NTNU – Trondheim
Norwegian University of Science and Technology

Statoil

RAMS
Reliability, Availability
Maintainability and Safety

Remaining useful lifetime modeling of motor insulation

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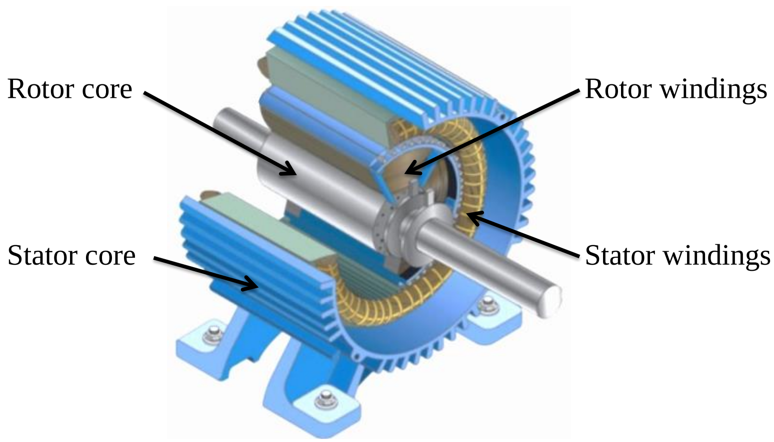
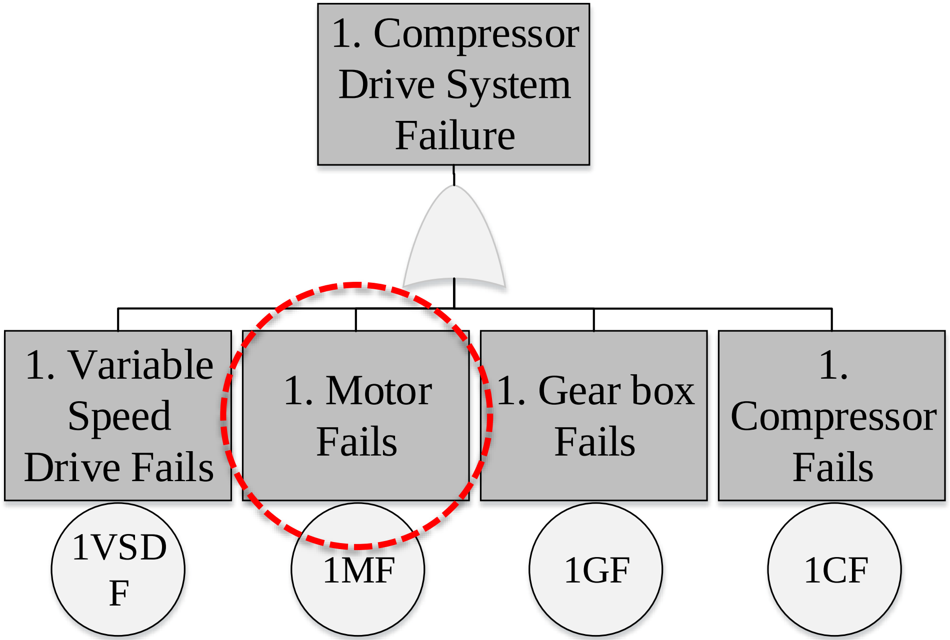
Problem description

- Aging compressor drive system





- Aging analysis of the electrical motor
  - Insulation
    - Degradation mechanisms
      - Partial discharge
        - Good health indicator

$$Y(t) = PD(t)$$

Insulation condition

Insulation condition	Partial Discharge [nC]	State
Excellent	<2	5
Good	>2<4	4
Average	>4<10	3
Still Acceptable	>10<15	2
Inspection Recommended	>15<25	1
Unreliable	>25	0

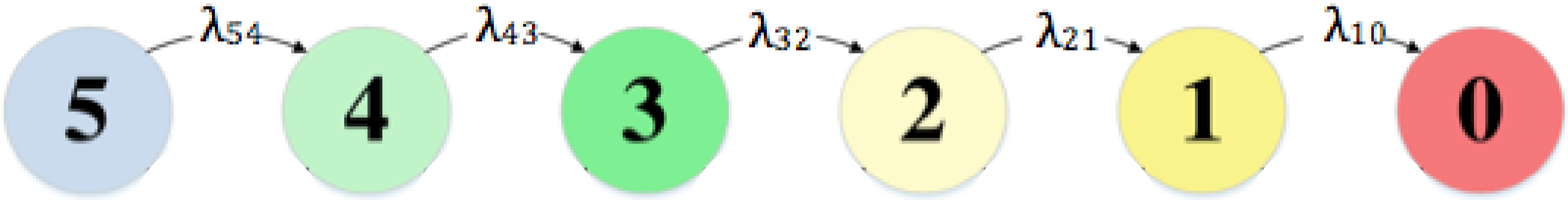
Insulation quality measured in PD [nC), Karsten Moholt

Model selection

- Probabilistic model
- SINTEF article model suggestions
  - lifetime distributions, ROCOF, Markov, PoF

- Why Markov?
- Markov property
  - Continuous time homogenous Markov process
  - Discrete states
  - Time spent in each state exp.dist. $T_i \sim \lambda_i e^{-\lambda_i t}$

Modeling with Markov



Insulation condition	Partial Discharge [nC]	State
Excellent	<2	5
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Average	>4<10	3
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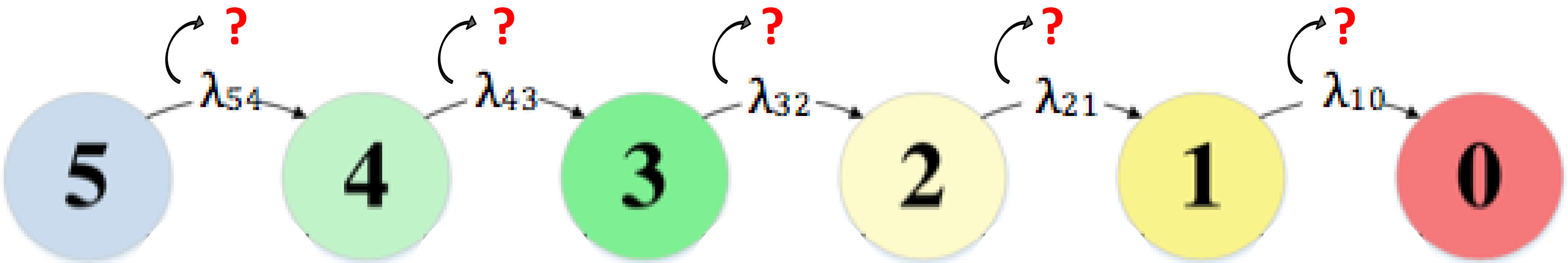
Unreliable	>25	0

Fictional failure rates

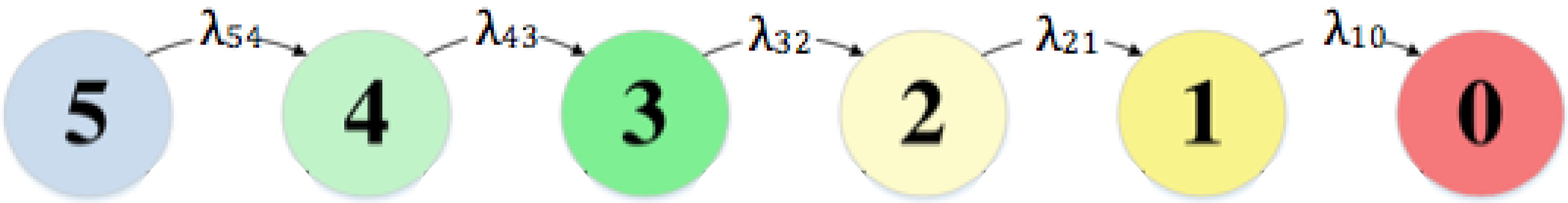


- The expected remaining lifetime is assumed to be 5 years

$$\frac{1}{\lambda_{tot}} = \frac{5}{365 * 24} \Rightarrow \lambda_{tot} = 0.000571$$

$$\lambda_{avg} = \frac{\lambda_{tot}}{5} = \frac{0.000571}{5} = 0.000114$$

Modeling with Markov



- MINITAB
- Simulation of random data
  - 5 values
  - Exponential distributed
  - Mean equal to 0.000114

$$\lambda_{10} = 0.0003517$$
$$\lambda_{21} = 0.0001423$$
$$\lambda_{32} = 0.0000876$$
$$\lambda_{43} = 0.0000470$$
$$\lambda_{54} = 0.0000378$$

Transition matrix

$$
\mathbf{A} = \begin{array}{c} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \\ \mathbf{5} \end{array}
\begin{array}{cccccc}
\mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_{10} & -(\lambda_{10}) & 0 & 0 & 0 & 0 \\
0 & \lambda_{21} & -(\lambda_{21}) & 0 & 0 & 0 \\
0 & 0 & \lambda_{32} & -(\lambda_{32}) & 0 & 0 \\
0 & 0 & 0 & \lambda_{43} & -(\lambda_{43}) & 0 \\
0 & 0 & 0 & 0 & \lambda_{54} & -\lambda_{54}
\end{array}
$$

$$
\mathbf{A} = \begin{array}{c} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \\ \mathbf{5} \end{array}
\begin{array}{cccccc}
\mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0.0003517 & -0.0003517 & 0 & 0 & 0 & 0 \\
0 & 0.0001423 & -0.0001423 & 0 & 0 & 0 \\
0 & 0 & 0.0000876 & -0.0000876 & 0 & 0 \\
0 & 0 & 0 & 0.0000470 & -0.0000470 & 0 \\
0 & 0 & 0 & 0 & 0.0000378 & -0.0000378
\end{array}
$$

Transition matrix

$$\begin{bmatrix} P_0(t), P_1(t), P_2(t), P_3(t), P_4(t), P_5(t) \end{bmatrix} \times \begin{array}{c|cccccc|} & 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \lambda_{10} & -(\lambda_{10}) & 0 & 0 & 0 & 0 \\ 2 & 0 & \lambda_{21} & -(\lambda_{21}) & 0 & 0 & 0 \\ 3 & 0 & 0 & \lambda_{32} & -(\lambda_{32}) & 0 & 0 \\ 4 & 0 & 0 & 0 & \lambda_{43} & -(\lambda_{43}) & 0 \\ 5 & 0 & 0 & 0 & 0 & \lambda_{54} & -\lambda_{54} \end{array} = \begin{bmatrix} \dot{P_0}(t), \dot{P_1}(t), \dot{P_2}(t), \dot{P_3}(t), \dot{P_4}(t), \dot{P_5}(t) \end{bmatrix}$$

- Equations

Eq 1. $P_1(t)\times\lambda_{10} = \dot{P_0}(t)$

Eq 2. $P_1(t)\times(-\lambda_{10}) + P_2(t)\times\lambda_{21} = \dot{P_1}(t)$

Eq 3. $P_2(t)\times(-\lambda_{21}) + P_3(t)\times\lambda_{32} = \dot{P_2}(t)$

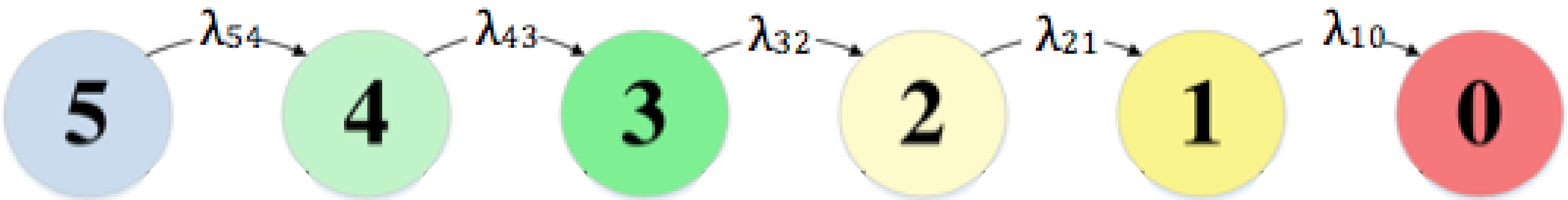
Eq 4. $P_3(t)\times(-\lambda_{32}) + P_4(t)\times\lambda_{43} = \dot{P_3}(t)$

Eq 5. $P_4(t)\times(-\lambda_{43}) + P_5(t)\times\lambda_{54} = \dot{P_4}(t)$

Eq 6. $P_5(t)\times(-\lambda_{54}) = \dot{P_5}(t)$

Eq7. $P_5(0) = 1,\ P_1(0) = 0,\ P_2(0) = 0, \ldots, P_0(0) = 0$

State probabilities



- State 5: $$P_5(t) = e^{-\lambda_{54}t}$$
- State 4: $$P_4(t) = \frac{\lambda_{54}}{\lambda_{43} - \lambda_{54}}(e^{-\lambda_{54}t} - e^{-\lambda_{43}t})$$
- State 3: $$P_3(t) = \frac{\lambda_{54}\times\lambda_{43}}{(\lambda_{43} - \lambda_{54})\times(\lambda_{32}-\lambda_{54})}(e^{-\lambda_{54}t} - e^{-\lambda_{32}t}) + \frac{\lambda_{54}\times\lambda_{43}}{(\lambda_{43} - \lambda_{54})\times(\lambda_{32}-\lambda_{43})}\times(e^{-\lambda_{32}t} - e^{-\lambda_{43}t})$$

State probabilities

- State 2:

$$P_2(t) = \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})}\times(e^{-\lambda_{54}t}-e^{-\lambda_{21}t})$$
$$+\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})}\times(e^{-\lambda_{21}t}-e^{-\lambda_{43}t})$$
$$+\left(\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})}-\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})}\right)\times(e^{-\lambda_{32}t}-e^{-\lambda_{21}t})$$

- State 1:

$$P_1(t) = \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})\times(\lambda_{10}-\lambda_{54})}\times(e^{-\lambda_{54}t}-e^{-\lambda_{10}t})$$
$$+\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})\times(\lambda_{10}-\lambda_{43})}\times(e^{-\lambda_{10}t}-e^{-\lambda_{43}t})$$
$$+\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{32})}\times(e^{-\lambda_{32}t}-e^{-\lambda_{10}t})$$
$$+\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{32})}\times(e^{-\lambda_{10}t}-e^{-\lambda_{32}t})$$
$$+\Big(\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})\times(\lambda_{10}-\lambda_{21})}$$
$$+\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{21})}\Big)\times(e^{-\lambda_{10}t}-e^{-\lambda_{21}t})$$
$$+\Big(\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})\times(\lambda_{10}-\lambda_{21})}$$
$$+\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{21})}\Big)\times(e^{-\lambda_{21}t}-e^{-\lambda_{10}t})$$

State probabilities

- State 0:

$$
\begin{aligned}
P_0(t) = {} & \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})\times(\lambda_{10}-\lambda_{54})\times(-\lambda_{54})}\times(e^{-\lambda_{54}t}-1) + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})\times(\lambda_{10}-\lambda_{43})\times(-\lambda_{43})}\times(1-e^{-\lambda_{43}t}) \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{32})\times(-\lambda_{32})}\times(e^{-\lambda_{32}t}-1) \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{32})\times(-\lambda_{32})}\times(1-e^{-\lambda_{32}t}) + (\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{21})} \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{21})})\times(e^{-\lambda_{21}t}-1) + (\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{21})} \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{21})})\times(1-e^{-\lambda_{21}t}) + (\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})\times(\lambda_{10}-\lambda_{43})\times(-\lambda_{10})} \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{32})\times(-\lambda_{10})})\times(e^{-\lambda_{10}t}-1) + (\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})\times(\lambda_{10}-\lambda_{54})\times(-\lambda_{10})} \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{32})\times(-\lambda_{10})})\times(1-e^{-\lambda_{10}t}) + (\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{43})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{10})} \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{10})})\times(1-e^{-\lambda_{10}t}) + (\frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{54})\times(\lambda_{21}-\lambda_{54})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{10})} \\
& + \frac{\lambda_{54}\times\lambda_{43}\times\lambda_{32}\times\lambda_{21}\times\lambda_{10}}{(\lambda_{43}-\lambda_{54})\times(\lambda_{32}-\lambda_{43})\times(\lambda_{21}-\lambda_{32})\times(\lambda_{10}-\lambda_{21})\times(-\lambda_{10})})\times(e^{-\lambda_{10}t}-1)
\end{aligned}
$$

State probabilities with Excel

	λ_{10}	λ_{21}	λ_{32}	λ_{43}	λ_{54}
Transition rates	0.0003517	0.0001423	0.0000876	0.0000470	0.0000378
Time (Hours)	8760				

State 0	State 1	State 2	State 3	State 4	State 5
0.0014	0.0018	0.0089	0.0414	0.2285	0.7181

State probabilities with MATLAB

```
For time t = 1 Year(s)

    State 0    State 1    State 2    State 3    State 4    State 5
    0.0014     0.0018     0.0089     0.0414     0.2285     0.7181
```

State 0	State 1	State 2	State 3	State 4	State 5
0.0014	0.0018	0.0089	0.0414	0.2285	0.7181

State probabilities with MATLAB

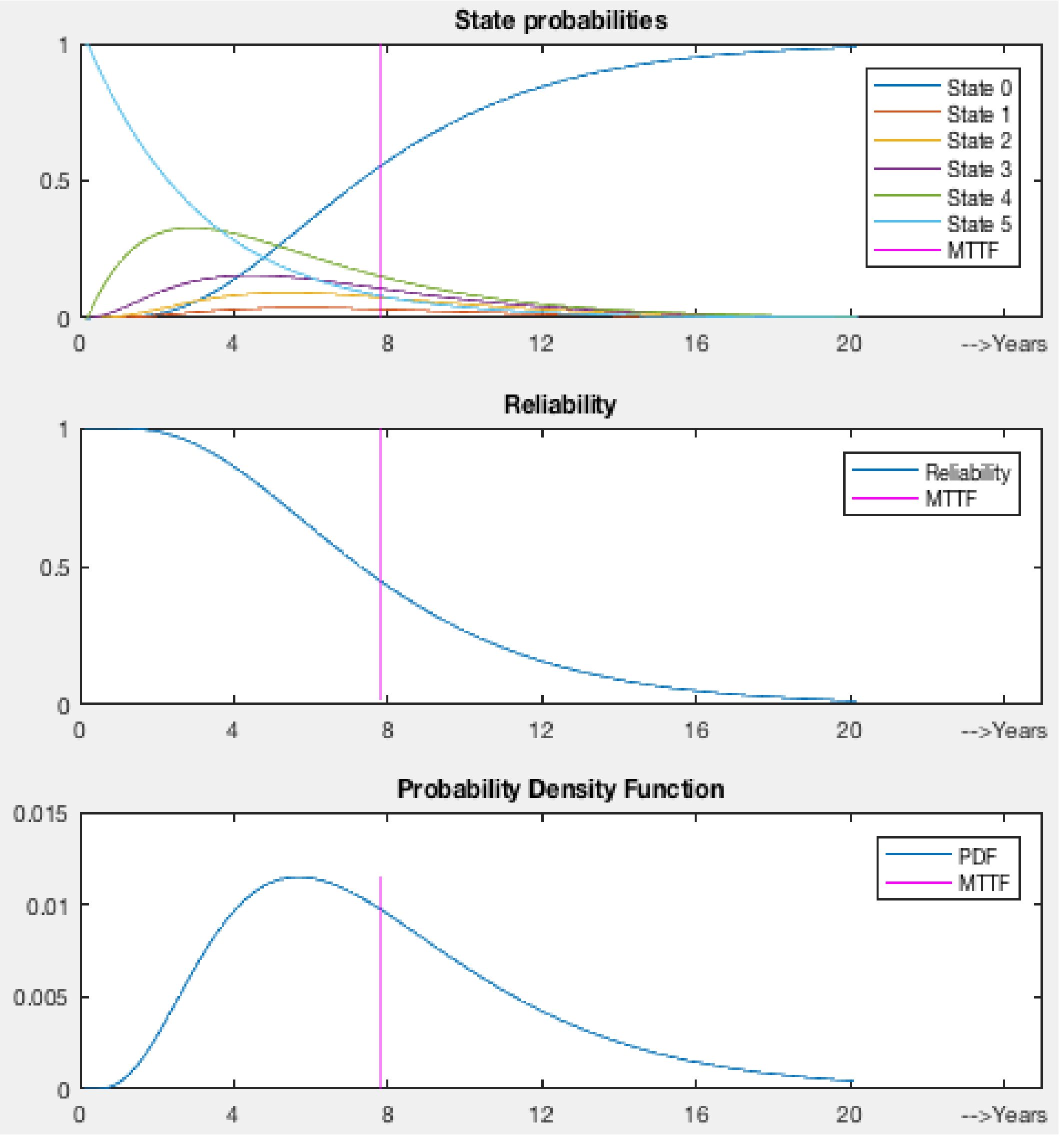
```
------------------------------------------------------------
For time t = 20 Years
------------------------------------------------------------
- The reliability, R(t), is 0.0142
- The Mean Time To Failure, MTTF, is 7.83 Years
- The state probabilities are:

    State 0   State 1   State 2   State 3   State 4   State 5
    0.9858    0.0014    0.0032    0.0039    0.0044    0.0013

- The expexted time (years) spent in each stat is:

    State 0   State 1   State 2   State 3   State 4   State 5
    0.0000    0.3246    0.8022    1.3031    2.4288    3.0200
```

State probabilities with MATLAB



Conditional reliability

- The reliability when we know that the motor is in a specific state at a given time

$$R(x \mid t) = \Pr(T > t + x \mid T > t) = \frac{\Pr(T > t + x)}{\Pr(T > t)}$$

$$= \frac{e^{-\lambda(t+x)}}{e^{-\lambda t}} = e^{-\lambda x}$$

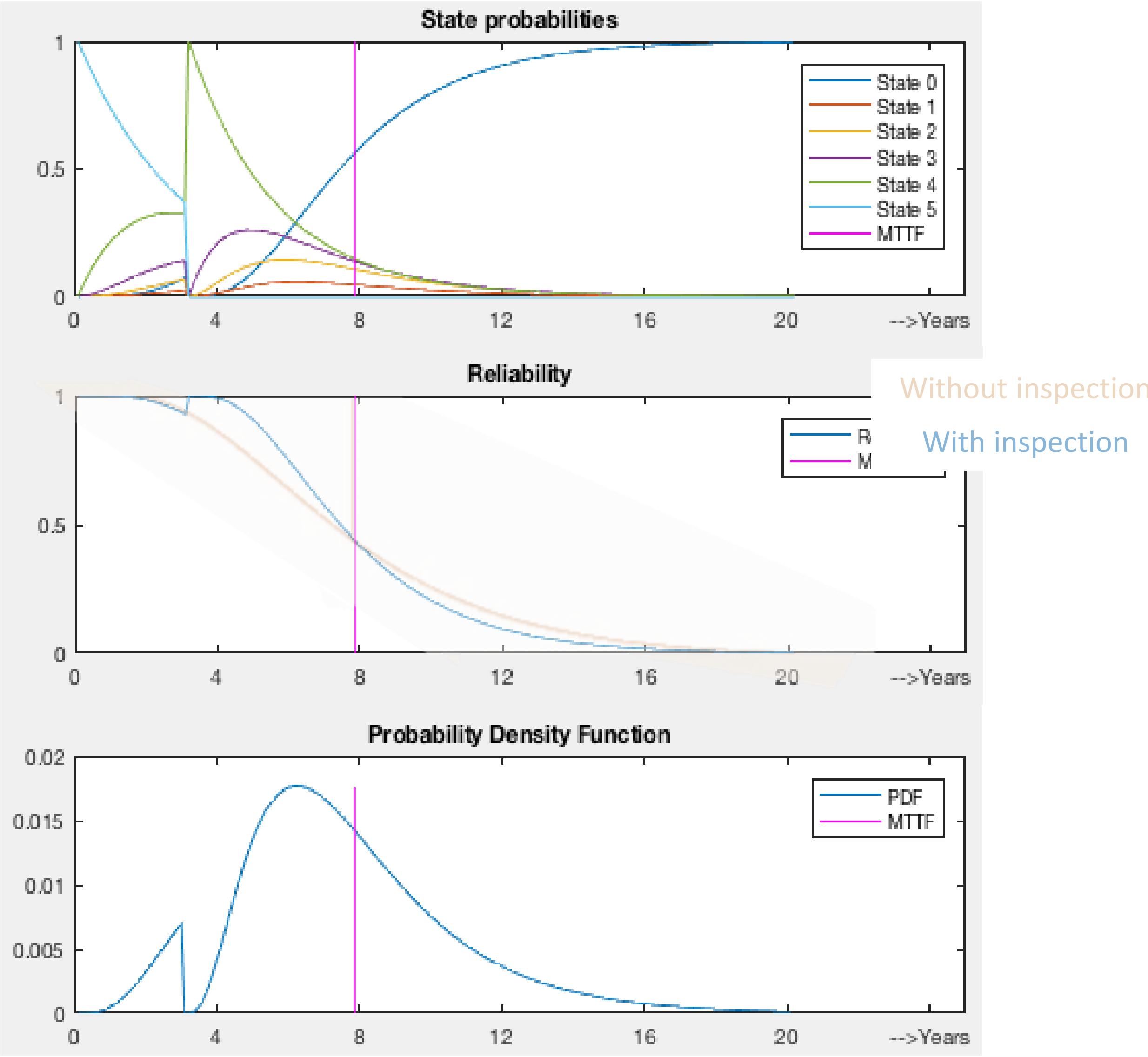
Conditional reliability with MATLAB

- Total time (t+x) = 20 years
- State 4 at time t = 3 years

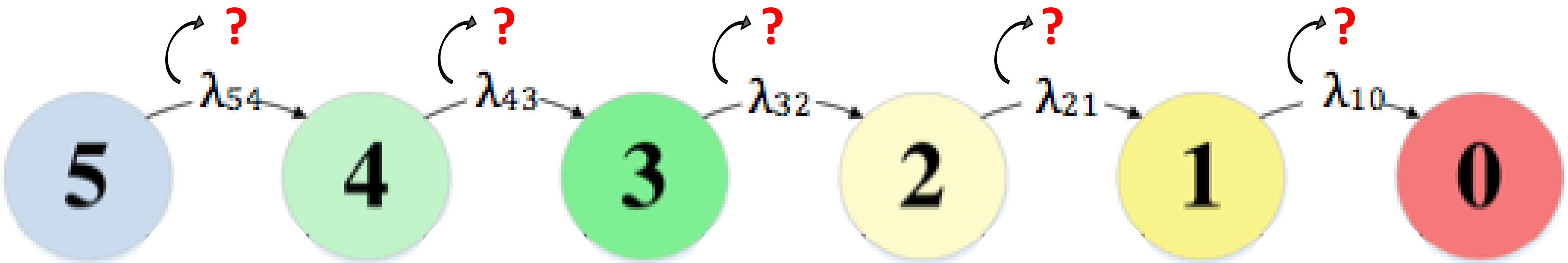
```
---------------------------------------------------------------
For a total time of t = 20 Years,
and a condition of state 4 at time t = 3 Years
---------------------------------------------------------------
- The reliability, R(t), is 0.0034
- When the motor is at state 4 after 3.00 years,
  The Mean Time To Failure, MTTF, is 4.85 Years.
- The state probabilities are:

    State 0    State 1    State 2    State 3    State 4    State 5
    0.9966     0.0005     0.0010     0.0011     0.0009          0
```

Conditional reliability with MATLAB



Modeling with censored data



- Right censored
- Left censored
- Interval censored

Sample nr	L	R
1	2	NA
2	NA	5
3	3	6
...	...	...

Data set extracted with MATLAB

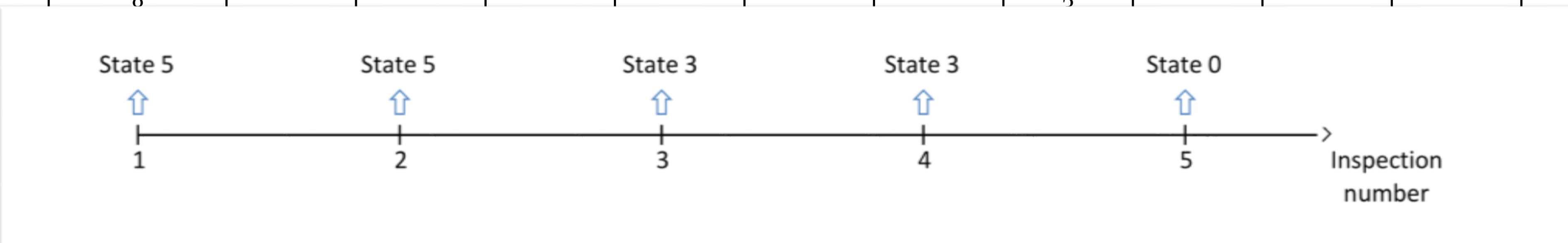
	Time of a transition into a more degraded state in years									
State observed	**Sample 1**	**Sample 2**	**Sample 3**	**Sample 4**	**Sample 5**	**Sample 6**	**Sample 7**	**Sample 8**	**Sample 9**	**Sample 10**
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	2.1795	1.8884	1.8424	0.7566	0.1099	0.4590	2.7774	2.7648	3.3130	0.6823
3	4.2294	3.6084	1.9298	3.6632	1.3927	3.3724	6.2383	3.1591	3.9048	1.3035
2	4.5300	4.1066	3.1600	4.4410	3.6484	3.6069	7.8159	3.7706	4.2275	1.3758
1	4.6955	4.3479	3.5365	4.6926	4.9849	4.5996	8.1563	4.1911	4.9067	1.3933
0	5.1724	4.4280	3.9621	4.7254	5.3707	4.6205	8.3690	4.2157	5.0676	1.3953

→

State observed	**Sample 11**	**Sample 12**	**Sample 13**	**Sample 14**	**Sample 15**	**Sample 16**	**Sample 17**	**Sample 18**	**Sample 19**	**Sample 20**
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.5399	4.8123	1.3427	2.1093	0.5063	0.5921	0.3724	2.2240	5.2767	0.4205
→ 3	0.6600	5.3023	4.1846	7.3839	1.8239	0.6004	5.6315	2.4232	5.5747	1.4306
2	0.7463	6.6348	4.6692	9.0667	1.8283	1.2139	6.6781	4.3691	6.1972	2.6259
1	1.0577	7.0829	4.9308	9.1304	3.6194	1.2318	7.6251	5.3055	6.6175	3.0946
0	1.0886	7.5941	5.0133	9.6653	3.8511	1.2375	7.6885	5.8535	7.1666	3.3539

Data set extracted with MATLAB

	Observed states for each sample with inspections every ΔT = 1 year									
Inspection nr	**Sample 1**	**Sample 2**	**Sample 3**	**Sample 4**	**Sample 5**	**Sample 6**	**Sample 7**	**Sample 8**	**Sample 9**	**Sample 10**
1	5	5	5	5	5	5	5	5	5	5
2	5	5	5	4	5	4	5	5	5	4
3	5	4	3	4	3	4	5	5	5	0
4	4	4	3	4	3	4	4	4	5	
5	4	3	0	3	2	2	4	2	3	
6	1	0		0	1	0	4	0	1	
7	0				0		4		0	
8							3			



3	0	5	4	5	2	0	4	5	5	3
4		5	4	4	2		4	3	5	2
5		5	4	4	0		4	3	5	0
6		4	1	4			4	2	5	
7		3	0	4			3	0	3	
8		2		4			2		1	
9		0		3			0		0	
10				3						
11				0						

Censored data table

Inspection nr	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7	Sample 8	Sample 9	Sample 10
	Observed states for each sample with inspections every ΔT = 1 year									
1	5	5	5	5	5	5	5	5	5	5
2	5	5	5	4	5	4	5	5	5	4
3	5	4	3	4	3	4	5	5	5	0
4	4	4	3	[illegible]	[illegible]	4	4	4	5	
5	4	3	0	[illegible]	[illegible]	2	4	2	3	
6	1	0		[illegible]	[illegible]	0	4	0	1	
7	0			[illegible]	[illegible]		4		0	
8				[illegible]	[illegible]		3			
9				[illegible]	[illegible]		2			
10				[illegible]	[illegible]		0			
11				[illegible]	[illegible]					

→

Inspection nr	Sample 11	Sample 12	Sample 13	Sa[illegible]	[illegible]	Sample 16	Sample 17	Sample 18	Sample 19	Sample 20
1	5	5	5	[illegible]	[illegible]	5	5	5	5	5
2	2	5	5	[illegible]	[illegible]	3	4	5	5	4
3	0	5	4	[illegible]	[illegible]	0	4	5	5	3
4		5	4	[illegible]	[illegible]		4	3	5	2
5		5	4	[illegible]	[illegible]		4	3	5	0
6		4	1	4			4	2	5	
7		3	0	4			3	0	3	
8		2		4			2		1	
9		0		3			0		0	
10				3						
11				0						

$l_k^1 = 5$

$l_k^2 = 11$

$l_k^3 = 17$

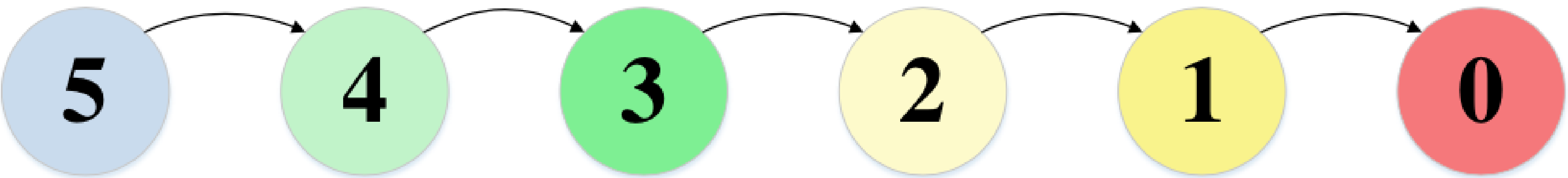
$l_k^4 = 32$

$l_k^5 = 46$

$\Delta T = 1\ year$

n = 20

Parameter estimation



$\lambda_{10} = 0.0003517$

$\lambda_{21} = 0.0001423$

$\lambda_{32} = 0.0000876$

$\lambda_{43} = 0.0000470$

$\lambda_{54} = 0.0000378$

Parameter	Error
$\tilde{\lambda}_{10}$	47.98%
$\tilde{\lambda}_{21}$	16.87%
$\tilde{\lambda}_{32}$	1.37%
$\tilde{\lambda}_{43}$	15.96%
$\tilde{\lambda}_{54}$	8.99%

$$error = \frac{|\lambda_{ij} - \tilde{\lambda}_{ij}|}{\lambda_{ij}} \times 100$$

Things to do if time

- Make a model based on the censored data
- Parameter estimation with more samples
- Include variance and confidence intervals
- Simulate with Monte Carlo
- Make a simple Markov for the whole system
- Finish and update what's done so far

Questions?