NTNU

Norwegian University of Science and Technology

NTNU

Statens vegvesen

Norwegian Public Roads Administration

A phase-type maintenance model considering condition-based inspections and delays before the repairs

By Tianqi Sun, Jørn Vatn, Yixin Zhao, Yiliu Liu

Norwegian University of Science and Technology (NTNU)

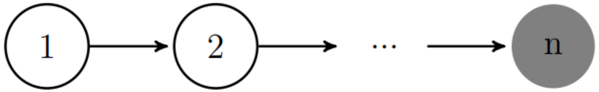
Agenda

1. Introduction
2. Model Description
3. Numerical results
4. Summary

1. Introduction

1.1 Background

- Markov Model
  - Widely used in maintenance modelling and system performance analysis
  - Computational efficient and analytically traceable
  - Limitation: exponentially distributed sojourn times, periodic/continuous inspections, immediate repairs



- Previous work
  - PH distribution for non-exponentially distributed sojourn times.
  - Condition-based inspections and deterministic delays before the repairs
    - Introducing extra matrices keep track of the probability mass for repair
    - Good estimation but complex
- This paper
  - Get rid of the matrices
  - Approximate the delays before repairs with PH distributions

1.2 Phase-Type (PH) Distribution

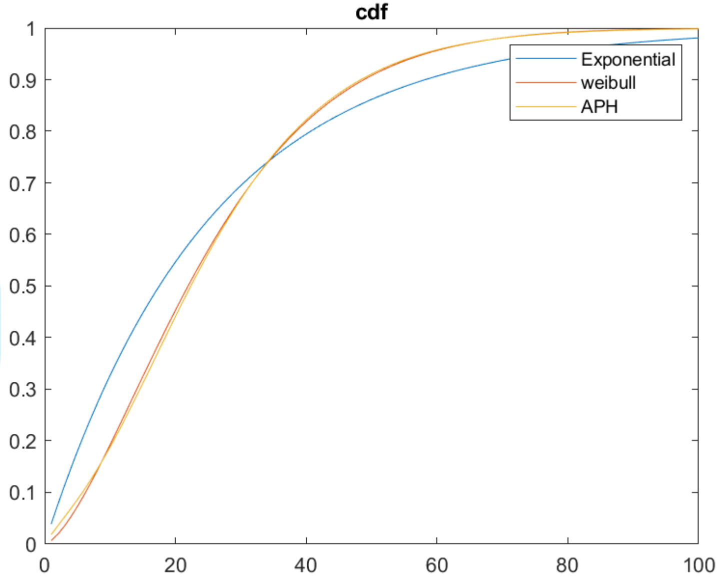
A phase-type (PH) distribution is Continuous Time Markov Chain (CTMC) of dimension m + 1, with m transient states and one absorbing state.

$$\mathbf{A} = \left(\begin{array}{c|c} \boldsymbol{S} & \boldsymbol{s} \\ \hline \mathbf{0} & 0 \end{array}\right)$$

- **Representation: $(\boldsymbol{\alpha}, \boldsymbol{S})$**
  - $\boldsymbol{\alpha}$: the initial probability vector among the transient states.
  - $\boldsymbol{S}$: the infinitesimal generator matrix among the transient states.

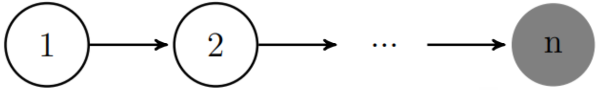
$$\boldsymbol{\alpha}_1 = [0.8384, 0, 0.1616], \boldsymbol{S}_1 = \begin{pmatrix} -0.0994 & 0.0994 & 0 \\ 0 & -0.0994 & 0.0994 \\ 0 & 0 & -0.1185 \end{pmatrix}$$

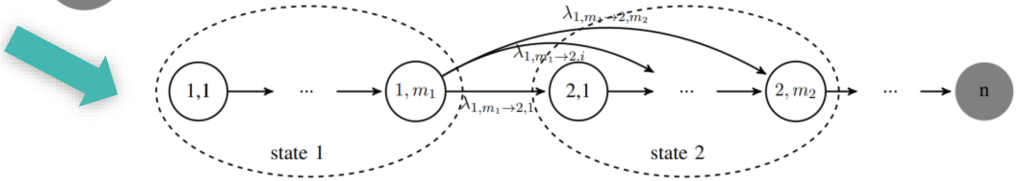
- **A Weibull distribution with shape parameter 1.5 and scale parameter 0.04**



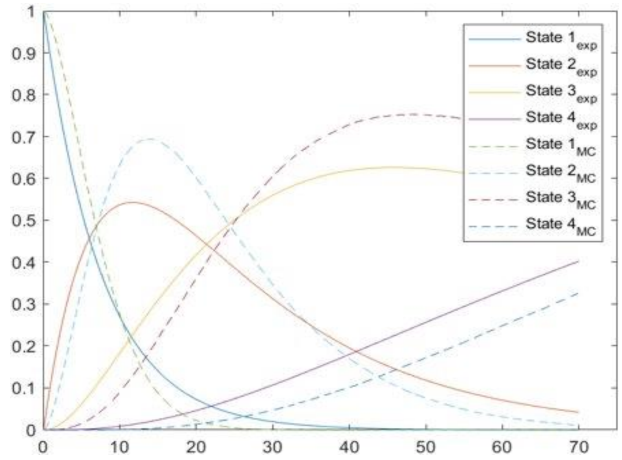
1.3 PH expansion of multi-state Markov models

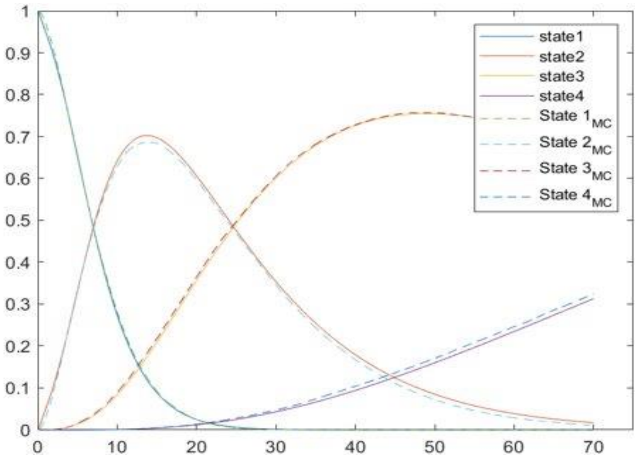
Extend a conventional Markov Chain to its PH expansion.





$$[\lambda_{j.m_j\to\{j+1\},1},\dots,\lambda_{j,m_j\to\{j+1\},m_{j+1}}] = \boldsymbol{\alpha}_{j+1}\cdot\lambda_{j\to j+1}$$

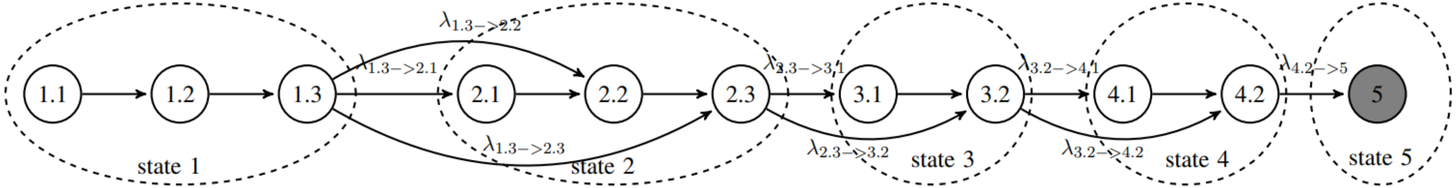




2. Model Description

2.1 Model Assumption

1. The bridge deterioration is described by five macro discrete condition states. The sojourn times for bridge deterioration are assumed Weibull-distributed.
2. The bridge condition can be revealed by condition-based inspections and after each repair.



$$\boldsymbol{A}_d = \begin{pmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_{1\to 2} & & & \\ & \boldsymbol{S}_2 & \boldsymbol{S}_{2\to 3} & & \\ & & \boldsymbol{S}_3 & \boldsymbol{S}_{3\to 4} & \\ & & & \boldsymbol{S}_4 & \boldsymbol{S}_{4\to 5} \\ & & & & 0 \end{pmatrix}$$

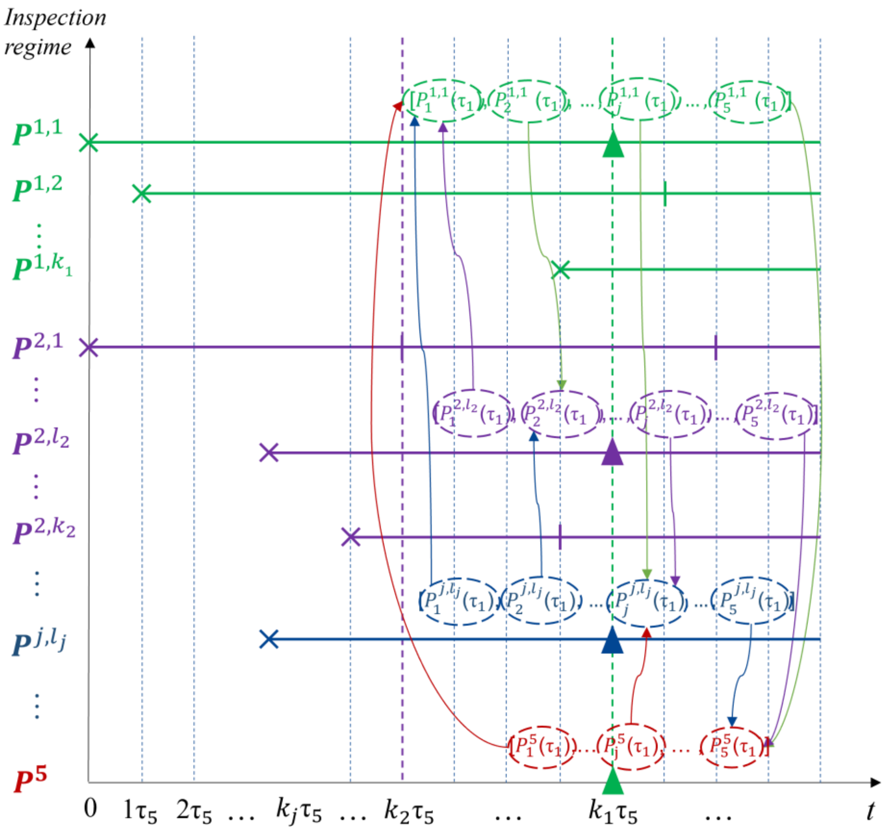
2.1 Model Assumption (Cont.)

3. The inspections are condition-based. The time for the next inspection is based on the bridge condition revealed at the current inspection.

   Let t_0 be the current inspection time, and j be the bridge condition revealed from an inspection; the time for the next inspection would be at $t_0 + \tau_j$.

   It is assumed that $\tau_j = k_j \tau_5$, where k_j are integers.

4. All inspections are perfect and can reveal the true system condition.



2.1 Model Assumption (Cont.)

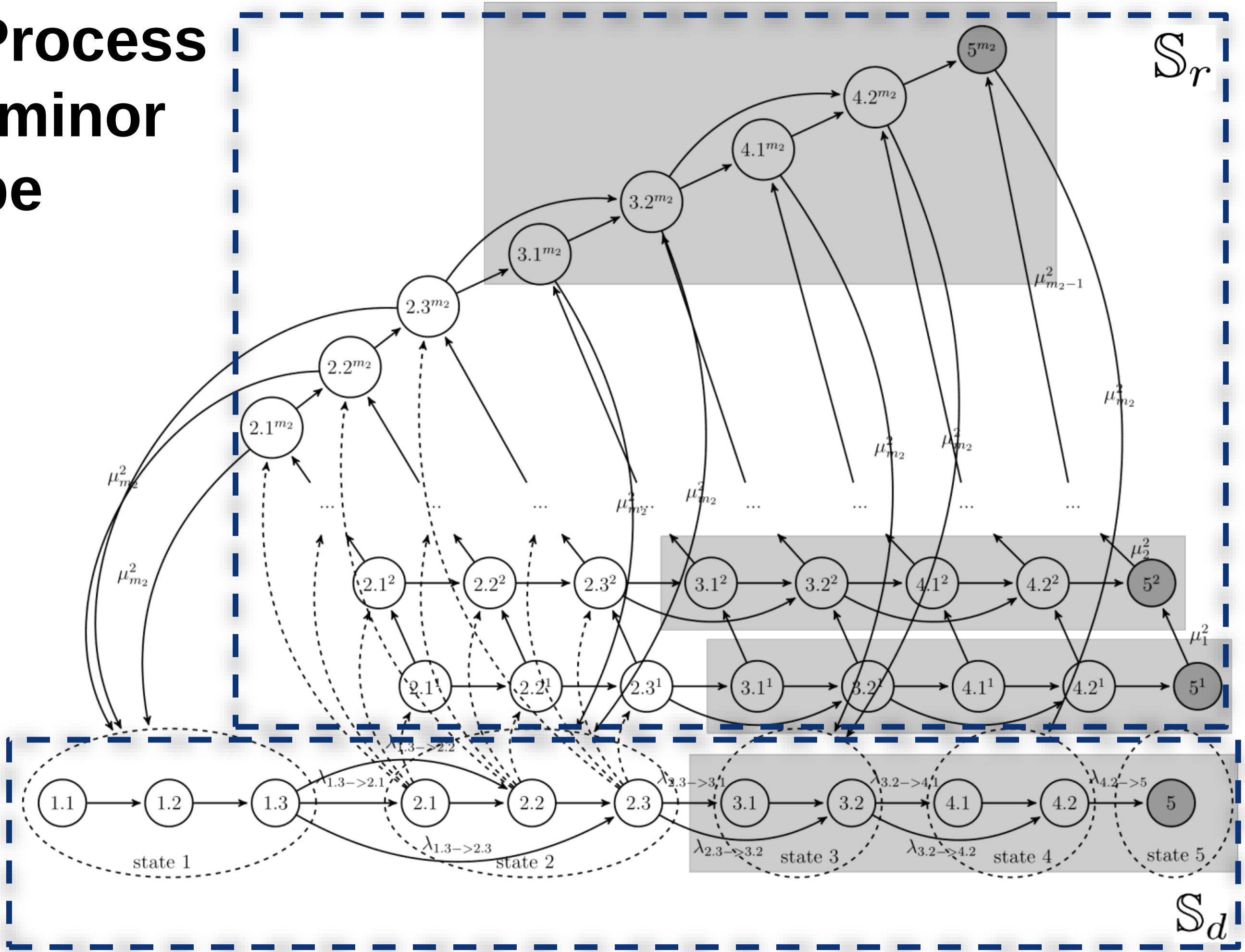
5. There are significant waiting times before the conduction of repairs. The waiting times depend on the bridge's condition revealed during an inspection and are assumed to be lognormal-distributed
6. There are three levels of repairs: minor repair (MiRep), major repair (MaRep) and rehabilitation (Rehab). The repairs will most restore the bridge to the desired state but may fail to achieve the planned bridge improvement in some cases.

Table 2. Different levels of repairs

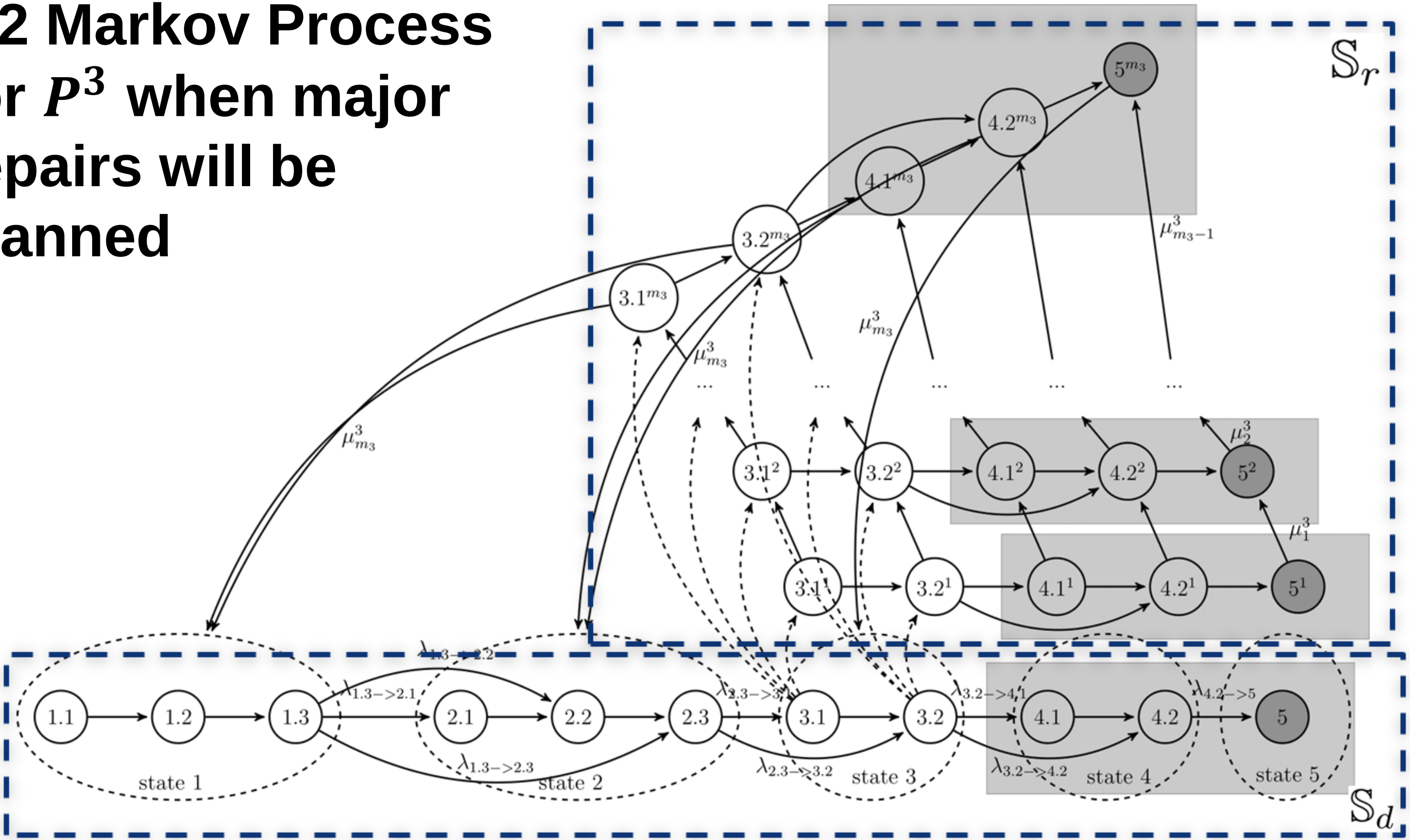
Level of repair	States improvement		Success probability
	Success	Failure	
MiRep	1	0	e_S
MaRep	2	1	e_M
Rehab	3	2	e_L

Note: State improvement = 1 means to improve the bridge condition by 1, e.g. from state 4 to state 3.

2.2 Markov Process for P^2 when minor repairs will be planned



2.2 Markov Process for P^3 when major repairs will be planned

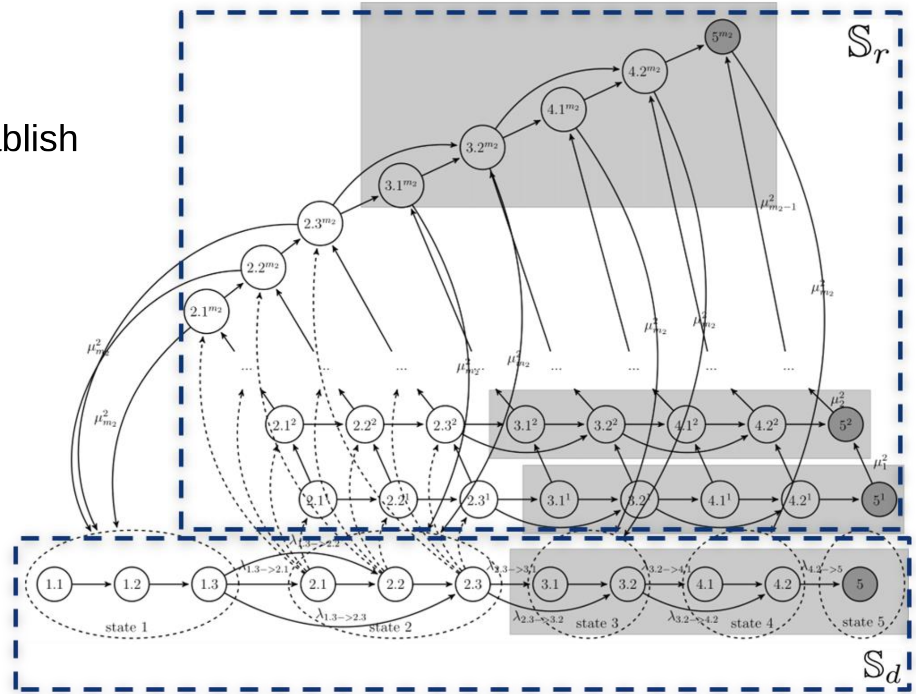


2.2 Transition Matrix

To make it consistent in the calculation, we establish transition matrices with the same size.

$$\boldsymbol{m}_r = \max(\boldsymbol{m}_2, \boldsymbol{m}_3, \boldsymbol{m}_4, \boldsymbol{m}_5).$$

$$\boldsymbol{A}^j = \begin{pmatrix} \boldsymbol{A}_d & & & & & & \\ & \boldsymbol{A}_{2-5} & \boldsymbol{A}^j_{r,1} & & & & \\ & & \boldsymbol{A}_{2-5} & \boldsymbol{A}^j_{r,2} & & & \\ & & & \ddots & & & \\ \boldsymbol{A}^j_{r,m_j} & & & & \boldsymbol{A}_{2-5} & & \\ & & & & & \ddots & \\ & & & & & & \boldsymbol{A}_{2-5} \end{pmatrix}$$



$\boldsymbol{A}_{2-5}$ - transition rates among macro state 2 to macro state 5 in $\mathbb{S}_d$.

$\boldsymbol{A}^j_{r,u} = \mathrm{diag}(\mu^j_u, \ldots, \mu^j_u)$ - Transition rates from the $\boldsymbol{u}_{th}$ phase to the $\boldsymbol{u}+\boldsymbol{1}_{th}$ phase of $\mathbb{S}_r$.

$\boldsymbol{A}^j_{r,m_j}$ - transition rates from the last phase of $\mathbb{S}_r$ to $\mathbb{S}_d$.

2.3 Modelling of condition-based inspections

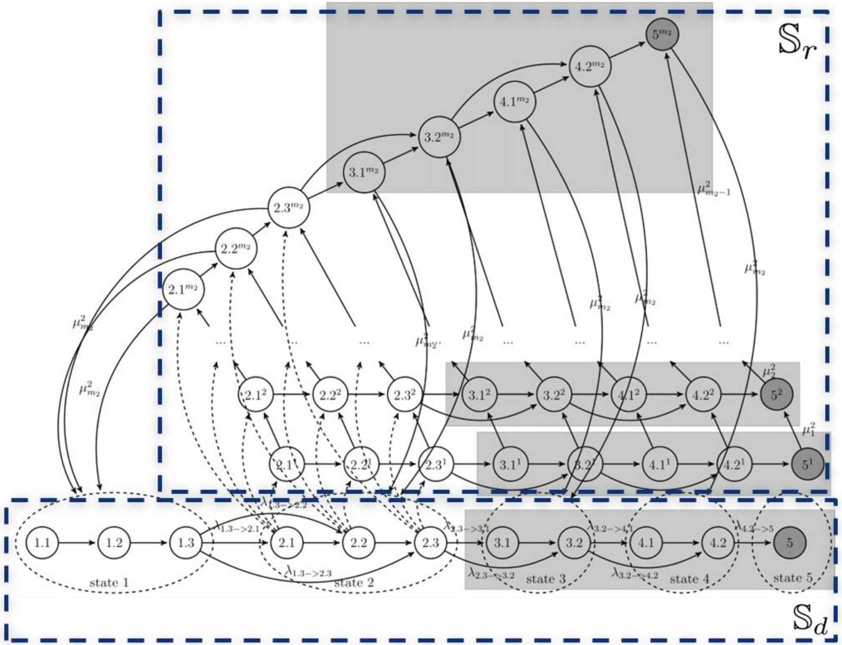
1. During the inspections, the probability mass in $\mathbb{S}_d$ is moved to $\mathbb{S}_r$, by inspection matrices $\boldsymbol{B}^j$.

$$\boldsymbol{B}^2 = \begin{pmatrix} \boldsymbol{I}_1 & & & & \\ & \boldsymbol{B}_1^2 \cdots \boldsymbol{B}_{m_2}^2 & \cdots & \boldsymbol{B}_{m_r}^2 \\ & & \boldsymbol{I}_{3-5} & \\ & & & \boldsymbol{I}_r \end{pmatrix}$$

$$\boldsymbol{B}_u^2 = \mathrm{diag}(\beta_u^2, \ldots, \beta_u^2)$$

2. The bridge can further deteriorate while waiting for repair. In this case, the bridge will follow the earlier time for repair between the original and the rescheduled one.

Updating the inspection matrices $\boldsymbol{B}^j$ by matching the expected waiting time.



The expected waiting time for the u_{th} phase of $\mathbb{S}_r$ in vector $\boldsymbol{P}^j$

$$\frac{1}{\mu_u^j} + \frac{1}{\mu_{u+1}^j} + \ldots + \frac{1}{\mu_{m_j}^j}$$

2.4 Maintenance Optimisation

Find the strategy with the minimum expected cost.

- **Cost Function**

$$E(C(T)) = \frac{\sum_{t=1}^{T} \sum_{\theta \in \Theta} C_\theta \cdot E(N_\theta(t))}{T} + d_f \cdot C_f$$

- **Expected number of actions**

$$E(N_{\text{Rehab}}) = \sum_{t=0}^{T} \sum_{\phi j > 2} \sum_{i \in \mathbb{S}_d} (P_i^{j,l_j}(t^+) - P_i^{j,l_j}(t^-))$$

$$E(N_{\text{MaRep}}) = \sum_{t=0}^{T} \sum_{\phi j = 2} \sum_{i \in \mathbb{S}_d} (P_i^{j,l_j}(t^+) - P_i^{j,l_j}(t^-))$$

$$E(N_{\text{MiRep}}) = \sum_{t=0}^{T} \sum_{\phi j = 1} \sum_{i \in \mathbb{S}_d} (P_i^{j,l_j}(t^+) - P_i^{j,l_j}(t^-))$$

$$E(N_{\text{Insp}}) = \sum_{t=0}^{T} \sum_{t=\tau_{\{j,l_j\}}} \sum_{i \in \mathbb{S}_d \cup \mathbb{S}_r} P_i^{j,l_j}(t)$$

3. Numerical Results

3.1 Input parameters

1. Weibull-distributed deterioration process
2. Lognormal-distributed waiting times before the repairs to match the current practice.

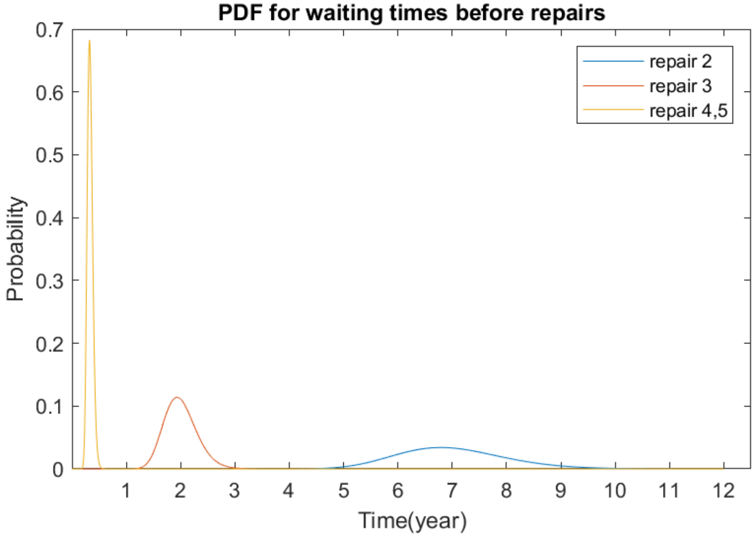
Condition Level	Degree of damage	Maintenance action
1	Small damage	No action
2	Medium damage	Within 4 -10 years
3	Major damage	Within 1 - 3 years
4	Critical damage	Within half year
5	Dangerous damage	Within half year

Table 4. Cost values

	C_{Insp}	C_{MiRep}	C_{MaRep}	C_{Rehab}
Cost (10^3NOK)	500	1,000	2,000	4,000

Table 3. Input parameters

State	Deterioration parameters		Repair parameters	
	Scale (yr)	Shape	Mean (yr)	Sigma
1	27.531	1.458	/	/
2	26.025	1.599	4.421	0.142
3	31.788	1.328	3.167	0.149
4	21.266	1.217	1.375	0.149
5	/	/	1.375	0.149



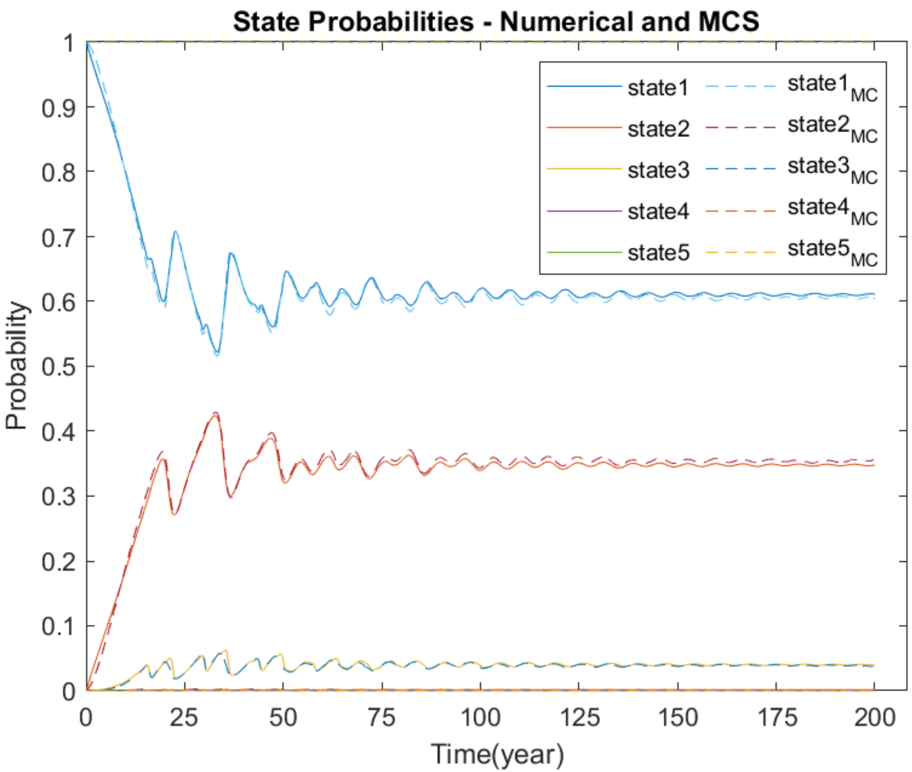
3.2 Verification With Monte Carlo Simulation

Consider the following strategy:

- Inspection intervals: $\tau_1 = 14$ years, $\tau_2 = 6$ years, $\tau_3 = 2$ years and $\tau_4 = 3$ months.
- All repairs intend to restore the bridge to state 1, with repair efficiency $e_L = 0.95$, $e_M = 0.9$ and $e_S = 0.85$.

Table 5. Expected number of actions and system performance

	Proposed Model	MCS
$E(N_{\text{Insp}})$	15.891	15.838
$E(N_{\text{MiRep}})$	3.727	3.689
$E(N_{\text{MaRep}})$	1.184	1.226
$E(N_{\text{Rehab}})$	0.081	0.086
d_f (yr)	0.00091	0.00084



3.3 Optimisation Result

- Genetic algorithm for the optimal solution
- Genetic algorithm toolbox in MATLAB
- Simulation time: 200 years
- Stopping condition: 30 stall generations
- $\tau_1 \in \{1, 2, \dots, 30\}$ years,

  $\tau_2 \in \{1, 2, \dots, 8\}$ years

  $\tau_3 \in \{1, 2, 3\}$ years

  $\tau_4 \in \{1, 2, \dots, 6\}$ months

Table 6. Optimisation results considering different C_f

C_f (10^3 NOK/yr)	20,000	50,000
Inspection Intervals	$\tau_1 = 12$ years, $\tau_2 = 7$ years, $\tau_3 = 2$ years, $\tau_4 = 6$ months	$\tau_1 = 10$ years, $\tau_2 = 5$ years, $\tau_3 = 2$ years, $\tau_4 = 6$ months
Repair Strategy	μ_2 = MiRep, μ_3 = MaRep, $\mu_4 = \mu_5$ = Rehab	
$E(N_{\text{Insp}})$	16.116	21.48
$E(N_{\text{MiRep}})$	4.269	4.261
$E(N_{\text{MaRep}})$	1.032	1.066
$E(N_{\text{Rehab}})$	0.004	0.004
d_f (yr)	6.193×10^{-4}	2.252×10^{-4}

4. Summary

4 Summary

- This paper proposed a PH model considering non-Markovian deterioration process, condition-based inspections and repair delays.
- In contrast to the deterministic delay times in our previous work, the delays in this paper is assumed lognormal distributed and modelled with PH distributions.
- An illustration case of road bridges is presented to demonstrate the modelling approach and its potential use in maintenance optimisation.

Thanks for your attention

Contact information

Tianqi Sun: sun.tianqi@ntnu.no