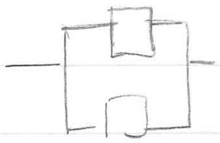
I) Reliability of a system with ageing unit and const repairs duration



→ Availability of the system as a function of the unit availability. → ok

→ Reliability ?

→ ok if exponential laws.

→ if Weibull lif. + constant repair ?

Pb: Reliability of systems with ageing units and repair

↳ Monte Carlo simulation

↳ PDMP

II) What is a PDMP

This is an hybrid process (X_t, M_t) where X_t is discrete with values in a finite state E and M_t is continuous in B.

X_t and M_t interact with each others.

X_t = system state

M_t = environmental conditions, age of units, durations.

jump at t

DISCRETE JUMPS

$$(X_{t^-}, M_{t^-}) = (x, m) \xrightarrow{} (X_{t^+}, M_{t^+}) = (x', m')$$

The transition rate from x to x' depends on m.
The distribution function from m to m' ——— x, m, x'

Transition kernel from (x,m) to (x',m') is :

$$k\left[(x,m),(x',m')\right] = \underbrace{t(x,x',m)}_{\text{transition rate}}\ \underbrace{d_{(x,x',m)}}_{\text{distribution}}(dm')$$

Between jumps

→ X_t is constant $= x$

→ $M_t = m(t)$ is solution of a differential equation :

$$\frac{dm(t)}{dt} = f(x, m(t)) \quad \text{for a } \underline{\text{given } x}.$$

Here $m(t) = t + g(x)$.

CONTINUOUS JUMPS

In case B is bounded with boundary Γ, jumps may be induced by the reaching of Γ

Then the after jump distribution is denoted

$$k\left[(x,m),(x',m')\right] = q\left[(x,m),(x',dm')\right]$$

III Numerical calculation of the law of the process

→ **Availability.**

Time is discretise (counter part for no simulation)

$$A(nd) = \sum_i \sum_j p_n(i,j)$$

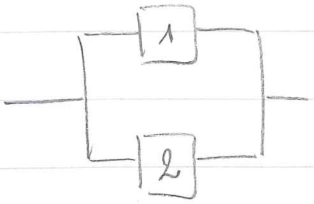
$p_n(i,j)$ is the probability to be in the available state (i,j) at time nd.

$$p_{n+1}(i,j) = \sum_k \sum_l p_n(k,l) \underbrace{G_n[(k,l)(i,j)]}_{\text{Transition propability}}$$

1. Calculate $G_n[(k,l)(i,j)]$ step by step.
2. Use the recursive equation

(1) → Define states
→ Calculate transitions.

IV Come back to the example

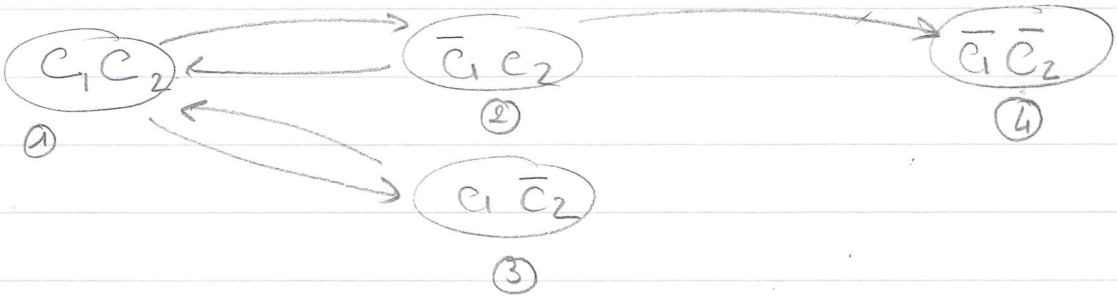


Weibull laws + Repair duration deterministic.

→ Ageing - Weibull law - Identical. → $\lambda(t) = \alpha\lambda^{\alpha} t^{\alpha-1}$

→ Repair duration = constant = m.

→ Different states (no merge) because of ageing



$R(t) = 1 - P_4(t)$.

$X_t = x_k \in \{\underset{k=1}{x_1}, \underset{k=2}{x_2}, x_3, x_4\}$

$\Pi_t = (m_1(t), a_1(t), m_2(t), a_2(t))$

$(X_t, \Pi_t) = (x_k, m_1(t), a_1(t), m_2(t), a_2(t))$

$m_i(t)$ = time spent under repair for unit i

$m_i(t) = 0$ ⇒ no maintenance on-going

$m_i(t) = m_i$ ⇒ maintenance finished.

For $k = 1$

$$G_n\left[(x_1, 0, a_1(n), 0, a_2(n))\,(x_1, 0, a_1(n)+d, 0, a_2(n)+d)\right]$$

$$\triangleq 1 - \left(\lambda(a_1(n)) + \lambda(a_2(n))\right)d$$

$$G_n\left[(x_1, 0, \quad\text{———}\quad)\,(x_2, 0, a_1(n), 0, a_2(n)+d)\right]$$

$$\triangleq \lambda(a_1(n))d$$

Idem for $x_1 \to x_3$

For $k = 2$

If $m_1(n) < (m_1 - 1)d$

$$G_n\left[(x_2, m_1(n), a_1(n), 0, a_2(n))\,(x_2, m_1(n)+d, a_1(n), 0, a_2(n)+d)\right]$$

$$\triangleq 1 - \lambda(a_2(n))d$$

$$G_n\left[(\qquad)\,(x_4, m_1(n)+d, a_1(n), 0, a_2(n))\right]$$

$$\triangleq \lambda(a_2(n))d$$

If $m_1(n) = (m_1 - 1)d$

$$G_n\left[(x_2, m_1(n), a_1(n), 0, a_2(n))\,(x_1, 0, 0, 0, a_2(n+1))\right]$$

$$\triangleq 1 - \lambda(a_2(n))d$$

$(x_4, \ldots\ )$

For $k = 3$

Idem

For $k = 4$

$$G_n\left[(x_4, \ldots)(x_i, \ldots)\right] = 0 \qquad i \neq 4$$

Absorbing state

$$G_n\left[(x_4, \ldots)(x_4, \ldots)\right] = 1$$

Explicite Euler Method

$$y(t_{n+1}) = y(t_n + h) \simeq y(t_n) + h y'(t_n)$$

Implicite Euler Method

$$y(t_n) = y(t_{n+1} - h) \simeq y(t_{n+1}) - h y'(t_{n+1})$$