Criticality in Dynamic Arrest: Correspondence between Glasses and Traffic

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Statistical Mechanics

micro particles $\Rightarrow$ macro properties

Statistical mechanics: many particles, equilibrium: $\exp(-E/kT)$

- out of equilibrium or few particles: no powerful formalism

The most important and interesting things are moving

- transport of matter, energy, momentum (diffusion, friction, viscosity, heat conductivity)
- fluctuations (small-ish systems)
- $\Rightarrow$ not in equilibrium
- stuck with ad-hoc approaches

transport is everywhere

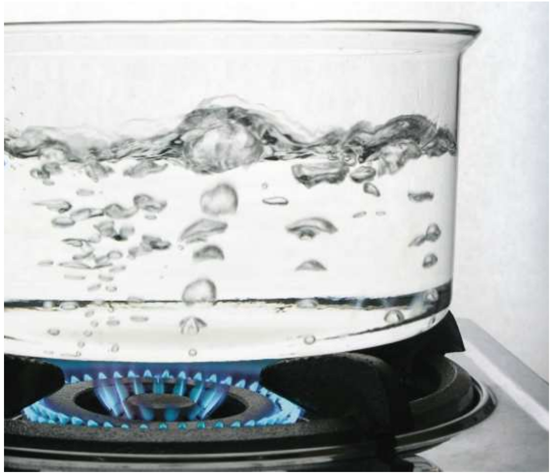
Applied to:

- material science
- surface science, especially tribology
- (soft) condensed matter
- ...
- biology (cells)
- traffic jams (cars)
- ...

overview

- introduction: phase transitions
- introduction: dynamic arrest
- kinetically constrained models for glasses
- traffic $\leftrightarrow$ glasses
  - traffic jams and nervous drivers $\leftrightarrow$ low temperature
  - use tools for glasses
  - dynamic criticality in traffic
- conclusions

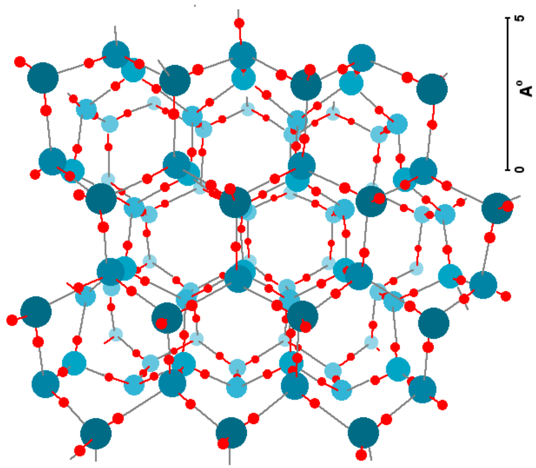
thermodynamics: phase transitions



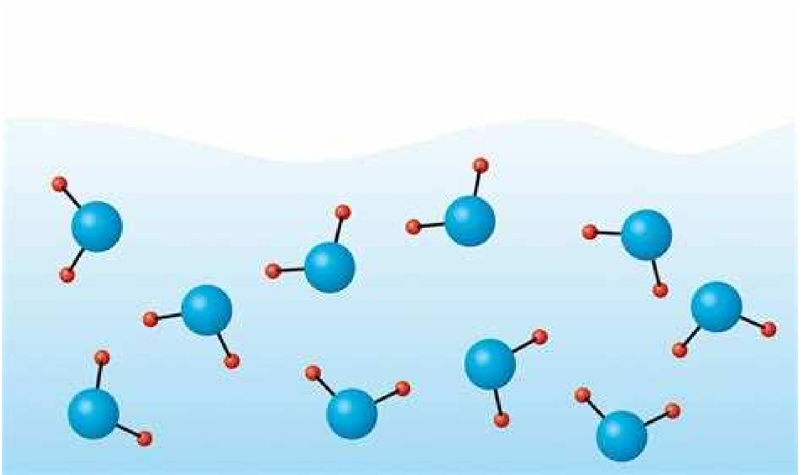
- solid ice (below 0°C)
- liquid water (between 0°C and 100°C)
- gaseous water vapour (above 100°C) [ambient pressure]
- small change in temperature: equilibrium completely different

order

solid



liquid

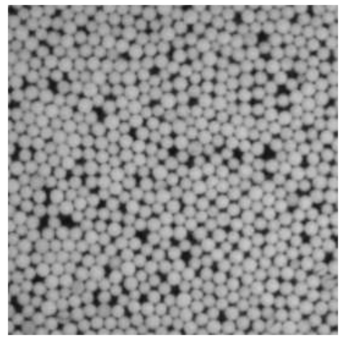


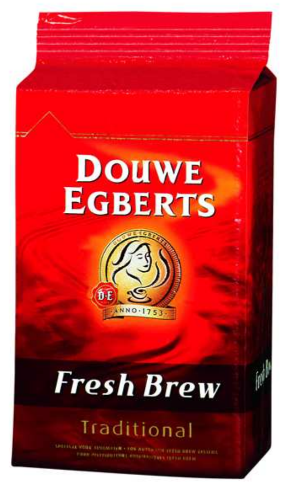
- solid ice: ordered lattice
- liquid water: high density, disorder
- gas: low density, disorder
- change in ordering or structure: order parameters
- correlation
- criticality: diverging correlation

dynamic arrest and jamming

jamming transition:
increase in density $\Rightarrow$ particles stop moving

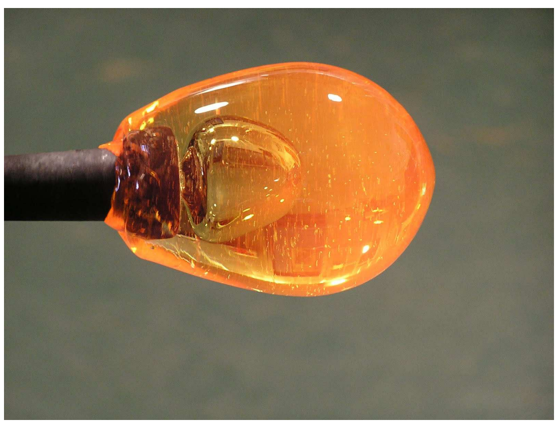
- granular matter:
  sand, coffee grounds
- cars in traffic
- etc...





dynamic arrest in glasses

- viscosity of supercooled liquid diverges with decrease in temperature: behaves like solid
- amorphous, no long-range order, looks like liquid
- criticality at $T \to 0$



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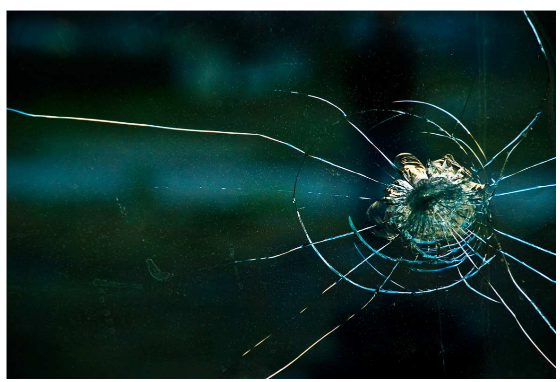
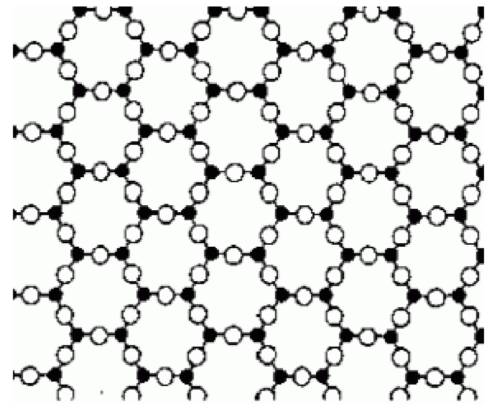
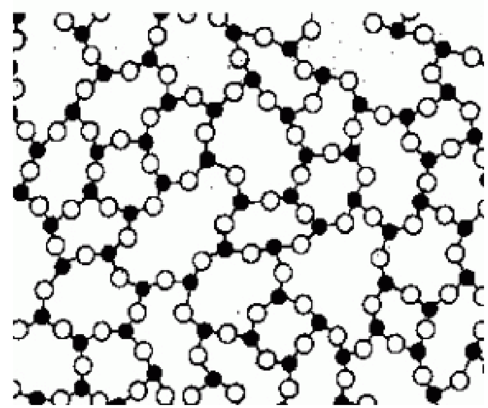
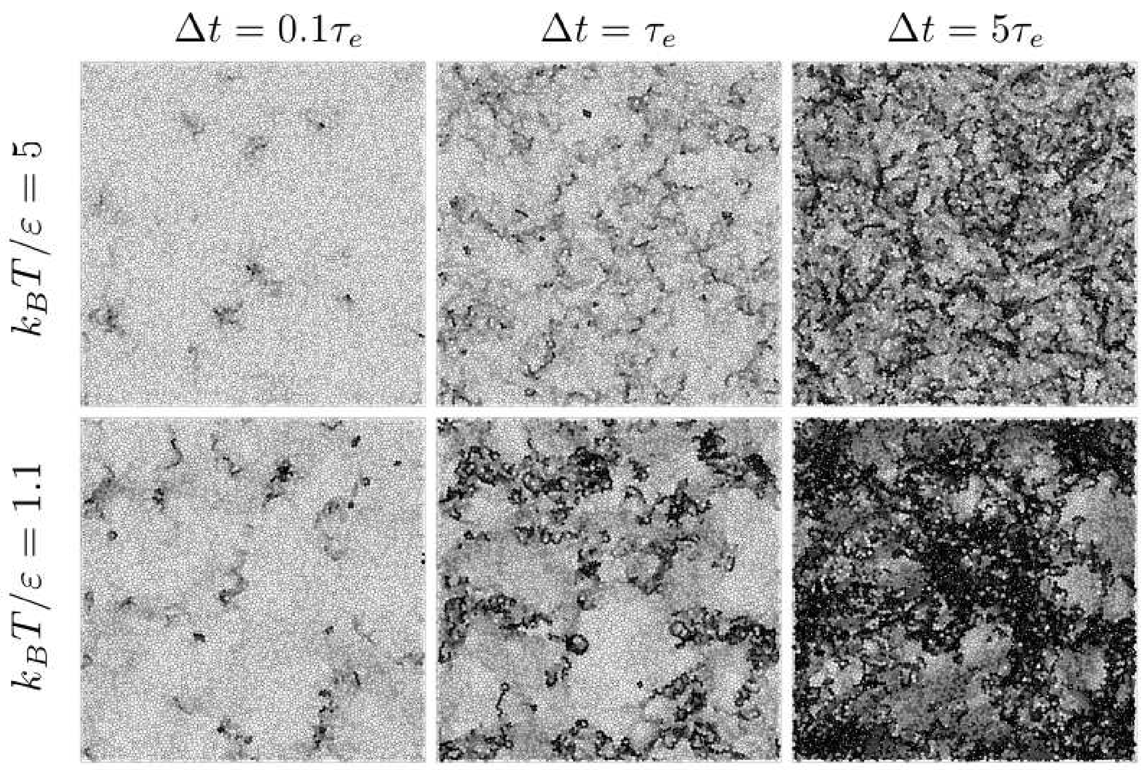






dynamic heterogeneity in glasses

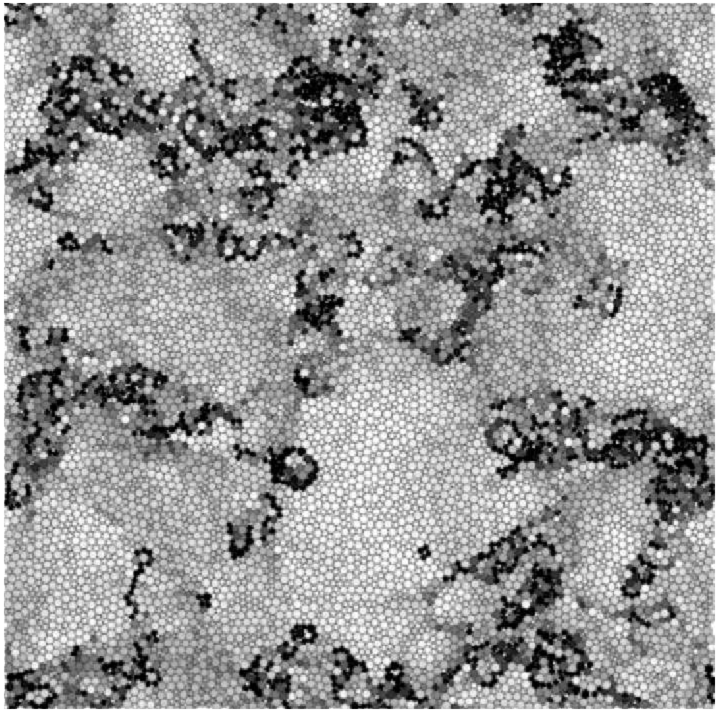


picture from Chandler and Garrahan, Annu. Rev. Phys. Chem. 61, 191 (2010).

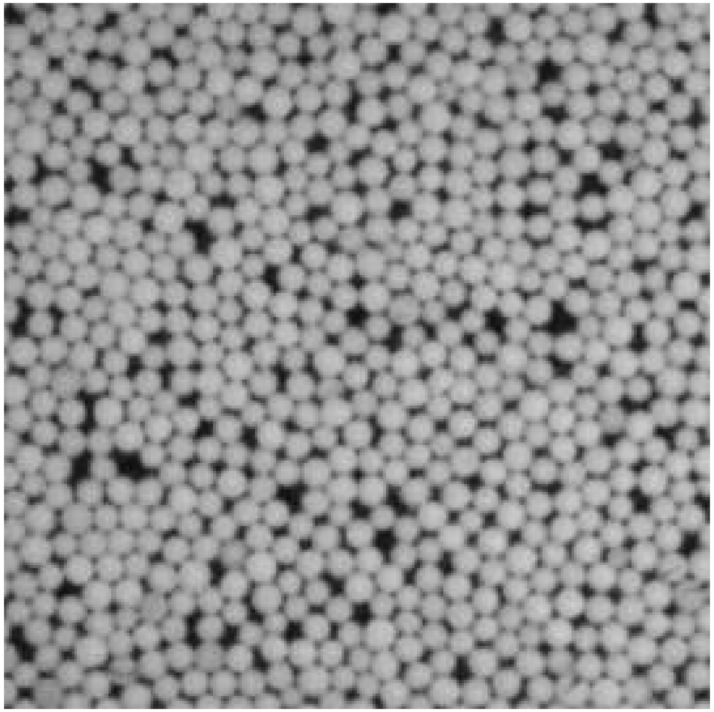
as T decreases

- relaxation slows down (energy barrier, rearrangement of atoms)
- mobility becomes heterogeneous (available space)

dynamic arrest



slowing down in glasses



jamming in granular matter

there is no general description for dynamic arrest

- different dynamics
- out of equilibrium

dynamic phase transition?

- analogous to equilibrium phase transition
- dynamic order parameter
- similar signatures, f.e. diverging correlations

- dynamical critical point in traffic flow
- $\Leftrightarrow$ critical slowing down in glasses

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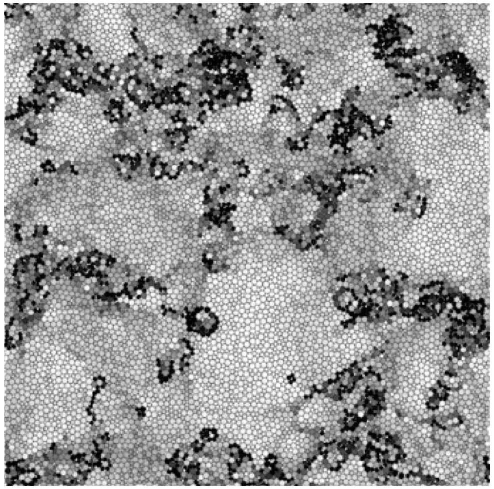
glasses: Kinetically Constrained Models (KCMs)



- successful at describing glasses
- discrete-time lattice models (amorphousness not essential for glassy behaviour!)
- stochastic
- criticality at $T \to 0$ (as supposed to be)
- kinetic constraint: activity only if local constraint is met
- f.e. spin-facilitated Ising models
  - $\uparrow$ active, low-density region
  - $\downarrow$ inactive, high-density region
  - $E = -J \sum_{\rm nn} S_i S_j + \sum S_i$
  - flip only possible if enough space, i.e. enough neighbours $\uparrow$

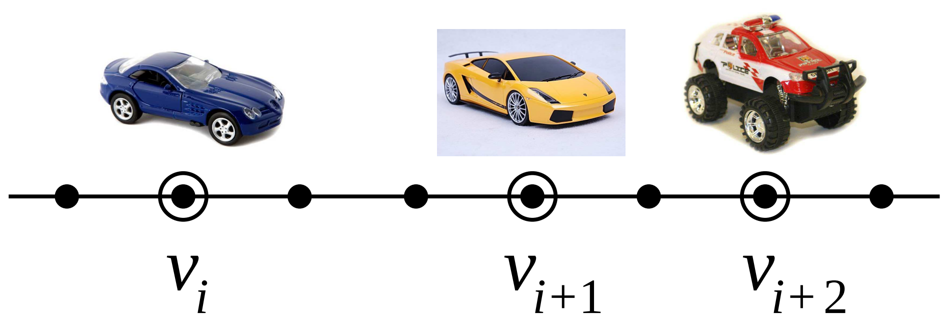
traffic jams

- annoying to be stuck in
- a lot of time and fuel wasted, pollution
- bad weather makes it worse
- low activity (speed), because car in front inactive
- high activity (speed) only possible if car in front active
- $\Rightarrow$ kinetic constraint

Nagel-Schreckenberg model

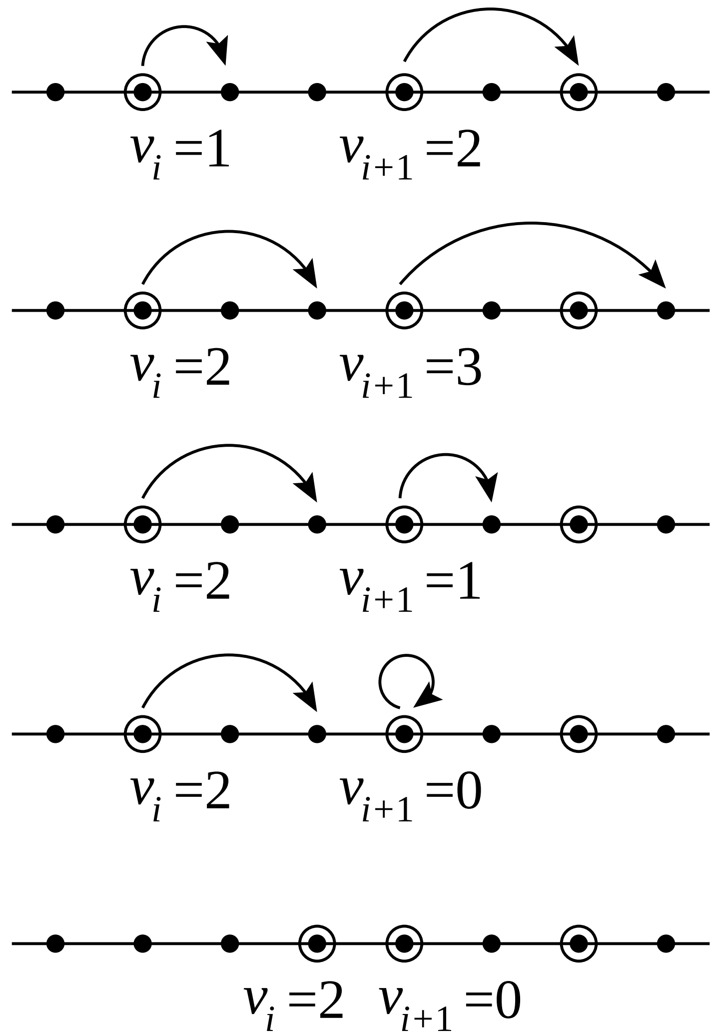
- developed in 80's
- widely used with all kinds of modifications
- catches much qualitative behaviour of traffic jams
- kinetic constraint

- 1d chain of road sites
- cars with velocities located at lattices sites
- discrete time



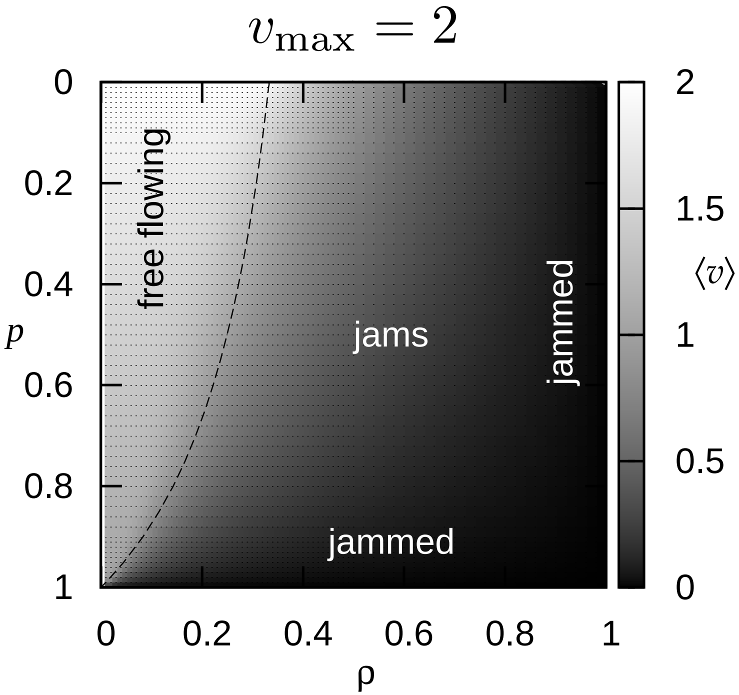
Nagel-Schreckenberg model

- parameters:
  - density of cars ρ
  - maximum velocity $v_{\max} \approx 5$
  - driver stochasticity p.
- discrete time update:
  1. cars accelerate:
     $v_i \rightarrow \max(v_i + 1, v_{\max})$
  2. unless there is no space (kinetic constraint):
     $v_i \rightarrow \min(v_i, x_{i+1} - x_i - 1)$
  3. random deceleration with probability p:
     $v_i \rightarrow \max(v_i - 1, 0)$
  4. move: $x_i \rightarrow x_i + v_i$



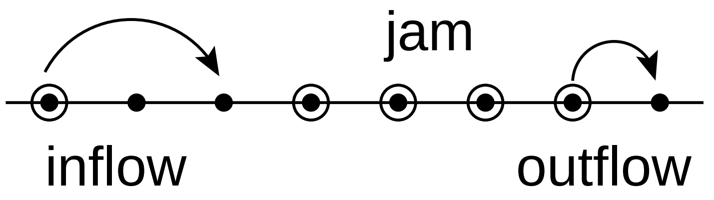
Nagel-Schreckenberg model

- spontaneous percolation of jams
- backwards moving jams
- no phase transition at $p \in \langle 0, 1 \rangle$ (agreed upon after much fighting)
  no phase separation: typical jam size is finite
- Kinetically Constrained Model:
  - v_i is activity
  - step 2 is kinetic constraint
  - p equivalent to temperature
  - deterministic $1 - p \to 0$
    is low temperature



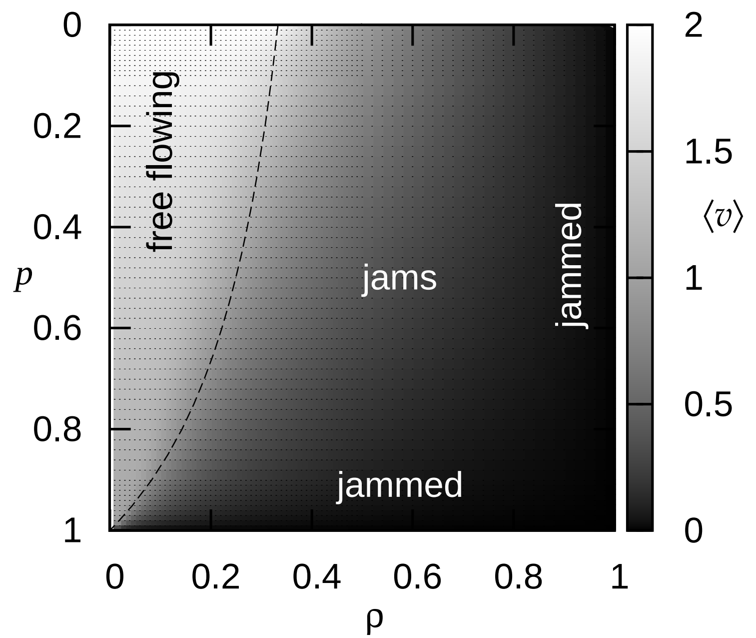
transition density

When do traffic jams persist?
inflow = outflow

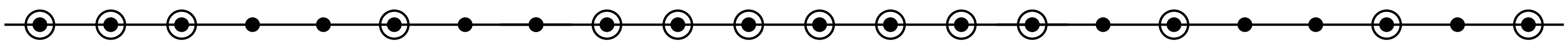


- average outflow rate: $\approx 1 - p$
- $=$ average backwards velocity of back of jam
- average inflow velocity: $v_{\max} - p$
- cars approach jam with speed $v_{\max} + 1 - 2p$

$$\rho_{\text{tra.}} = \frac{1-p}{v_{\max}+1-2p} \propto 1 - p$$

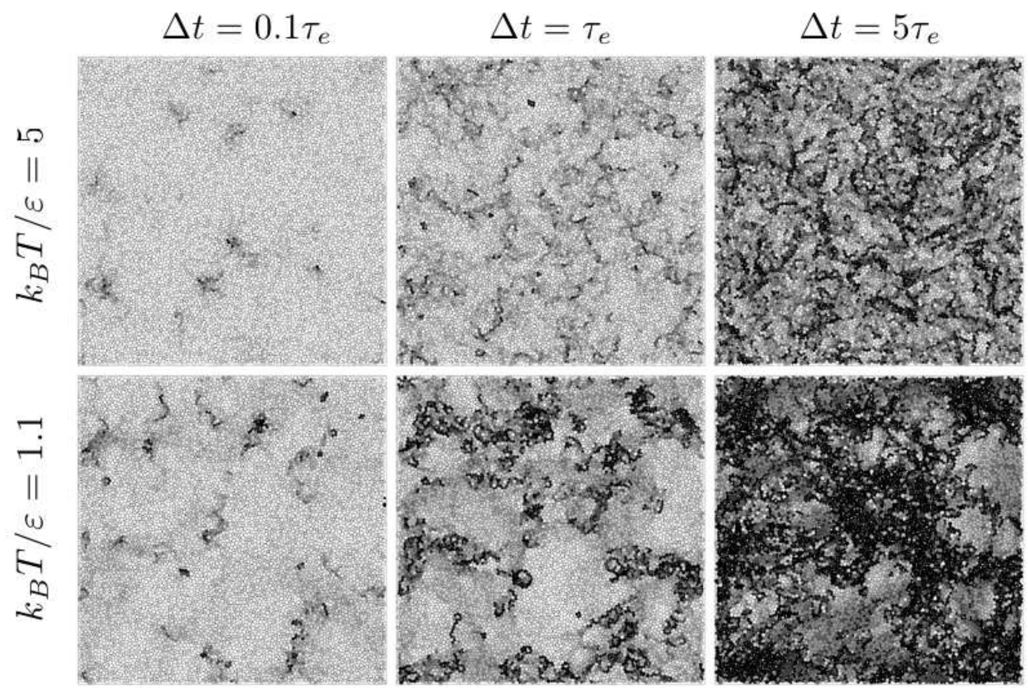


dynamic heterogeneity



What does the jammed regime look like?

- road consists of jams and free flow at $\rho_{\mathrm{tra.}}$
- jam size is finite, no phase separation
- glasses:
  dynamic heterogeneity
- quantify with
  dynamic susceptibility



correlated activity

- average activity over time between 0 and t

$$c(i;t) = (1/(t+1)) \sum_{t'=0}^{t} v_i(t')$$

in glasses: $c = [x(t) - x(0)]^2/t$.

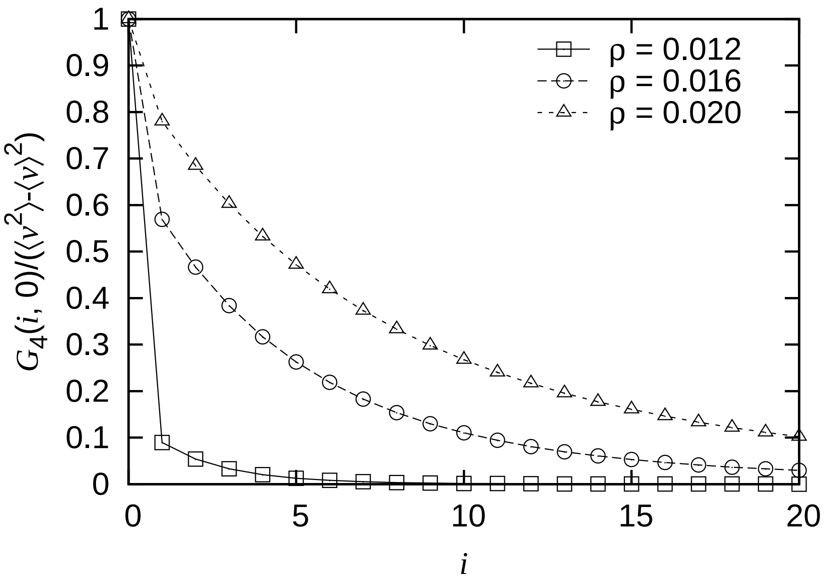
- dynamic correlation of activity between two sites distance i apart

$$G_4(i,t) = \langle c(i;t)c(0;t)\rangle - \langle c(0;t)\rangle^2$$

- associated susceptibility

$$\chi_4(t) = \frac{1}{\langle v^2\rangle - \langle v\rangle^2} \sum_{i=0}^{N-1} G_4(i;t)$$

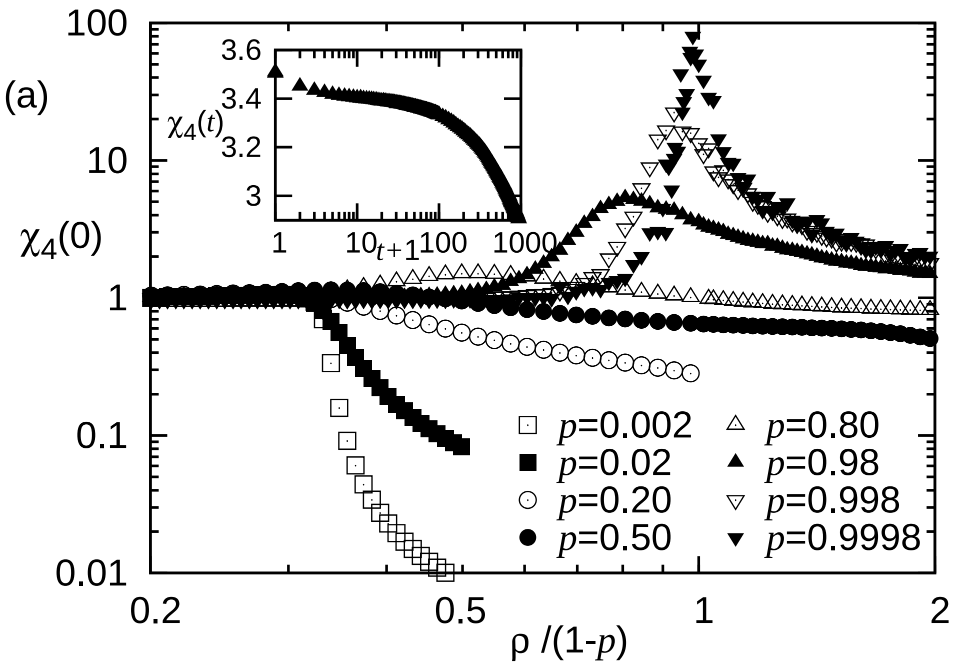
$v_{\max} = 2, p = 0.98, \rho_{\text{tra.}} \approx 0.02$



simulation details

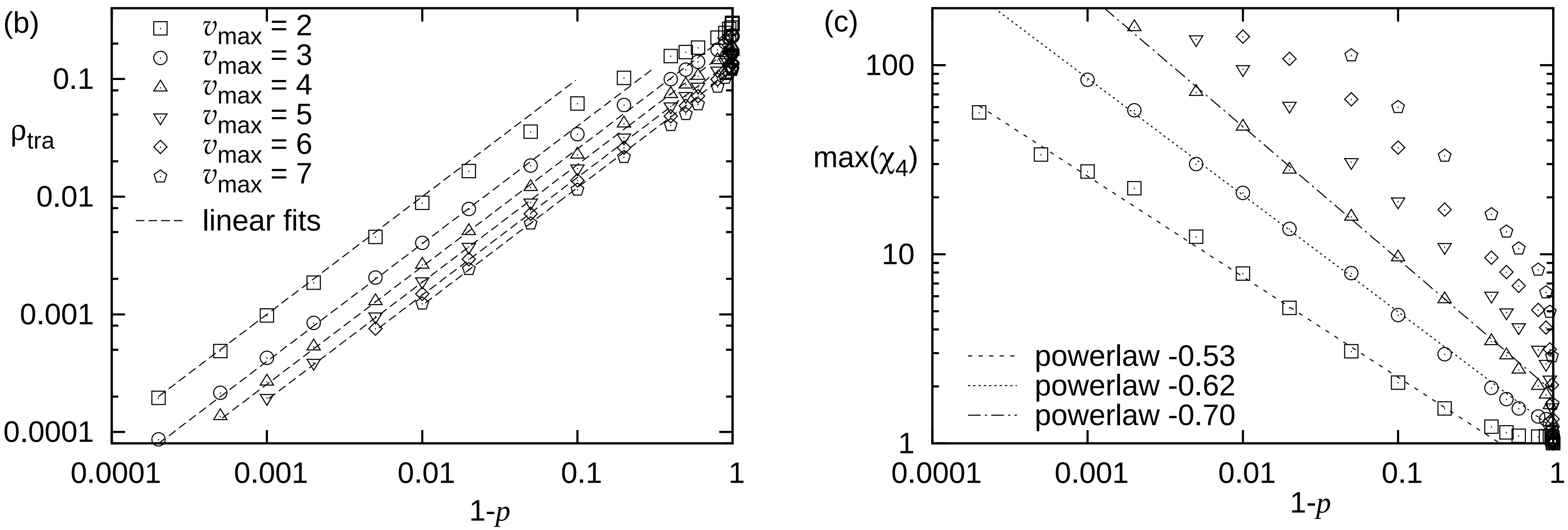
- periodic boundary condition (circular road)
- $2^{14} = 16384$ cars
- up to 5×10^8 time steps

spatial correlation



- $\chi_4(t)$ maximum at $t = 0$ (cars cannot escape environment independently)
- $\rho_{\rm tra.} \propto 1 - p \Rightarrow$ rescaled density
- reproduce discontinuous point at $p = 0$
- peak at $\rho_{\rm tra.}$
- at $\rho > \rho_{\rm tra.}$, $\chi_4(0) =$ typical number of subsequent cars not in jam

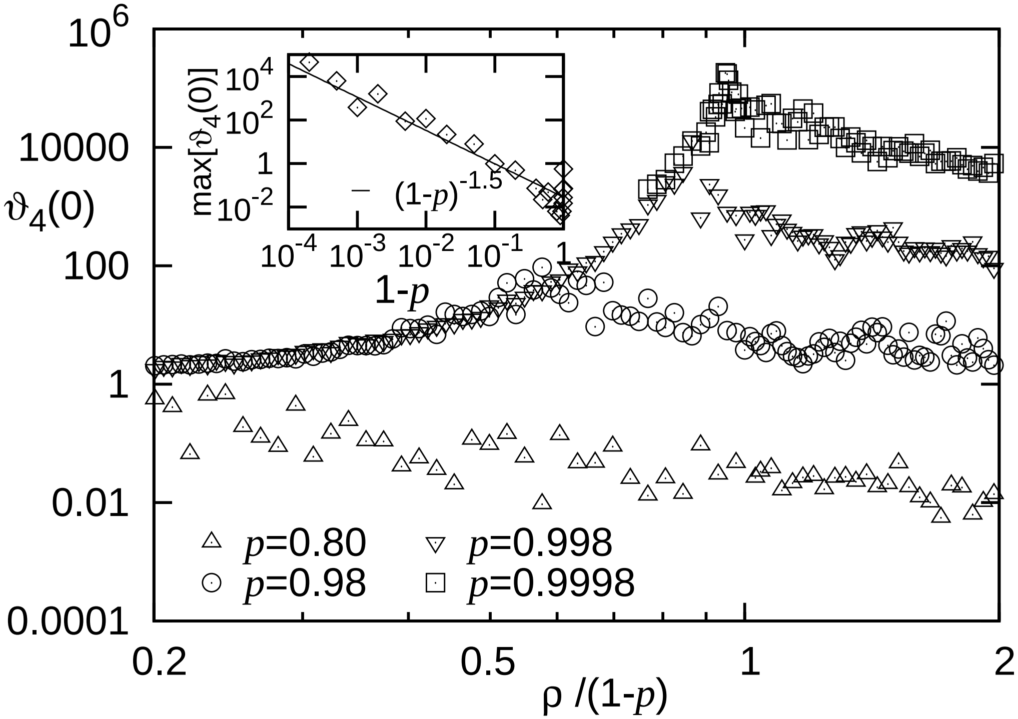
transition density and correlation length



- powerlaws near $p \to 1$
- correlation length $\propto (1-p)^{-\nu}$
- $\Rightarrow$ dynamic critical point at $p \to 1$
- ν depends on $v_{\max}$

temporal correlations

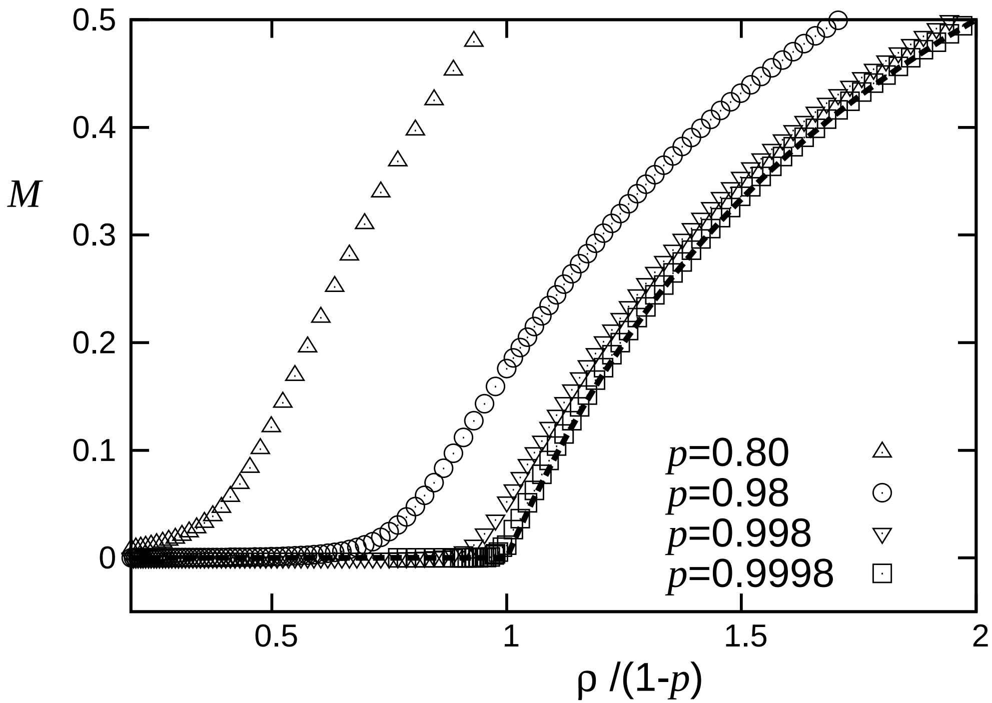
- interchange time and car index $\chi_4(t) \to \vartheta_4(i)$



- peak near $\rho_{\mathrm{tra.}}$
- as $p \to 1$:

  $\max[\vartheta_4(0)] \propto (1-p)^{-\mu}$
- $\mu \approx 1.5$
- diverging correlation time in traffic jams is not nice!

dynamic order parameter



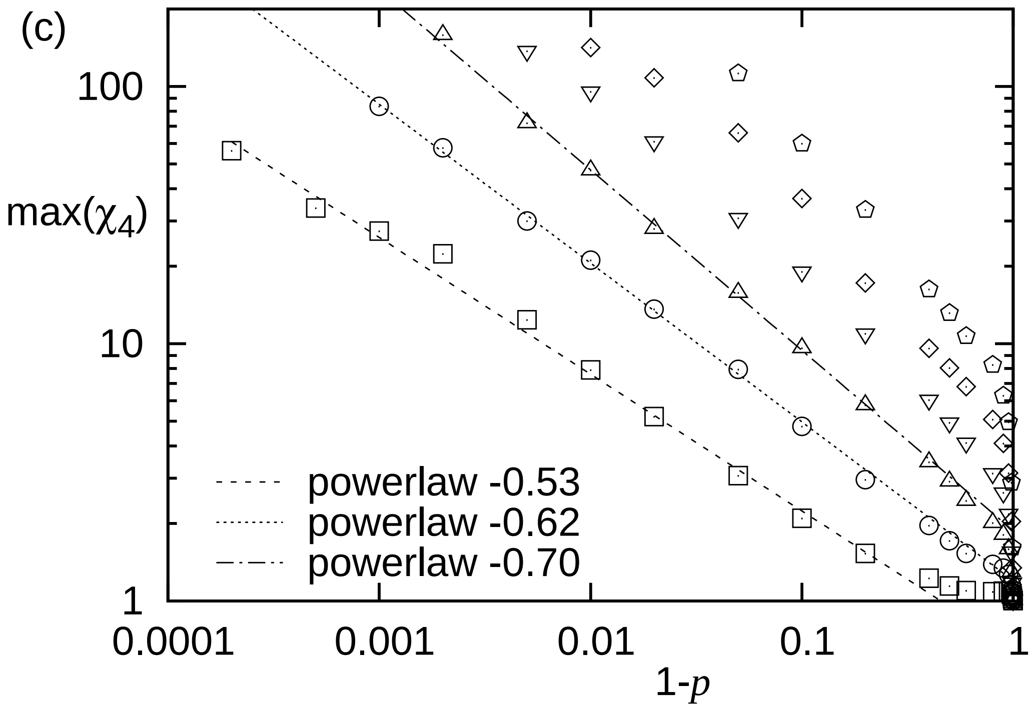
- dynamic order parameter

$$M = \frac{v_f - \langle v \rangle}{v_f} \; ,$$

$$v_f = v_{\max} - p$$

- $\rho < \rho_{\text{tra.}}$:
  $\langle v \rangle \approx v_f \Rightarrow M \approx 0$
- $\rho > \rho_{\text{tra.}}$,
  simple coexistence:
  $\langle v \rangle \approx v_f \rho_{\text{tra.}} / \rho$
  $\Rightarrow M \approx \frac{\rho - \rho_{\text{tra.}}}{\rho}$
- kink as $p \to 1$

critical exponents and universality



- glasses: directed percolation for $d > 2$ [Whitelam et al., PRL 92, 185705 (2004)]
- critical exponents depend on parameters
- $\Rightarrow$ more complicated [Miedema et al., PRE 89, 062812 (2014)]

conclusions

- Nagel-Schreckenberg model for traffic
  $\leftrightarrow$ kinetically constrained models for glasses
  - tools usually used for glasses (dynamic susceptibility)
  - dynamic critical point at $p \to 1$ $(T \to 0)$, $\rho \to 0$,
    diverging correlation length and time
  - some differences (critical exponents)
- traffic jams behave a lot like a glass
- $\Rightarrow$ universal nature of dynamic arrest

$\equiv$

de Wijn et al., PRL 109, 228001 (2012),
Miedema et al., PRE 89, 062812 (2014).