

## **Distributed Detection and Localization**

A Statistical Signal Processing Approach

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## Detection Single Sensor



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## Detection Single Sensor



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## Distributed Detection Sensor Network



## Distributed Detection Sensor Network



## Distributed Detection Sensor Network with Fusion Center





## Distributed Detection Sensor Network with Fusion Center

































#### **Applications**



- Collaborative Spectrum Sensing in Cognitive Radio
- Event Detection in Underwater Acoustic Sensor Networks
- Leak Detection and Localization in Oil&Gas production/distribution systems



# **Sensor Modeling**

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Part I - Detection





True Hp \ Estimated Hp	$\mathcal{H}_{0}$	$\mathcal{H}_1$
$\mathcal{H}_{0}$	Correct Decision	Type I Error (False Alarm)
$\mathcal{H}_1$	Type II Error (Missed Detection)	Correct Decision (Detection)

$$P_D = p(d = \mathcal{H}_1 | \mathcal{H}_1)$$

- $P_F = p(d = \mathcal{H}_1 | \mathcal{H}_0)$
- $P_M = p(d = \mathcal{H}_0 | \mathcal{H}_1)$  $= 1 P_D$





## Receiver Operating Characteristic (ROC)







#### Sensor Model





True Hp \ Estimated Hp	$\mathcal{H}_{0}$	$\mathcal{H}_1$
$\mathcal{H}_{0}$	Correct Decision	Type I Error (False Alarm)
${\mathcal H}_1$	Type II Error (Missed Detection)	Correct Decision (Detection)

$$P_D = p(d = \mathcal{H}_1 | \mathcal{H}_1)$$

$$P_F = p(d = \mathcal{H}_1 | \mathcal{H}_0)$$



## Local Test – Optimum Test – Likelihood Ratio Test (LRT)

- LRT is the optimum test in the Neyman-Pearson framework and in the Bayesian framework
  - $y|\mathcal{H}_0{\sim}p(y|\mathcal{H}_0)$
  - $y|\mathcal{H}_1{\sim}p(y|\mathcal{H}_1)$
- Compute the likelihood ratio or equivalently the log-likelihood ratio (LLR)

$$- \lambda(y) = \ln\left(\frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)}\right)$$

- Compare the LLR with a threshold
  - $\lambda(y) \gtrless \gamma$
- Requires complete knowledge of the conditional probabilities  $p(y|\mathcal{H}_0)$  and  $p(y|\mathcal{H}_1)$

#### LRT – Example 1 (Shift in Mean)



- Statistical Signal Model
  - $y|\mathcal{H}_0{\sim}\mathcal{N}\left(0;\sigma^2\right)$
  - $y|\mathcal{H}_1 \sim \mathcal{N}(\mu; \sigma^2)$

$$p(y|\mathcal{H}_0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}$$
$$p(y|\mathcal{H}_1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

Compute the LLR

$$- \lambda(y) = \ln\left(\frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)}\right) = \frac{\mu}{\sigma^2}y - \frac{\mu^2}{2\sigma^2}$$

- LRT is equivalent to Level Test
  - $y \gtrless \gamma$

$$p(y|\mathcal{H}_0)$$
  $p(y|\mathcal{H}_1)$ 

### LRT – Example 2 (Shift in Variance)



- Statistical Signal Model
  - $\hspace{0.1 cm} y | \mathcal{H}_0 {\sim} \mathcal{N}(0; \sigma_0^2)$



- $y|\mathcal{H}_1 \sim \mathcal{N}(0; \sigma_1^2)$
- Compute the LLR

$$- \lambda(y) = \ln\left(\frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)}\right) = \frac{1}{2} \frac{\sigma_1^2 - \sigma_0^2}{\sigma_0^2 \sigma_1^2} y^2 + \frac{1}{2} \ln\left(\frac{\sigma_0^2}{\sigma_1^2}\right)^2$$

- LRT is equivalent to Energy Test
  - $y^2 \gtrless \gamma$



#### **Practical Tests**



#### • (Optimum) LRT

$$\ln\left(\frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)}\right) \gtrless \gamma$$

- Test commonly employed in absence of other relevant information
  - Level Test

 $y \gtrless \gamma$ 

- Energy Test

 $y^2 \gtrless \gamma$ 



## (Wireless) Sensor Networks Part I – Distributed Detection

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#### Sensor-Network Architecture



Global Test and Global Performance



#### Sensor-Network Architecture



- · Possible assumptions on the information processing at sensor location
  - Hard decisions, local binary decision  $d_k \in \{0,1\}$
  - **Soft decisions**, level of confidence, multibit quantization of the LRT  $d_k \in \{0, 1, ..., 2^n 1\}$
  - Analog information, in the ideal case of infinite precision, LLR information is sent (e.g.  $d_k = \lambda_k(y_k)$ )
- Possible assumptions on the reporting channel
  - Perfect channel:  $r_k = d_k$
  - Parallel Access Channel (**no interference**):  $r_k = f_k(d_k)$
  - Multiple Access Chanel (**interference**):  $r = f(d_1, d_2, ..., d_K)$
  - MIMO Chanel (interference and multiple antennas):  $r_n = f_n(d_1, d_2, ..., d_K)$
  - Common channel models:
    - Binary Symmetric Channel
    - Additive White Gaussian Noise Channel
    - Rayleigh-Fading Channel
- The fusion center takes a global decision depending on a specific **fusion rule**:  $\lambda(r_1, r_2, ..., r_N) \ge \gamma$

#### MIMO Decision Fusion in WSNs



- D. Ciuonzo, G. Romano, P. Salvo Rossi, "Channel-Aware Decision Fusion in Distributed MIMO Wireless Sensor Networks: Decode-and-Fuse vs. Decode-then-Fuse," *IEEE Trans. Wireless Commun.* (2012)
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## Why MIMO in WSNs?



- Introduces spatial diversity
  - Fading mitigation
- Is spectrally efficient
  - Resource saving
- Comes (almost) for free
  - Exploiting interference
  - No additional cost except for appropriate processing



#### System Model



y = Hx + w

 $\lambda(\mathbf{y}) \gtrless \gamma$  $Q_D = p(\lambda > \gamma | \mathcal{H}_1)$  $Q_F = p(\lambda > \gamma | \mathcal{H}_0)$ 

•  $\boldsymbol{H} = \left( \left( H_{1,1}, \dots, H_{1,K} \right)^T, \dots, \left( H_{N,1}, \dots, H_{N,K} \right)^T \right)^T$  is the **channel matrix** 

•  $y_n \in \mathbb{C}$  is the complex-valued signal received at the *n*th RX antenna

- $H_{n,k} \sim \mathcal{N}_{\mathbb{C}}(0; 1)$  is the channel coefficient between the *k*th sensor and the *n*th RX antenna
- $\mathbf{x} = (x_1, ..., x_K)^T$  is the **transmitted vector**

•  $y = (y_1, ..., y_N)^T$  is the received-signal vector

- $x_k \in \{-1, +1\}$  is the BPSK symbol transmitted by the *k*th sensor
- $\boldsymbol{w} = (w_1, \dots, w_N)^T$  is the **noise vector**
- $w_n \sim \mathcal{N}(0; \sigma_w^2)$  is the AWGN at the *n*th RX antenna

 $SNR_{tx} = \frac{K}{\sigma_w^2} \qquad SNR_{tx}^* = \frac{1}{\sigma_w^2}$  $SNR_{rx} = \frac{KN}{\sigma_w^2} \qquad SNR_{rx}^* = \frac{N}{\sigma_w^2}$ 

#### **Performance Benchmarks**

Observation Bound: noisy sensing with perfect reporting

$$Q_F = \sum_{k=g}^{K} {K \choose k} P_F^k (1 - P_F)^{K-k} \qquad Q_D = \sum_{k=g}^{K} {K \choose k} P_D^k (1 - P_D)^{K-k}$$

- Communication Bound: perfect sensing with noisy reporting
- Optimal Fusion Rule: LRT

$$\lambda(\mathbf{y}) = \ln \left( \frac{\sum_{x \in \{\pm 1\}^K} e^{-\frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma_w^2}} \prod_{k=1}^K p(x_k | \mathcal{H}_1)}{\sum_{x \in \{\pm 1\}^K} e^{-\frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma_w^2}} \prod_{k=1}^K p(x_k | \mathcal{H}_0)} \right)$$

- High computational complexity: exponential with the number of sensors  $\sigma(2^K N)$
- Numerical instability: large dynamic range is problematic with fixed-point implementations
- Excessive knowledge requirements: local performance, channel matrix, noise variance



#### **Alternative Fusion Rules**



- Decode-and-Fuse approach
  - Maximum Ratio Combining
    - (optimum at low SNR, linear complexity  $\sigma(N)$ , requires partial channel)
  - Equal Ratio Combining

(no optimality, linear complexity  $\sigma(N)$ , requires less partial channel)

- Max-Log

(optimal, reduced exponential complexity, full knowledge)

- Decode-then-Fuse approach
  - Chair-Varshney Rule with Maximum Likelihood Estimation (optimum at high SNR, reduced exponential complexity, requires local performance and channel matrix)
  - Chair-Varshney Rule with Minimum Mean Square Error Estimation (no optimality, polynomial complexity  $\sigma(NK^2 + N^2)$ , full knowledge)

#### Performance









# **Sensor Modeling**

Part II – Distributed Detection and Localization

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#### Sensor Model





True Hp \ Estimated Hp	$\mathcal{H}_{0}$	$\mathcal{H}_1$
$\mathcal{H}_{0}$	Correct Decision	Type I Error (False Alarm)
$\mathcal{H}_1$	Type II Error (Missed Detection)	Correct Decision (Detection)

$$P_D = p(d = \mathcal{H}_1 | \mathcal{H}_1)$$

$$P_F = p(d = \mathcal{H}_1 | \mathcal{H}_0)$$

#### Sensing Model



 $y = \theta \cdot g(\mathbf{x}; \mathbf{x}_T) + w$ 

- y is the **measurement** at the sensor
- $w \sim \mathcal{N}(0; \sigma_w^2)$  is the **noise** at the sensor
- x is the location of the sensor
- $\theta$  is the **intensity** of the target to be detected
  - Unknown and Deterministic:  $\theta \in \Omega_{\theta}$ 
    - e.g.  $\theta \in [-\theta_0, +\theta_0]$
  - Unknown and Stochastic:  $\theta \sim p(\theta)$ 
    - e.g.  $\theta \sim \mathcal{N}(0; \sigma_T^2)$
- $g(\cdot; \cdot)$  is the (distance-dependent) amplitude attenuation function (AAF) or spatial signature
- $x_T$  is the target location

#### Amplitude Attenuation Function (AAF)



- Comes from domain knowledge
- Represents the physical phenomenon and related propagation
- Common AAF with EM signals:
  - Exponential AAF

$$g^{2}(\mathbf{x};\mathbf{x}_{T}) = e^{-\frac{\|\mathbf{x}-\mathbf{x}_{T}\|^{2}}{\eta^{2}}}$$

- Power-Law AAF

$$g^{2}(\mathbf{x}; \mathbf{x}_{T}) = \frac{1}{1 + \frac{\|\mathbf{x} - \mathbf{x}_{T}\|^{2}}{\eta^{2}}}$$



#### Local Test



- Statistical Signal Model
  - $y|\mathcal{H}_0 \sim \mathcal{N}(0; \sigma_w^2) \qquad \qquad p(y|\mathcal{H}_0) = \frac{1}{\sqrt{2\pi\sigma_w^2}} e^{-\frac{y^2}{2\sigma_w^2}}$

$$- y|\mathcal{H}_1 \sim \mathcal{N}(0; \sigma_T^2 g^2(\mathbf{x}; \mathbf{x}_T) + \sigma_w^2)$$

$$p(y|\mathcal{H}_1) = \frac{1}{\sqrt{2\pi(\sigma_T^2 g^2(x_k; x_T) + \sigma_W^2)}} e^{-\frac{y^2}{2(\sigma_T^2 g^2(x; x_T) + \sigma_W^2)}}$$

Compute the LLR

$$- \lambda(y) = \ln\left(\frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)}\right) = \frac{\Gamma_s}{2} \frac{g^2(x;x_T)}{\sigma_T^2 g^2(x;x_T) + \sigma_w^2} y^2 + \frac{1}{2} \ln\left(\frac{1}{1 + \Gamma_s g^2(x;x_T)}\right) \qquad \Gamma_s \triangleq \frac{\sigma_T^2}{\sigma_w^2} \quad \text{sensing SNR}$$

- LRT is equivalent to Energy Test
  - $y^2 \gtrless \gamma$

#### Local Performance



- Assume fixed local FA probability
- Assume fixed AAF
- · Evaluate local detection probability vs target distance



• 
$$P_F = 2Q\left(\sqrt{\frac{\gamma}{\sigma_w^2}}\right)$$
  
•  $P_D = 2Q\left(\sqrt{\frac{\gamma}{\sigma_T^2 g^2(x;x_T) + \sigma_w^2}}\right)$ 

Local Performance

- Assume fixed local FA probability
- Assume fixed AAF
- · Evaluate local detection probability vs target distance
- Performance improves with sensing SNR

• 
$$P_F = 2Q\left(\sqrt{\frac{\gamma}{\sigma_w^2}}\right)$$
  
•  $P_D = 2Q\left(\sqrt{\frac{\gamma}{\sigma_T^2 g^2(x;x_T) + \sigma_w^2}}\right)$ 





#### Local Performance (ROC)



- Performance worsens with distance
- Performance improves with sensing SNR

• 
$$P_F = 2Q\left(\sqrt{\frac{\gamma}{\sigma_w^2}}\right)$$
  
•  $P_D = 2Q\left(\sqrt{\frac{\gamma}{\sigma_T^2 g^2(x;x_T) + \sigma_w^2}}\right)$ 





## (Wireless) Sensor Networks Part II – Distributed Detection and Localization

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#### MIMO Decision Fusion in WSNs





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- D. Ciuonzo, P. Salvo Rossi, "Distributed Detection of a Non-Cooperative Target via Generalized Locally-Optimum Approaches," *Elsevier Inform. Fusion* (2017)
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#### Sensor-Network Architecture



Global Test and Global Performance



## Counting Rule (CR)



- · Simple and intuitive strategy is to count the number of reported detections
  - $-\lambda = \sum_{k=1}^{K} d_k$
- Advantages
  - System knowledge not required (e.g. local performance, sensing SNR, etc.)
  - It is optimal in the case of homogeneous sensor networks

 $P_{F,k} = P_F$  and  $P_{D,k} = P_D$ 

- Disadvantages
  - Poor performance in practical scenarios of interest
  - No localization provided

## Ring Scenario





## Performance of CR in Ring WSNs



- Assume a WSN with K sensors
- · All sensors have the same distance from the target
- Performance improves with K



- Unrealistic assumption
  - if present the target is in known position
  - good approximation for large spreading factors

## Performance of CR in Ring WSNs



- Assume a WSN with K sensors
- · All sensors have the same distance from the target
- Performance improves with K
- Performance improves with sensing SNR

exp - K=10 0.9 power - K=10 exp - K=50 0.8 power - K=50 global detection probability  $(\mathbb{Q}_{\mathbf{D}})$ 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 0.5 2.5 15 2 З ٦Ū. distance (m)

- Unrealistic assumption
  - if present the target is in known position
  - good approximation for large spreading factors

#### **Randomly-Deployed Sensors**







#### Performance of CR in Random WSNs



- Assume a WSN with K sensors
- · Sensors are randomly generated in the sensor area
- Target (if present) is randomly generated in the target area
- Performance improves with sensing SNR
- Performance improves with  $\boldsymbol{\eta}$



### Optimum Rule - (Clairvoyant) LRT



Compute the LLR

$$- \lambda = \ln\left(\frac{p(\boldsymbol{d}|\mathcal{H}_1)}{p(\boldsymbol{d}|\mathcal{H}_0)}\right) = \sum_{k=1}^{K} \left[d_k \ln\left(\frac{P_{D,k}}{P_{F,k}}\right) + (1 - d_k) \ln\left(\frac{1 - P_{D,k}}{1 - P_{F,k}}\right)\right]$$

- Advantages
  - Optimum performance
- Disadvantages
  - Cannot be implemented in practice

Requires knowledge of both  $P_{F,k}$  and  $P_{D,k}$  which is unrealistic (because depending on  $x_T$  and  $\sigma_T^2$ )

## Generalized LRT (GLRT)



• Compute the LGLR using ML estimation

$$- \lambda = \ln\left(\frac{\max_{\boldsymbol{x}_T;\sigma_T^2} p(\boldsymbol{d}|\mathcal{H}_1;\boldsymbol{x}_T;\sigma_T^2)}{p(\boldsymbol{d}|\mathcal{H}_0)}\right) = \sum_{k=1}^K \left[d_k \ln\left(\frac{P_{D,k}(\widehat{\boldsymbol{x}_T};\widehat{\sigma_T^2})}{P_{F,k}}\right) + (1-d_k) \ln\left(\frac{1-P_{D,k}(\widehat{\boldsymbol{x}_T};\widehat{\sigma_T^2})}{1-P_{F,k}}\right)\right]$$

$$- \left(\widehat{\boldsymbol{x}_{T}}; \widehat{\sigma_{T}^{2}}\right) = \operatorname*{argmax}_{\boldsymbol{x}_{T}; \sigma_{T}^{2}} p(\boldsymbol{d} | \mathcal{H}_{1}; \boldsymbol{x}_{T}; \sigma_{T}^{2})$$

- Advantages
  - System knowledge not required (e.g. local performance, sensing SNR, etc.)
  - Excellent performance for both detection and localization tasks
- Disadvantages
  - Requires optimization procedure for ML estimation (e.g. grid search)

#### Randomly-Deployed Sensors









#### Performance of GLRT and CR in Random WSNs

- · Sensors are randomly generated in the sensor area
- Target (if present) is randomly generated in the target area
- · Performance improves with sensing SNR
- The improvement of GLRT wrt CR is apparent





#### **Alternative Fusion Rules**



- Bayesian approach
  - Bayesian LLR
- Locally Optimum Detection (LOD) approach
  - Generalized LOD (GLOD)
- Hybrid approach
  - Bayesian/GLLR
  - Bayesian/LOD



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