th generation $c_j = c_0 - j \cdot c_d$ and the ratio of the component $r_i = l_{ij}/c_j$.
5. Go to the next generation and iterate from step 2.



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Quantitative analysis with multi-state CASCADE model

• Distribution of the **total number** of components in different states

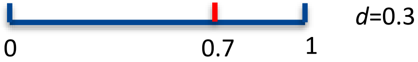
The state of the component follows a multinomial distribution $X \sim PN(\mathrm{N}: p_f, p_o, p_w)$, In each generation, the probability that there are n_{fj} components failed, n_{oj} components overloading and n_{wj} components normally working is

$$P[X_1 = n_f, X_2 = n_o, X_3 = n_w] = C_n^{n_f} C_{n-n_{fj}}^{n_o} p_f^{n_f} p_o^{n_o} p_w^{n_w} \tag{1}$$

where $p_f \geq 0, p_o \geq 0, p_w \geq 0, p_f + p_o + p_w = 1.$

After the initial disturbance d is applied, the load of each component is $l_i + d$ and the capacity of each component is c_0

- If the load/capacity ratio of a component exceeds 1, the component fails.



$$0 \leq l_i \leq 1 \text{ and } 1 < \frac{l_i + d}{c_0} \longrightarrow p_f = 1 - c_0 + d$$

- According to our definition, if the load/capacity ratio of a component lies in [r*, 1], this component is overloading.

$$0 \leq l_i \leq 1 \text{ and } r^* < \frac{l_i + d}{c_0} < 1 \longrightarrow p_o = c_0(1 - r^*)$$

- When the load/capacity ratio of a component is smaller than r^*, the component works normally.

$$0 \leq l_i \leq 1 \text{ and } \frac{l_i + d}{c_0} < r^* \longrightarrow p_w = c_0 r^* - d$$

The three probabilities are respectively $p_f = d, p_o = 1 - r^*, p_w = r^* - d$ in generation 0.

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Quantitative analysis with multi-state CASCADE model

- Distribution of the **total number** of components in different states

The total number of failed components and the total number of overloading components have different meanings in our case. Let $s_j = (n_{fj}, n_{oj}, n_{wj})$, for $j = 0,1, \dots$ and write

$$u_j = n_{f0} + n_{f1} + \cdots + n_{fj} \text{ and } v_j = n_{oj} \quad (2)$$

for $j = 0,1, \dots$.

An extended quasi-multinomial distribution is applied in

$$P[U = u, V = v] = \begin{cases} C_n^u C_{n-u}^v \varphi(d)\varphi(p_f)^{u-1} \varphi(p_o)^v \varphi(p_w)^{n-u-v}, & u = 0,1, \dots, n-1 \\ 1 - \sum_{u=0}^{u=n-1} P(U = u, V = v), & u = n \end{cases} \quad (3)$$

where $\varphi(x)$ is a saturation function representing the probability

$$p = \varphi(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases} \quad (4)$$

After cascading process going through some generations, we have (5) to calculate the distributions of the total number of components in different states.

$$P[U = u, V = v] = \begin{cases} C_n^u C_{n-u}^v \varphi(d)\varphi(1 - c_j + d + ul_f + vl_o)^{u-1} \varphi\left(c_j(1 - r^*)\right)^v \varphi\left(c_j r^* - (d + ul_f + vl_o)\right)^{n-u-v}, u = 0,1, \dots, n-1 \\ 1 - \sum_{u=0}^{u=n-1} P(U = u, V = v), u = n \end{cases} \quad (5)$$

❑ Quantitative analysis with multi-state CASCADE model

- Distribution for three **stop scenarios**

The probability that there are n_{fj} components failed, n_{oj} components overloading and n_{wj} components normally working in the jth generation is

$$P[X_{1j}=n_{fj},X_{2j}=n_{oj},X_{3j}=n_{wj}]=C_n^{n_{fj}}C_{n-n_{fj}}^{n_{oj}}p_f^{n_{fj}}p_o^{n_{oj}}p_w^{n_{wj}} \quad (6)$$

Let

$$\alpha_j=\varphi(p_{fj}), \qquad \beta_j=\varphi(p_{oj}), \qquad \gamma_j=\varphi(p_{wj})$$

In generation 0, $\alpha_1=0,\beta_1=0,\gamma_0=1$ for $j=0$.

The probability that there are n_{f0} failed components and n_{o0} overloading components is

$$P(S_0=s_0)=P[X_1=n_{f0},X_2=n_{o0},X_3=n_{w0}]=C_n^{n_{f0}}C_{n-n_{f0}}^{n_{o0}}\alpha_0^{m_{f0}}\beta_0^{m_{o0}}\gamma_0^{(n-n_{f0}-n_{o0})} \quad (7)$$

In first generation, for a loading dependent system considering decreasing capacity, the probability that the initial disturbance triggers one component failed or overloads is

$$\alpha_1=\varphi(1-c_0+d),\ \beta_1=\varphi(c_0(1-r^*)),\ \gamma_1=\varphi(c_0r^*-d)$$

and could be written as $\alpha_1=\varphi(d),\ \beta_1=\varphi(1-r^*),\ \gamma_1=\varphi(r^*-d)$ since $c_0=1$.

❑ Quantitative analysis with multi-state CASCADE model

- Distribution for three **stop scenarios**

In generation j+1, the additional loads from failed and overloading components could be assigned to the functioning components

$$l_j = n_{f(j-1)} l_f + n_{o(j-1)} l_o \quad (8)$$

For loading dependent system considering capacity decrement of the components, we have

$$\alpha_j = \varphi\left(\frac{1 - d - u_{(j-2)} l_f - v_{(j-2)} l_d - c_j + l_j}{1 - d - u_{(j-2)} l_f - v_{(j-2)} l_d}\right)$$

$$\beta_j = \varphi\left(\frac{c_j (1 - r^*)}{1 - d - u_{(j-2)} l_f - v_{(j-2)} l_d}\right)$$

$$\gamma_j = \varphi\left(\frac{c_j r^* - l_j}{1 - d - u_{(j-2)} l_f - v_{(j-2)} l_d}\right)$$

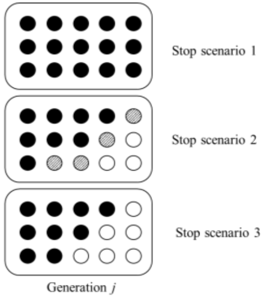
for $j = 2,3, \ldots$, and $u_{-1} = 0, v_{-1} = 0$.

For $u_j + v_j \le n$, the probability distribution for propagation in the jth generation

$$P[S_j = s_j, \ldots, S_0 = s_0] = \frac{n!}{n_{f0}! n_{o0}! n_{w0}!} \alpha_0^{n_{f0}} \beta_0^{n_{o0}} \gamma_0^{n_{w0}} \frac{(n-u_0)!}{n_{f1}! n_{o1}! n_{w1}!} \alpha_1^{n_{f1}} \beta_1^{n_{o1}} \gamma_1^{n_{w1}} \cdots \frac{(n-u_{(j-1)})!}{n_{fj}! n_{oj}! n_{wj}!} \alpha_j^{n_{fj}} \beta_j^{n_{oj}} \gamma_j^{n_{wj}} \quad (9)$$

❑ Quantitative analysis with multi-state CASCADE model

- Distribution for three **stop scenarios**



The probability distribution for propagation in the jth generation

$$P[S_j = s_j, \dots, S_0 = s_0] = \frac{n!}{m_{f0}!m_{o0}!m_{w0}!}\alpha_0^{m_{f0}}\beta_0^{m_{o0}}\gamma_0^{m_{w0}}\frac{(n-u_0)!}{m_{f1}!m_{o1}!m_{w1}!}\alpha_1^{m_{f1}}\beta_1^{m_{o1}}\gamma_1^{m_{w1}}\cdots\frac{(n-u_{(j-1)})!}{m_{fj}!m_{oj}!m_{wj}!}\alpha_j^{m_{fj}}\beta_j^{m_{oj}}\gamma_j^{m_{wj}} \quad (9)$$

Stop scenario 1: all components must fail in generation j. In this case

$$P[S_{j+1} = s_{j+1} | S_j = s_j, \dots, S_0 = s_0] = 1 \qquad \text{for} \quad n_{f(j+1)} = 0 \quad (10)$$

Stop scenarios 2 or 3: cascading process stops in generation j, and the loads of functioning components are uniformly distributed in $[d + u_{(j-1)}l_f + v_{(j-1)}l_d, c_j]$ conditioned on $n - u_j$ components not fail in generation j+1, then

$$P[S_{j+1} = s_{j+1} | S_j = s_j, \dots, S_0 = s_0] = C_{n-u_j}^{m_{o(j+1)}}\beta_{j+1}^{m_{o(j+1)}}\gamma_{j+1}^{m_{w(j+1)}} \quad (11)$$

Multiplying (9) and (11) we could obtain the distribution

$$P[S_{j+1} = s_{j+1}, \dots, S_0 = s_0] = \frac{n!}{m_{f0}!m_{o0}!m_{w0}!}\alpha_0^{m_{f0}}\beta_0^{m_{o0}}\gamma_0^{m_{w0}}\frac{(n-u_0)!}{m_{f1}!m_{o1}!m_{w1}!}\alpha_1^{m_{f1}}\beta_1^{m_{o1}}\gamma_1^{m_{w1}}\cdots\frac{(n-u_{(j-1)})!}{m_{fj}!m_{oj}!m_{wj}!}\alpha_j^{m_{fj}}\beta_j^{m_{oj}}\gamma_j^{m_{wj}} \cdot C_{n-u_j}^{m_{d(j+1)}}\beta_{j+1}^{m_{o(j+1)}}\gamma_{j+1}^{m_{w(j+1)}} \quad (12)$$

Contents

n = 100
r^* =0.8
l_f = 0.005, l_o = 0.001
d =0.001, 0.01, 0.05, 0.1

❑ Numerical examples

- Effect of initial disturbance

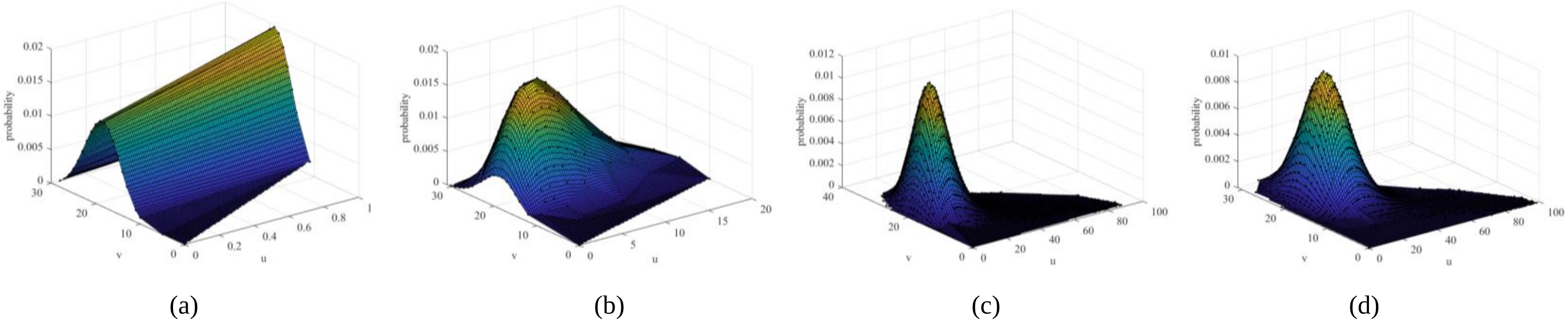


Fig. 3 Total number of failed and overloading components with different d. (a) d =0.001. (b) d =0.01. (c) d =0.05. (d) d =0.

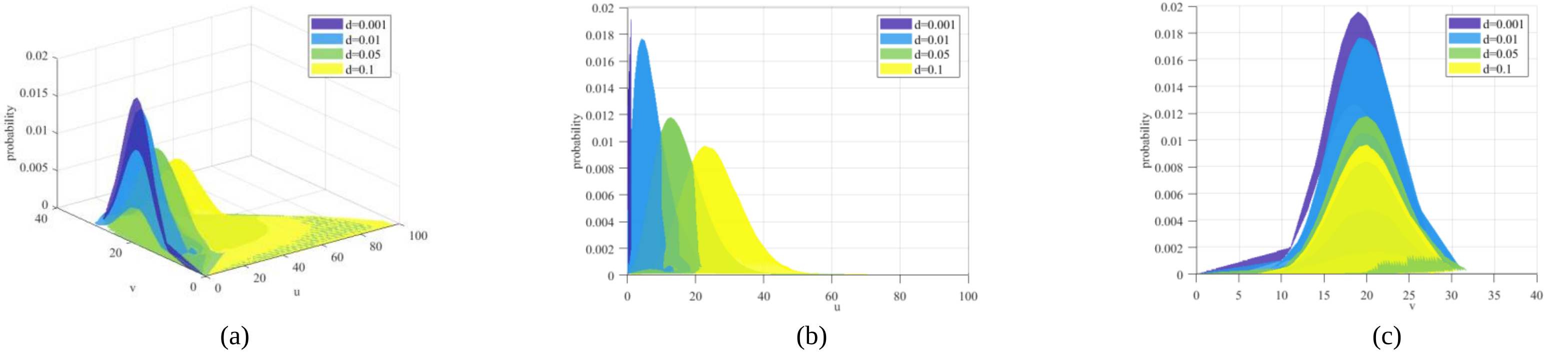
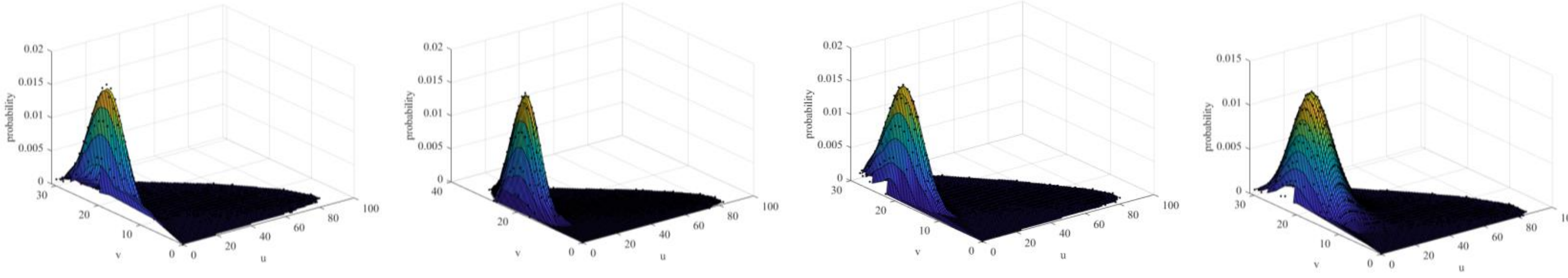


Fig. 4 Integration of probability distributions with different d. (a) Three-dimensional integration. (b) Integration of p-u. (c) Integration of p-v

n = 100
d =0.05
r^* =0.8
l_f = 0.0001, 0.0005, 0.001, 0.005 when l_o = 0.0001

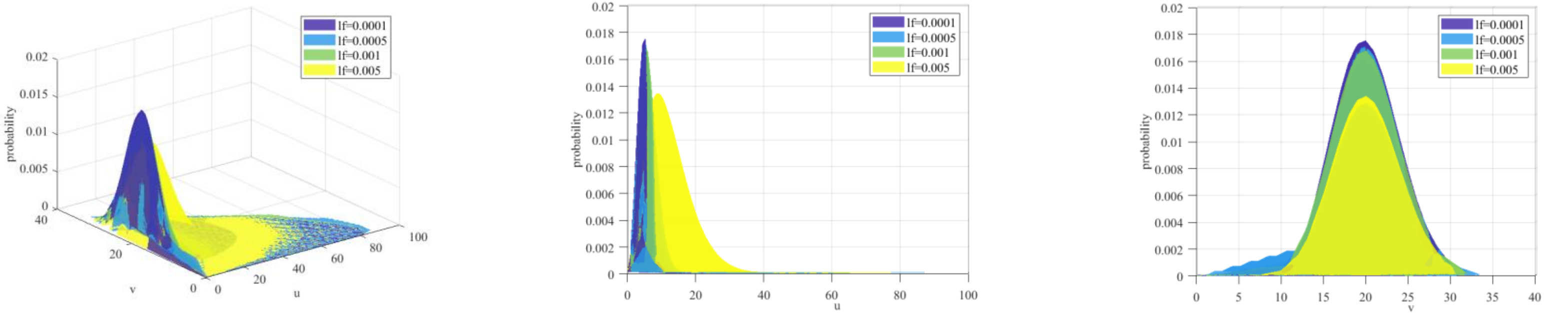
❑ Numerical examples

- Effect of loading increments from failed components



(a) (b) (c) (d)

Fig. 5 Total number of failed and overloading components with different l_f. (a) l_f =0.0001. (b) l_f =0.0005. (c) l_f =0.001. (d) l_f =0.005



(a) (b) (c)

Fig. 6 Integration of probability distributions with different l_f. (a) Three-dimensional integration. (b) Integration of *p*-*u*. (c) Integration of *p*-*v*

n = 100
d =0.05
r^* =0.8
l_o = 0.0001, 0.0005, 0.001, 0.005 when l_f = 0.005

Numerical examples

- Effect of loading increments from overloading components

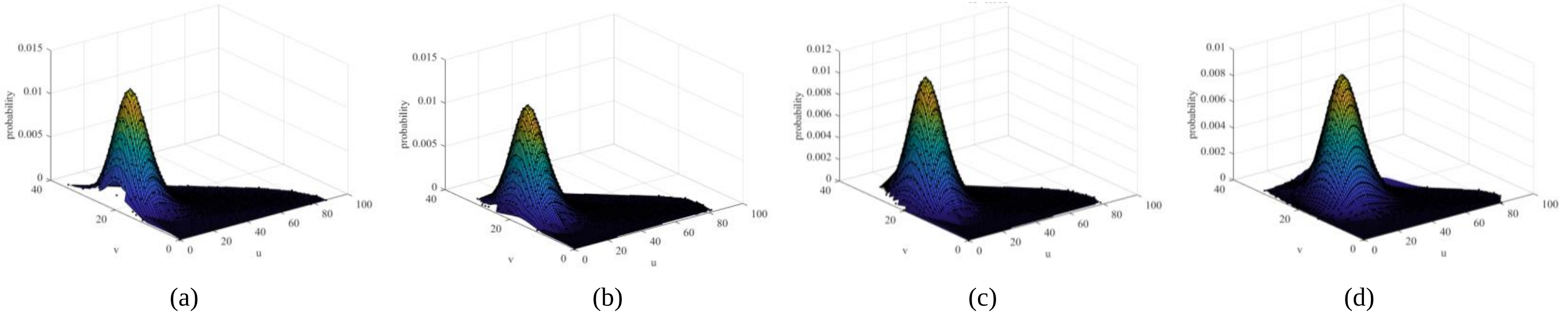


Fig. 7 Total number of failed and overloading components with different l_o. (a) l_o =0.0001. (b) l_o =0.0005. (c) l_o =0.001. (d) l_o =0.005

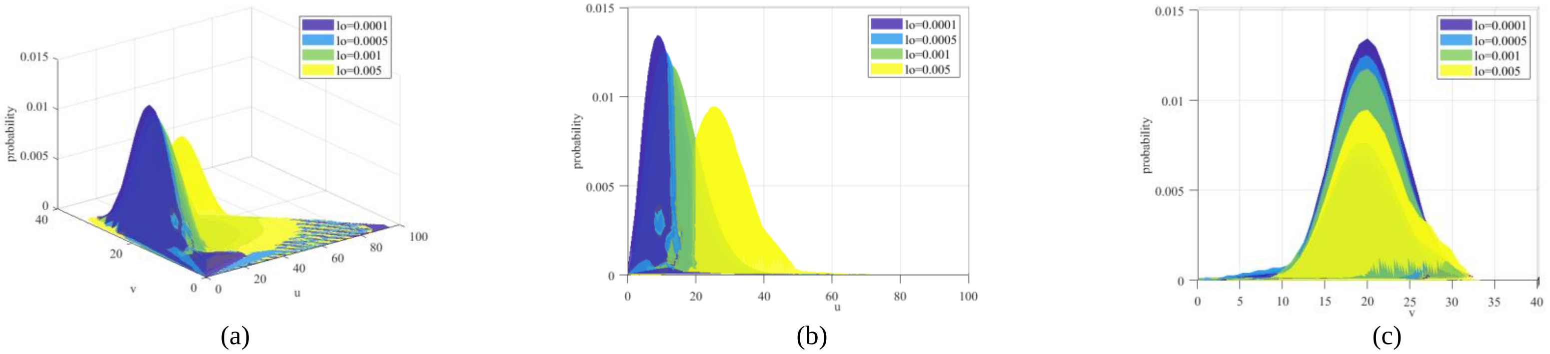
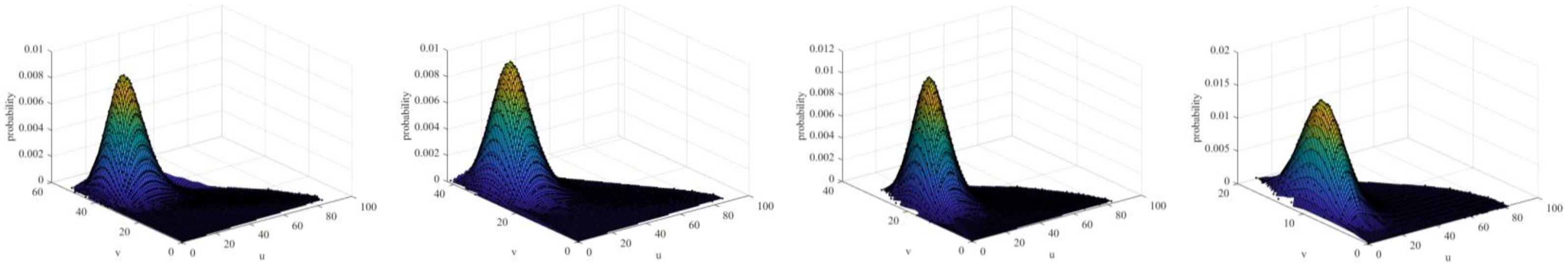


Fig. 8 Integration of probability distributions with different l_o (a) Three-dimensional integration. (b) Integration of p-u. (c) Integration of p-v

Numerical examples

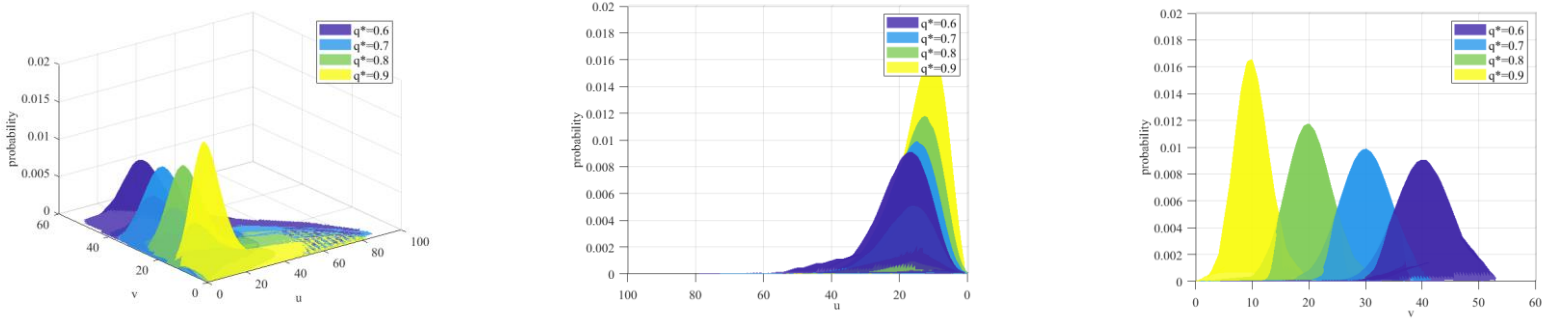
n = 100
d =0.05
l_f = 0.005, l_o = 0.001
r^* =0.6, 0.7, 0.8, 0.9

- Effect of overloading threshold



(a) (b) (c) (d)

Fig. 9 Total number of failed and overloading components with different q^*. (a) q^*=0.6. (b) q^* =0.7. (c) q^* =0.8. (d) q^* =0.9



(a) (b) (c)

Fig. 10 Integration of probability distributions with different q^* (a) Three-dimensional integration. (b) Integration of p-u. (c) Integration of p-v

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❑ Numerical examples

- Stop conditions and probability

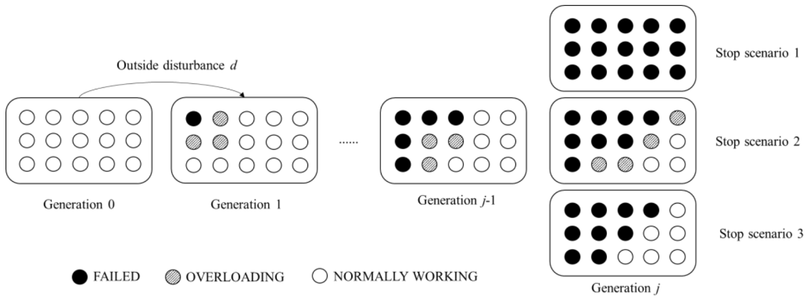

Fig. 2. Cascading process and stop scenarios of multi-state CASCADE Model.

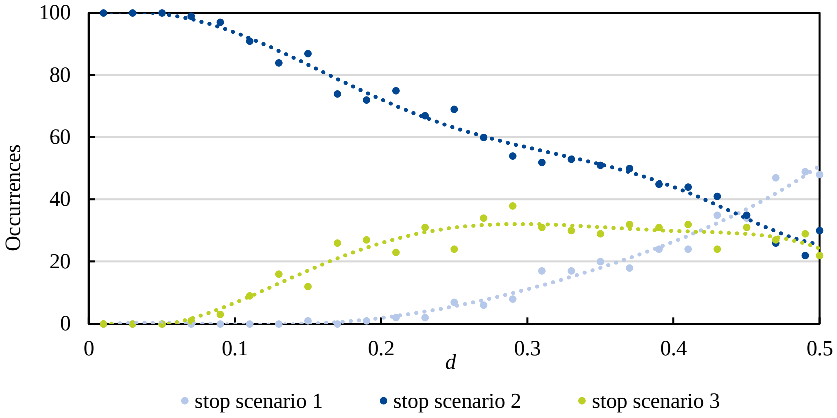

Fig. 11. Occurrences of three stop conditions

n = 100
d =0.05
q^* =0.8
l_f = 0.005, l_o = 0.001

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❑ Conclusions

- Developed a novel probabilistic model, multi-state CASCADE model, by extended quasi-multinomial distribution for loading dependent system with CAFs where overloading components affects the cascading process.

- Numerical examples are given to illustrate influencing factors on the probability distribution and occurrence possibilities of three stop scenarios.

- Some considerations also need to be explored. Since our proposed model is still limited in the multi-component system in simple configuration, further investigations on multi-state CASCADE Model for *k-out-of-n* system and engineering application are stimulated.

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Thanks for listening!

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