NTNU
Norwegian University of
Science and Technology

RAMS Seminar

18th February 2021

Mathematical Modeling for Remaining Useful Life Prediction

RUL prediction using Empirical Wavelet Transform (ongoing work)

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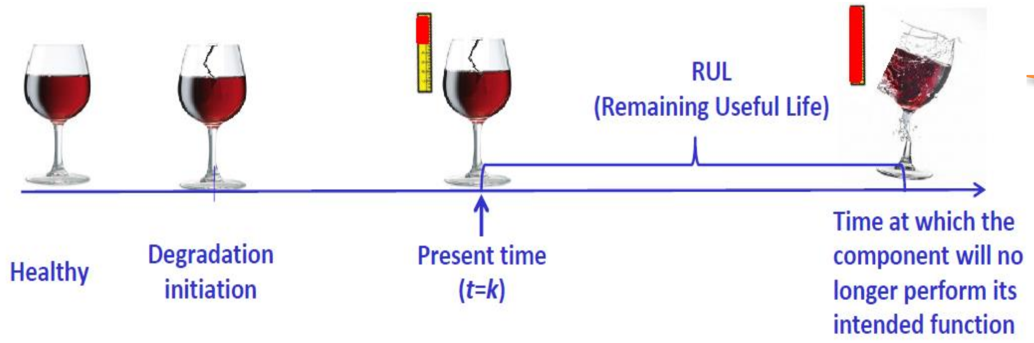
Start date: August 2019

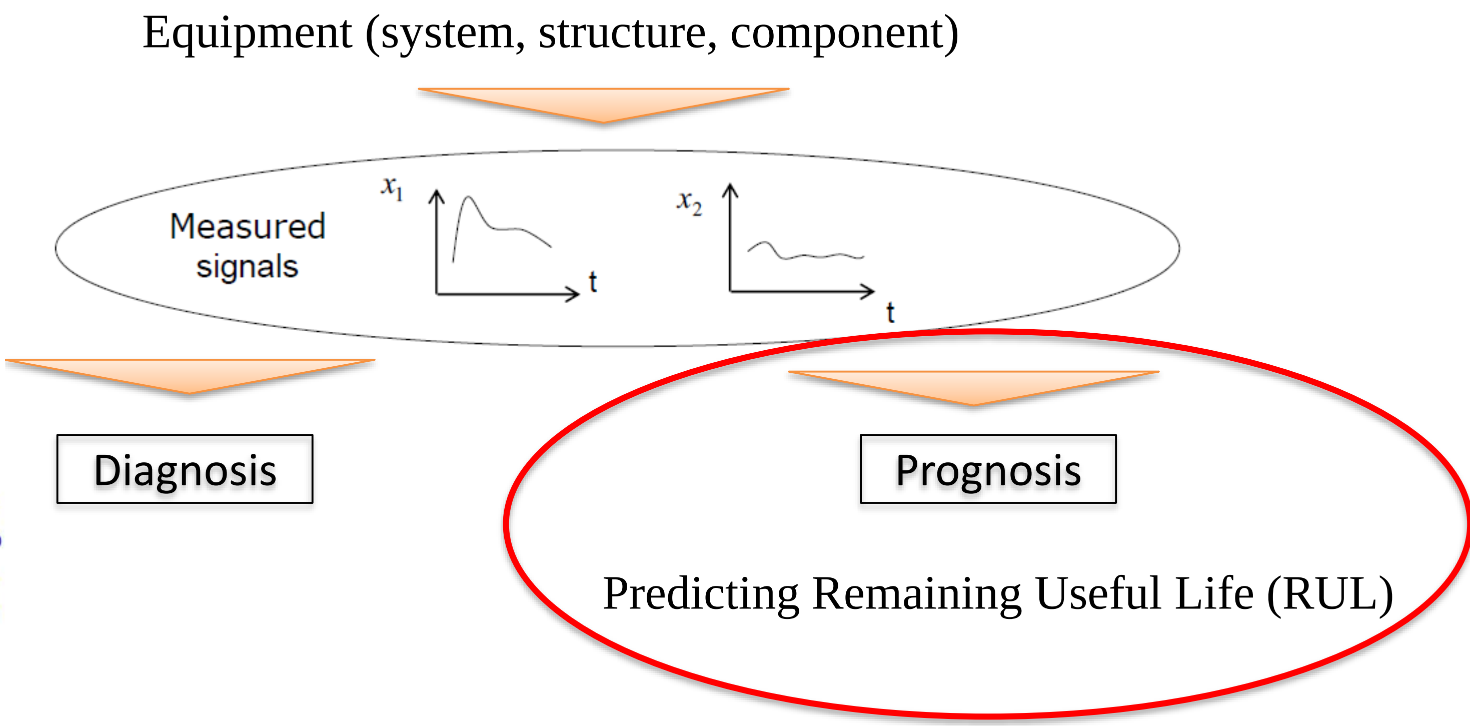
Agenda

- Introduction
- General Framework
- Experimental Vibration Setup and Data Description
- Feature Extraction and feature selection
- Modeling
- Plan for further work

Introduction and Background

- Roller bearings as critical components in rotating machinery
- Condition-monitoring of bearings
- Various parameters for diagnosis purpose (e.g., temperature, pressure, …), <u>vibration measurements</u>
- Mathematical modeling and RUL prediction of roller bearings





General Framework for RUL Prediction of Bearing

Collecting vibration data

Data collected at RAMS laboratory using two accelerometers

Preprocessing, Feature extraction and feature selection

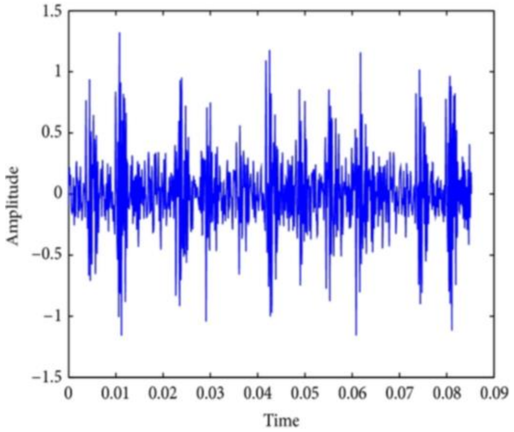
- Time-domain features
- Frequency-domain features
- Time-frequnecy representation techniques (TFR)

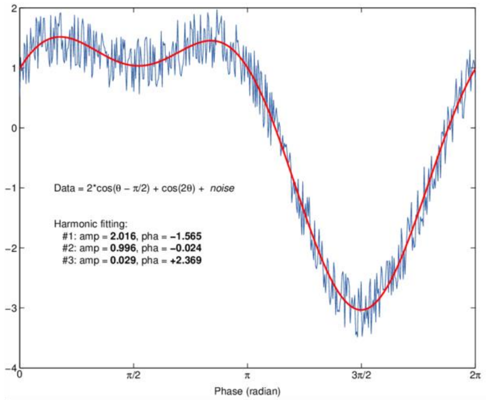
Feature trending and feature selection

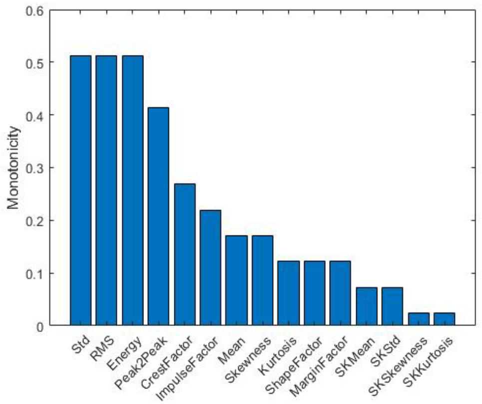
- PCA
- Correlation coefficient
- Monotonicity
- Trendability
- Prognosibility

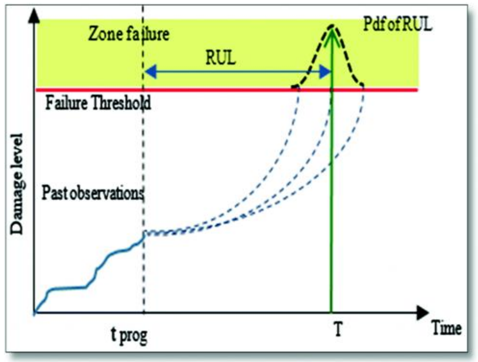
RUL prediction

Finding how much time is left until a failure occurs









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Experimental Vibration Setup

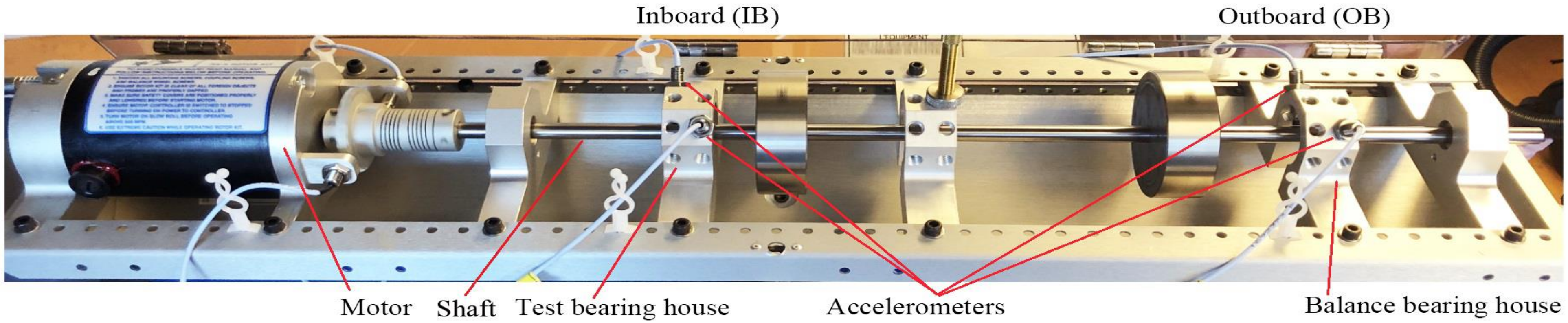


Fig. 1. Overview of the experimental setup

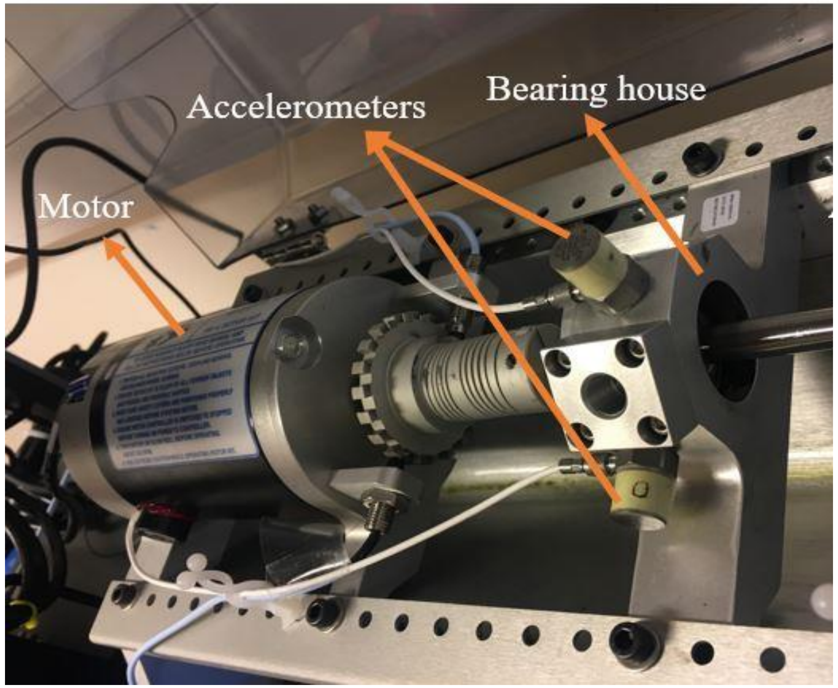


Fig. 2. Accelerometers, bearing house and motor

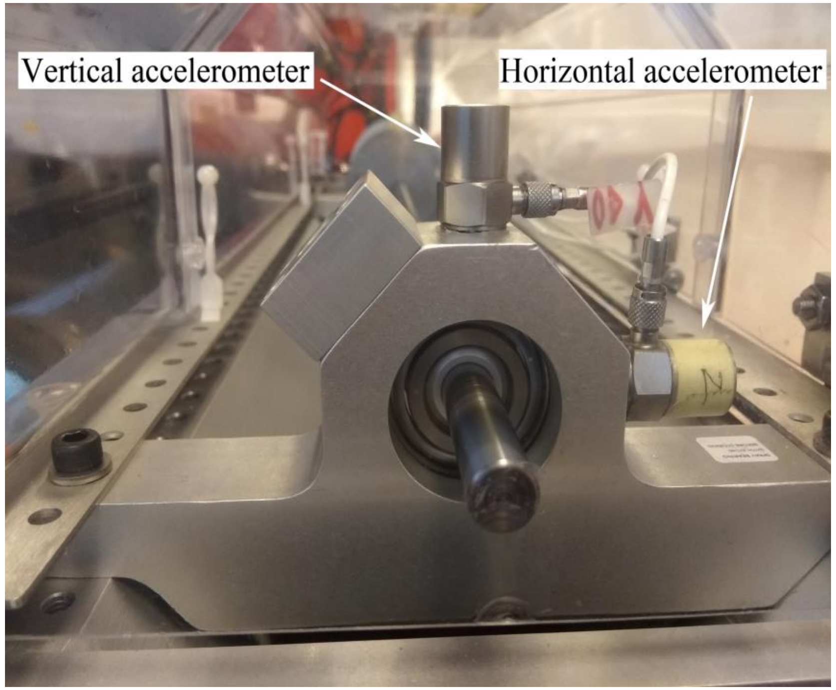


Fig. 3. Accelerometers

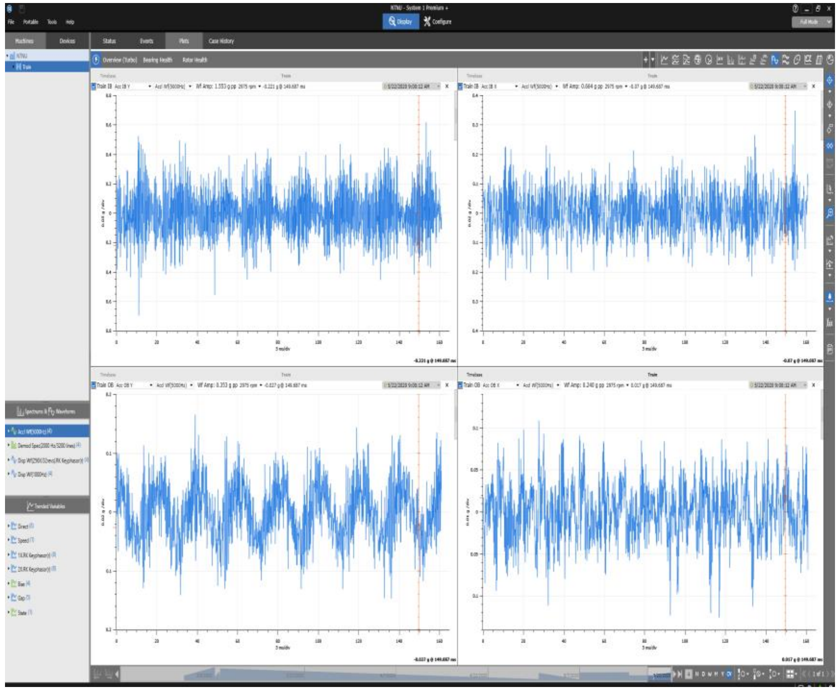


Fig. 4. System working page

Data Samples

- Bearing 1
- Bearing 2
- Bearing 3
- …
- Bearing 8
- Bearing 9

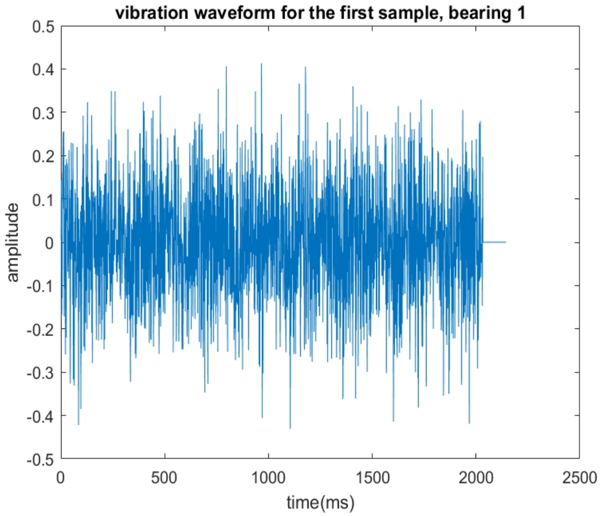


Fig. 5. The first sample (healthy bearing)

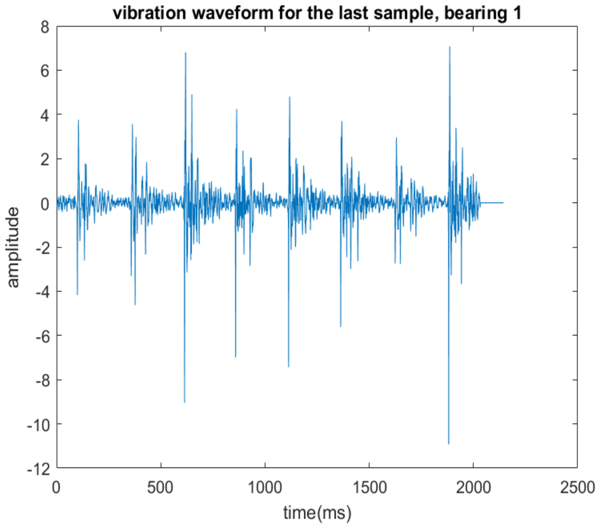
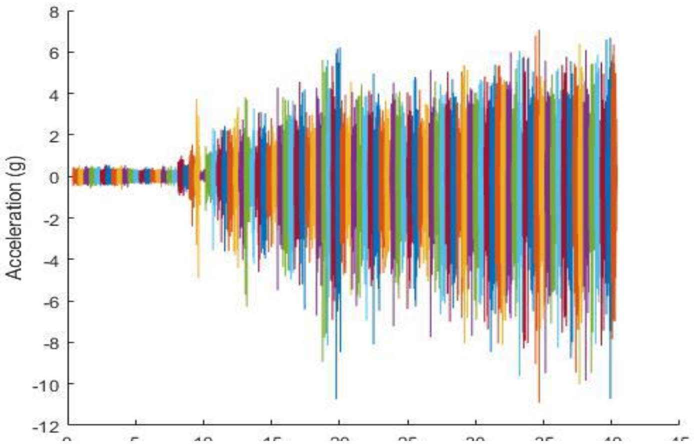


Fig. 6. The last sample (failed bearing)



Data Description

- Condition 1: Motor speed of 2975 rpm
- Condition 2: Motor speed of 3040 rpm
- Condition 3: Motor speed of 2000 rpm
- Condition 4: Motor speed of 1500 rpm

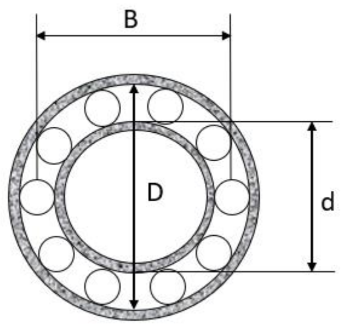


Table 1.Datasets

Operating Conditions				
	Condition 1	Condition 2	Condition 3	Condition 4
	Bearing 1-1	Bearing 2-1	Bearing 3-1	Bearing 4-1
	Bearing 1-2	Bearing 2-2		
	Bearing 1-3			
	Bearing 1-4			
Data sets	Bearing 1-5			
	Bearing 1-6			
	Bearing 1-7			
	Bearing 1-8			
	Bearing 1-9			

Table 2.Specifications of bearings

Data collected	Data collected
Number of balls	10 balls
Pitch diameter (B)	70 mm
Ball diameter	4.7 mm
Inner diameter (d)	15.9 mm
Outer diameter (D)	34.9 mm

General Framework

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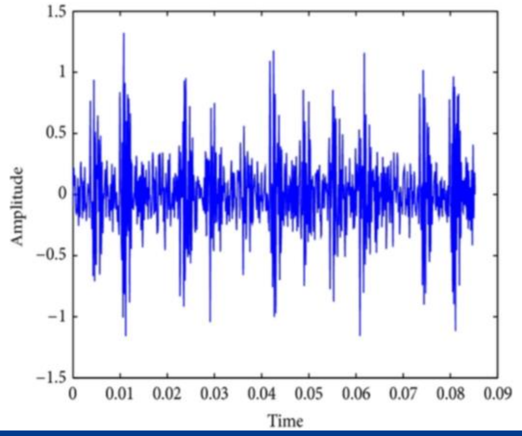
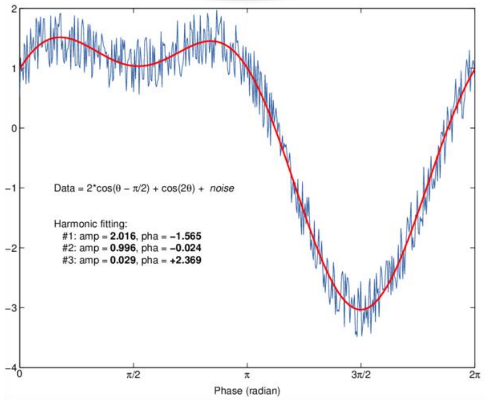
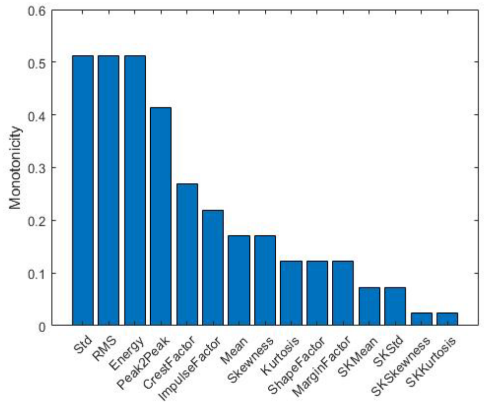
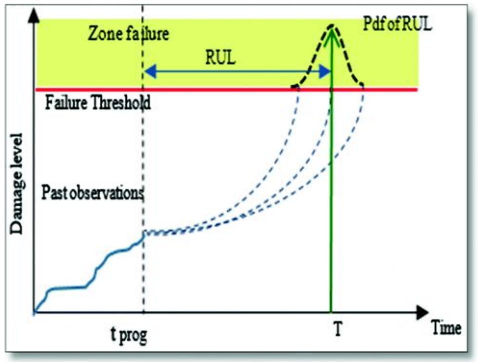
RUL prediction

Finding how much time is left until a failure occurs









Feature Extraction

Feature Extraction Methods

Time-domain features

- Kurtosis
- Root Mean Square (RMS)
- Mean
- Standard Deviation (SD)
- Crest Factor
- Shape Factor (SF)
- Impulse Factor (IF)
- Peak to Peak
- Margin Factor
- Skewness

Time-frequency representation techniques (TFR)

- Short-time Fourier Transform (STFT)
- Wavelet Transform (WT)
- Hilbert Haung Transform (HHT)

Frequency-domain features

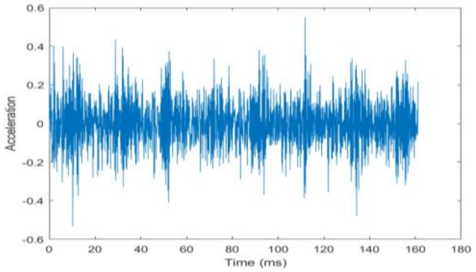
- Spectral kurtosis
- Spectral Entropy
- Shannon Entropy Feature
- Spectral skewness
- Frequency Center
- RMS Center

	Fourier	Wavelet	Hilbert
Nonlinear	No	No	Yes
Nonstationary	No	Yes	Yes
Mathematical theory	Yes	Yes	No

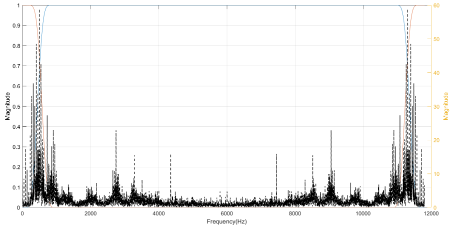
Empirical Wavelet Transform (EWT)

Empirical Wavelet Transform (EWT)

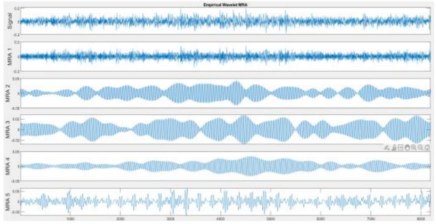
Step 1 — Vibration signal



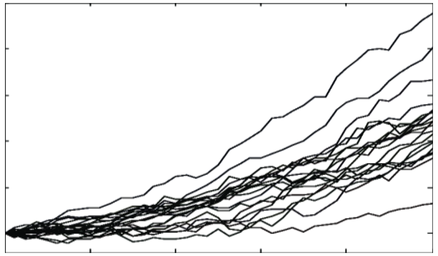
Step 2 — Taking the Fourier spectrum of the vibration signal



Step 3 — Decomposing signal into wavelet subbands

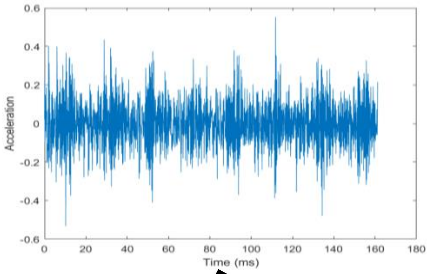


Step 4 — Extract statistical features from each subband

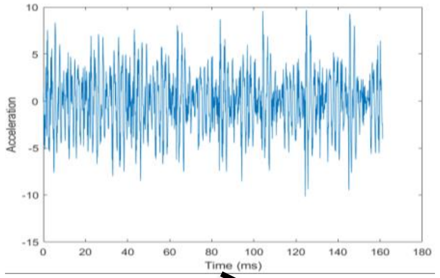


Empirical Wavelet Transform (EWT)

Step 1 and 2



Sample 1



Sample n

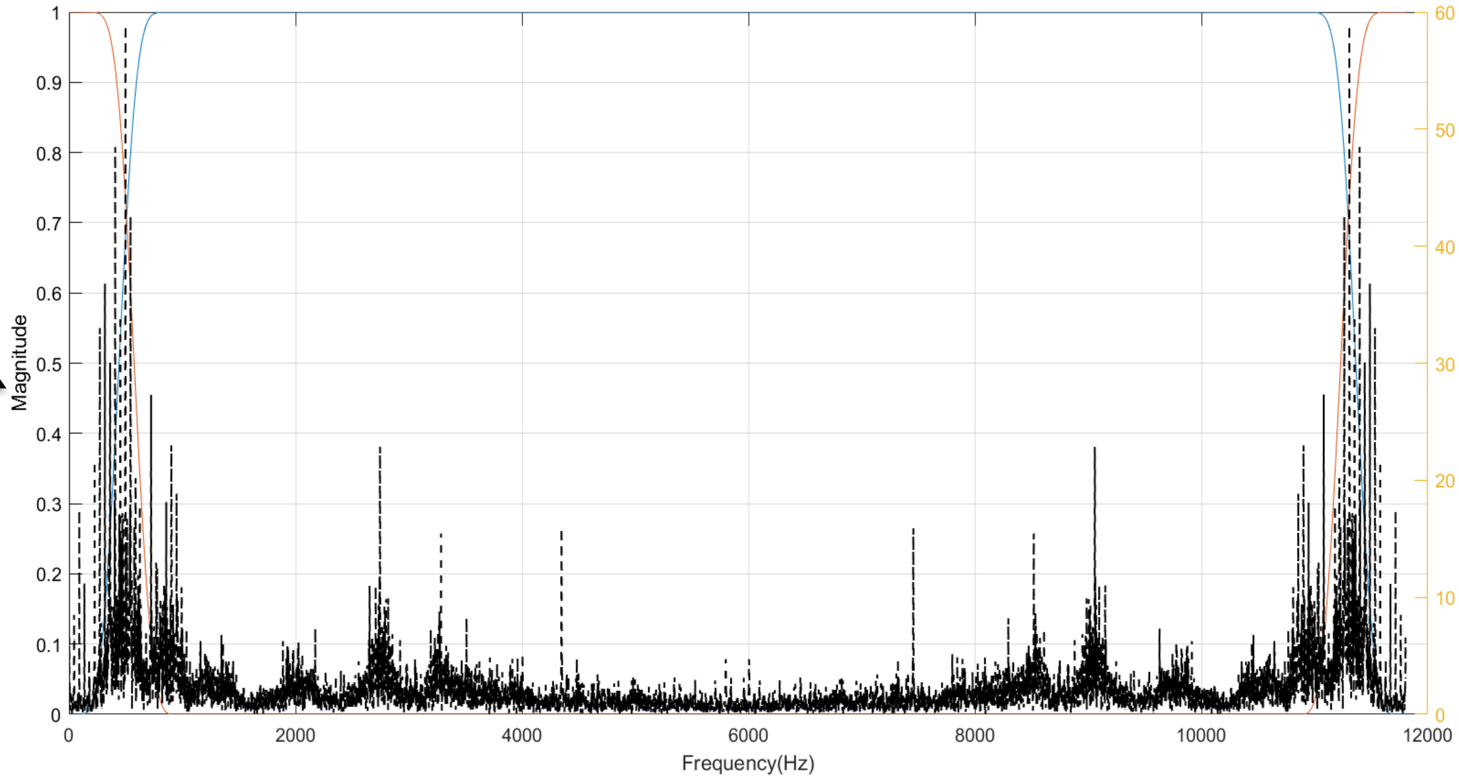
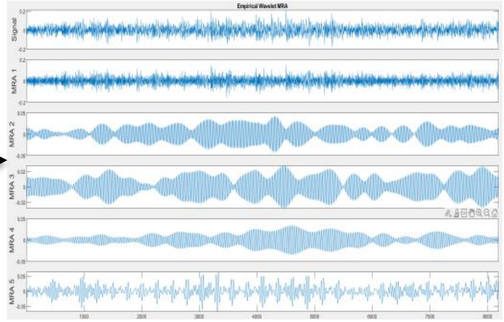
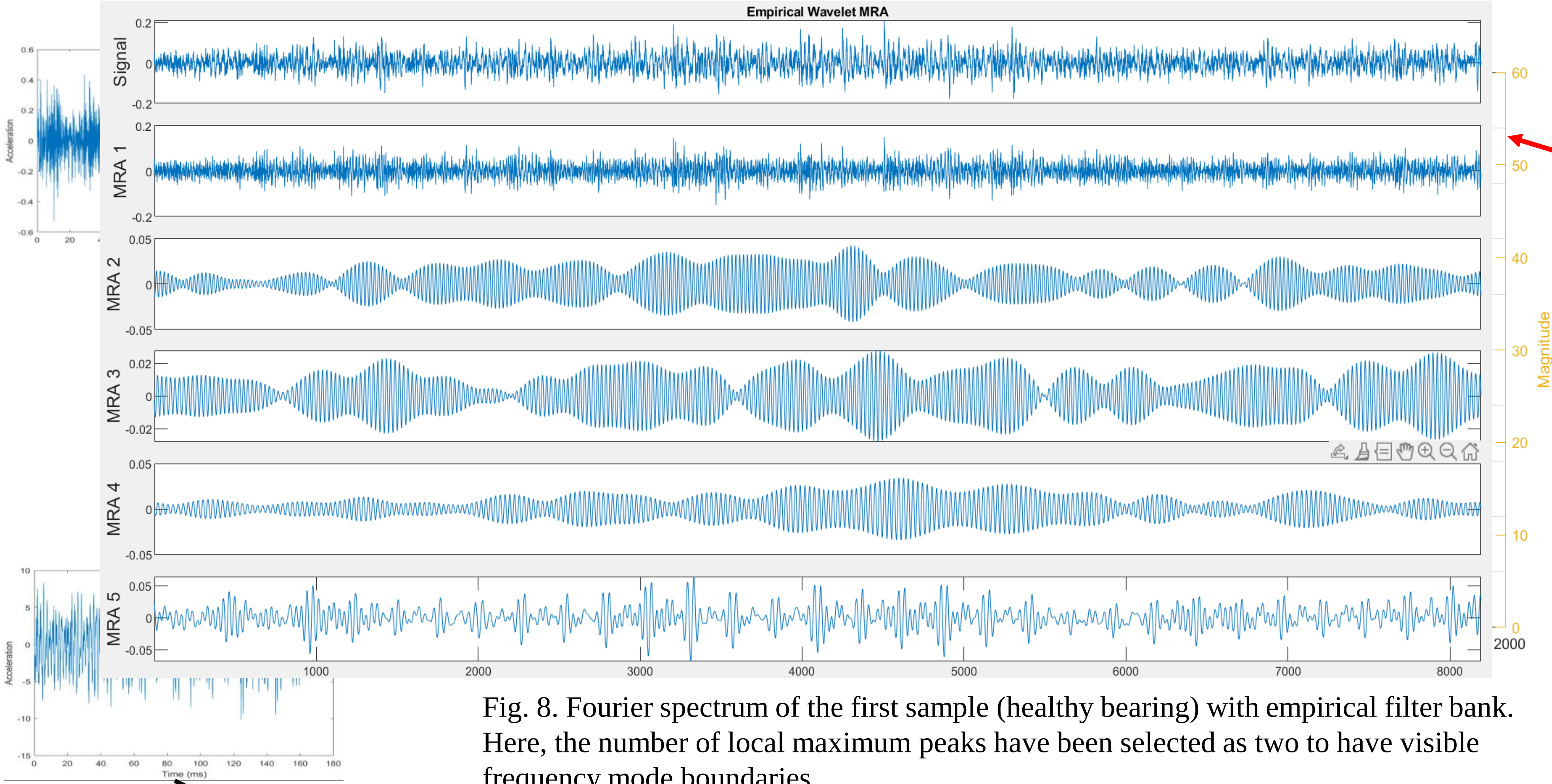


Fig. 8. Fourier spectrum of the first sample (healthy bearing) with empirical filter bank. Here, the number of local maximum peaks have been selected as two to have visible frequency mode boundaries



Empirical Wavelet Transform (EWT)

Step 3



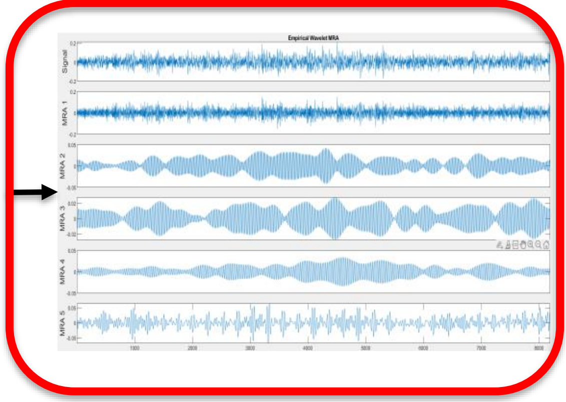


Fig. 8. Fourier spectrum of the first sample (healthy bearing) with empirical filter bank. Here, the number of local maximum peaks have been selected as two to have visible frequency mode boundaries

Sample n

Band-pass filters (Mode frequencies)

Step 4

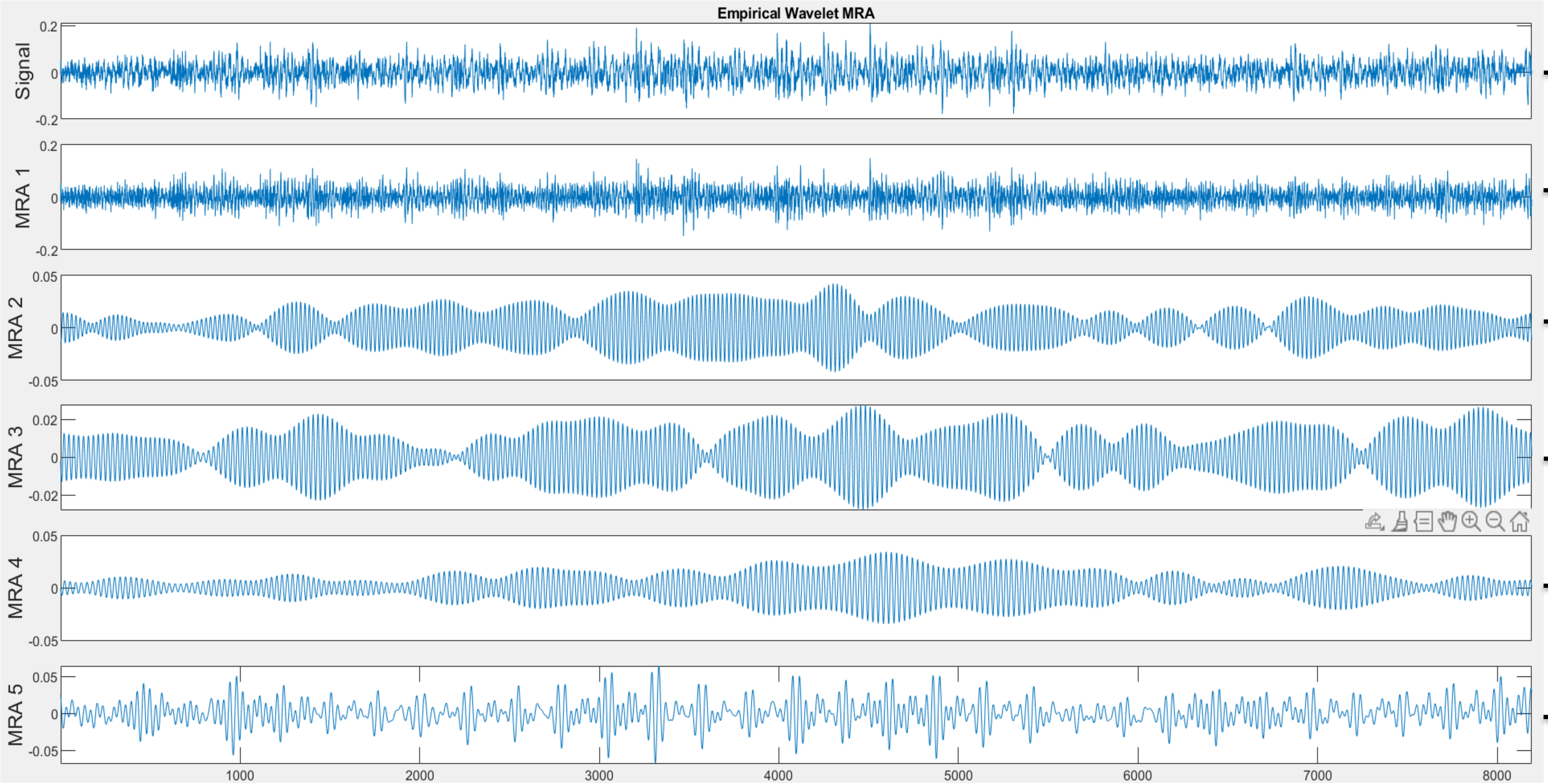


Fig. 9. Mode frequencies (MFs) of the first sample (healthy bearing)

Original signal
Sample number 1

- Kurtosis
- RMS
- Energy
- …

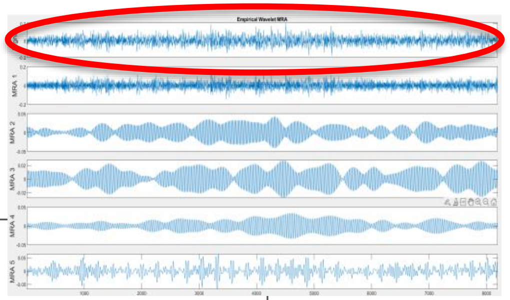
- Kurtosis
- RMS
- Energy
- …

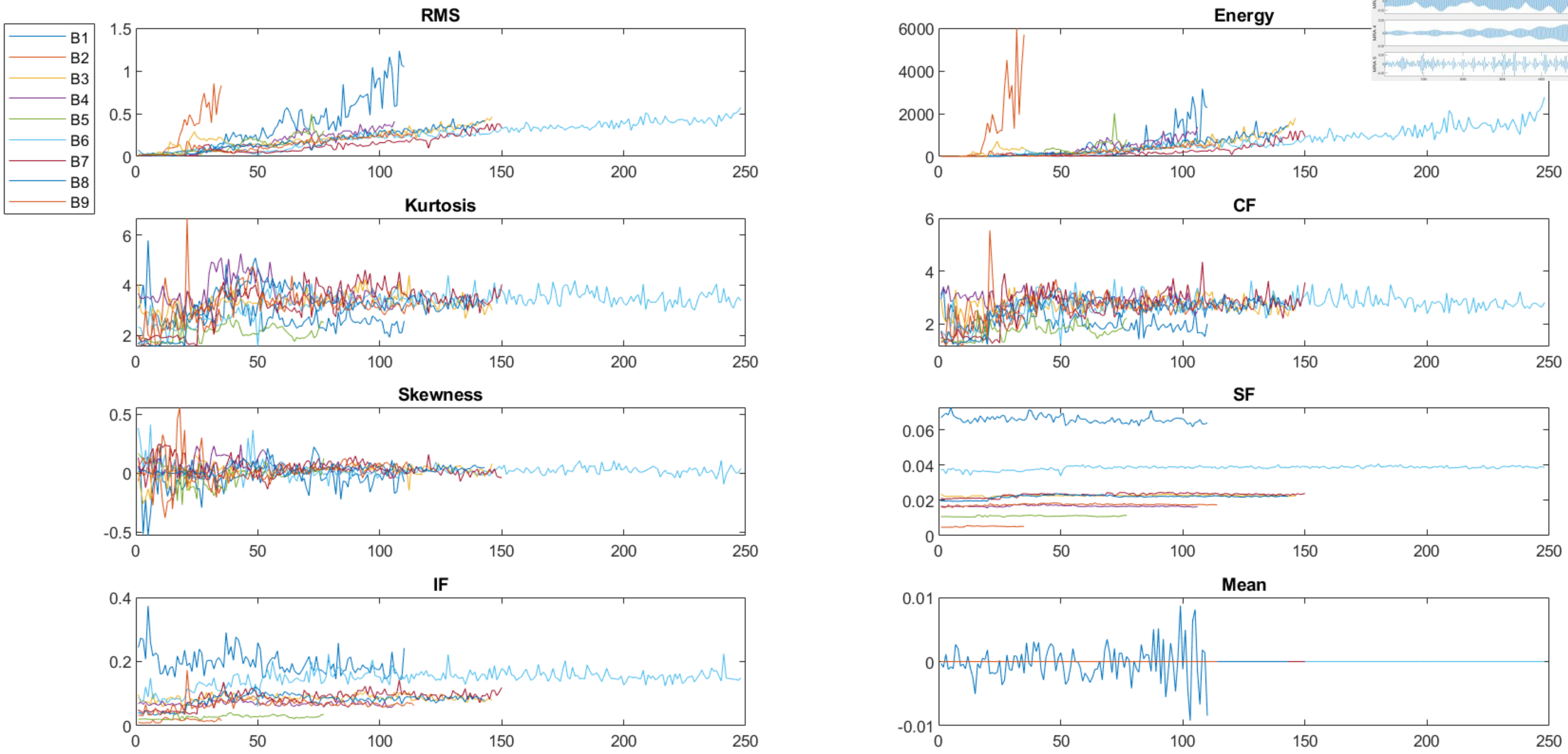
- Kurtosis
- RMS
- Energy
- …

- Kurtosis
- RMS
- Energy
- …

- Kurtosis
- RMS
- Energy
- …

Feature trending and feature selection





Statistical features for bandpass 1

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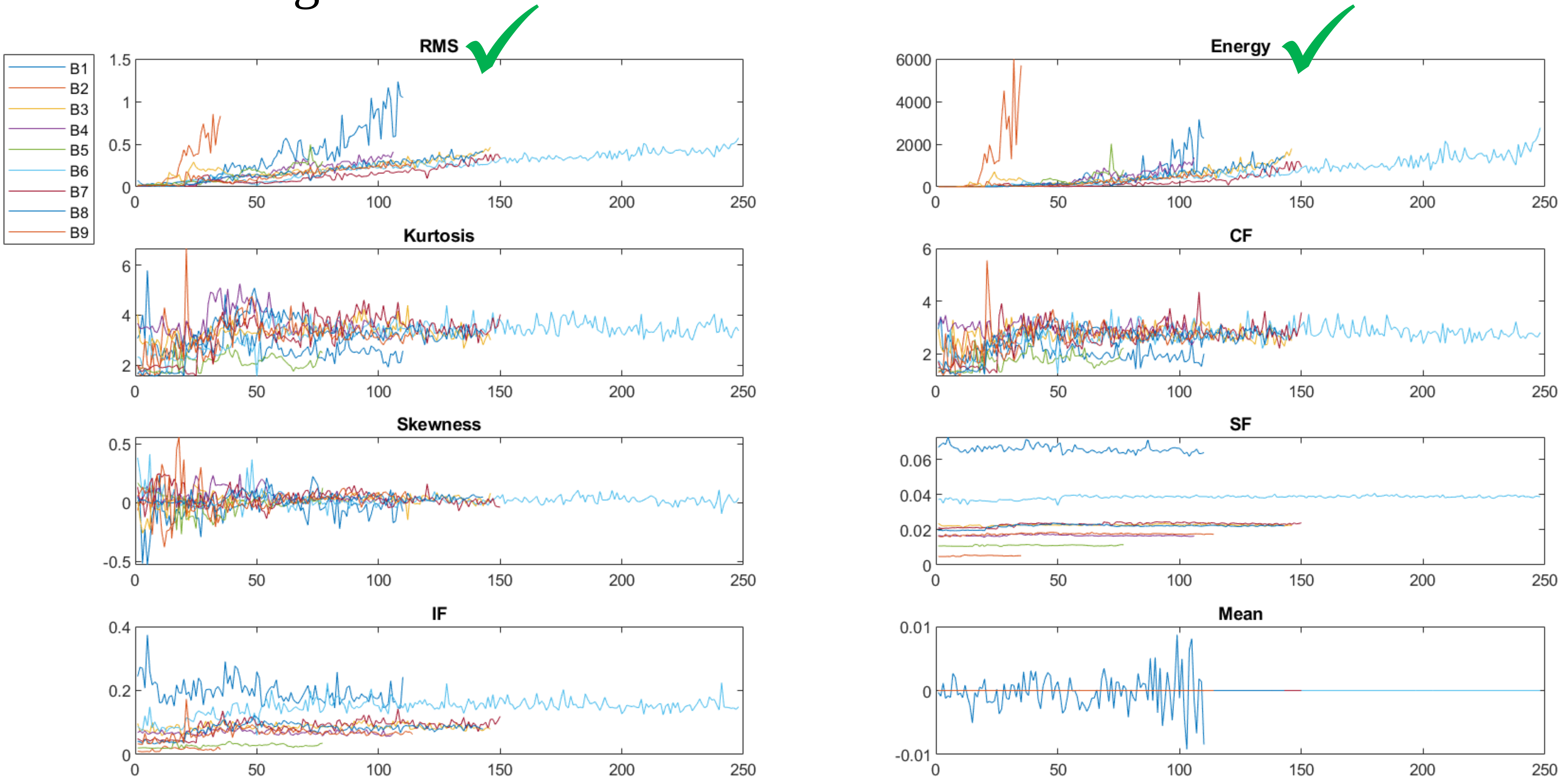
Correlation Coefficients with linear regression

$$\rho = \frac{\sum_{i=1}^{n}(A_i - \mu_A)(B_i - \mu_B)}{\sqrt{\sum_{i=1}^{n}(A_i - \mu_A)^2 (B_i - \mu_B)^2}}$$

Horizontal		B1	B2	B3	B4	B5	B6	B7	B8	B9
Bandpass1	RMS	0.9340	0.8759	0.8965	0.9290	0.8723	0.9384	0.8974	0.9789	0.9550
Bandpass2		0.8816	0.9312	0.6873	0.6668	0.8841	0.5717	0.6942	0.8291	0.6536
Bandpass3		0.8098	0.9275	0.7275	0.6764	0.8247	0.4973	0.7065	0.9052	0.8288
Bandpass4		0.7977	0.7903	0.7385	0.6119	0.8396	0.7431	0.8558	0.9159	0.7934
Bandpass5		0.9010	0.9217	0.8894	0.9594	0.8815	0.9576	0.9176	0.9707	0.9318
Bandpass1	Energy	0.7935	0.6754	0.8817	0.8365	0.7528	0.8930	0.7552	0.9322	0.9186
Bandpass2		0.7612	0.7933	0.6173	0.4920	0.7287	0.3802	0.5128	0.7333	0.4600
Bandpass3		0.5436	0.7883	0.6667	0.4172	0.6799	0.2834	0.5615	0.8108	0.7410
Bandpass4		0.5579	0.5622	0.6735	0.3593	0.7115	0.5740	0.7339	0.8179	0.6753
Bandpass5		0.7488	0.8119	0.8506	0.9033	0.7247	0.9279	0.8078	0.9124	0.8921

Feature trending and feature selection



Degradation Trajectories

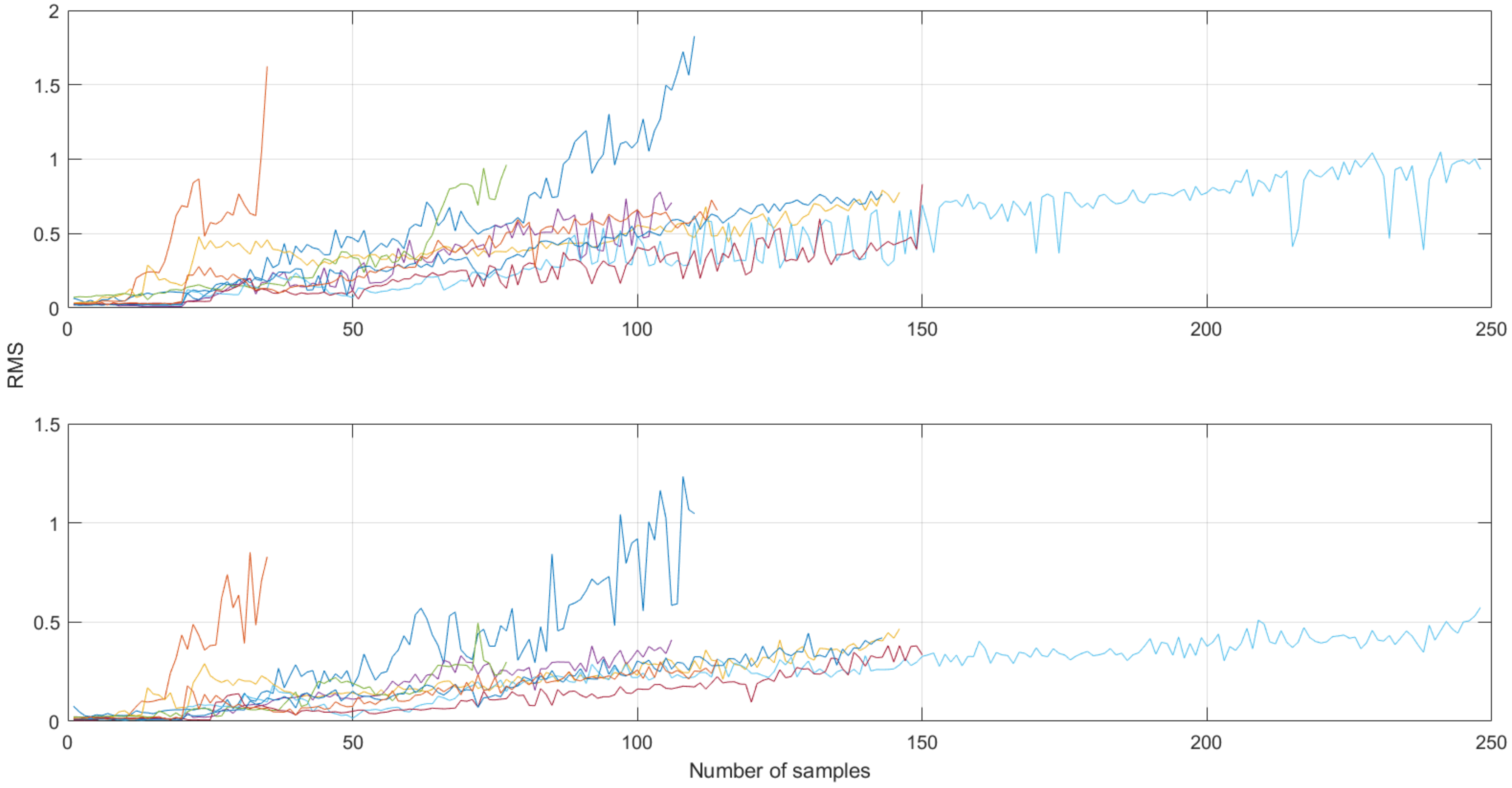


Fig. 11. Selected degradation trajectories. The blue line is the first EWT passband and the red line is the fifth EWT passband

Health stage division

Simple Algorithm:

1. Pick the first RMS window of size n $(P_1,\ P_2,\ \ldots,\ P_{n-1},\ P_n)$
2. Fit a linear regression model on n-size RMS values $(RMS = wt + b)$
3. Calculate the gradient of the fitted linear regression model $\left(w = \frac{\sum t_i RMS_i - \frac{\sum t_i RMS_i}{n}}{\sum t_i^2 - \frac{(\sum t_i)^2}{n}}\right)$

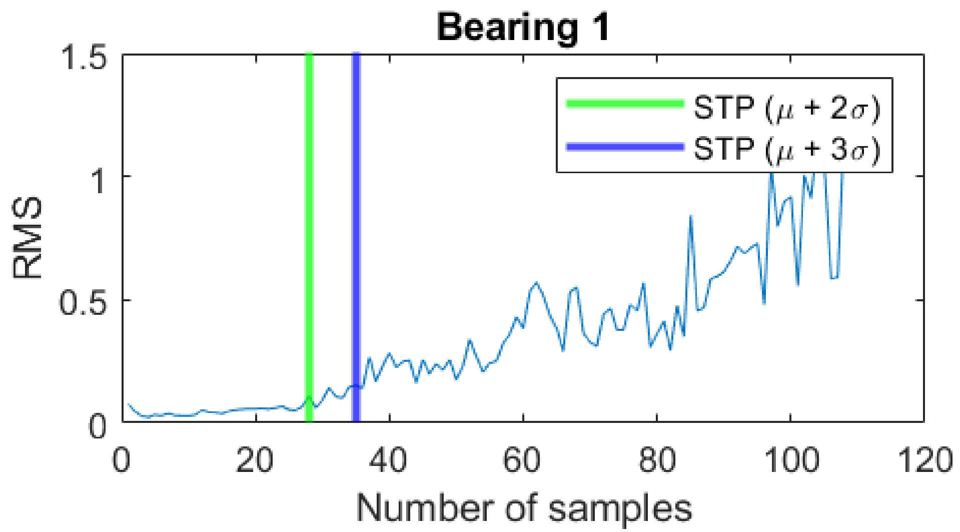
if $w_i > w_{th}$

$STP = P_n$

Else

$n = n + 1$

1. Repeat from step 2



Health stage division

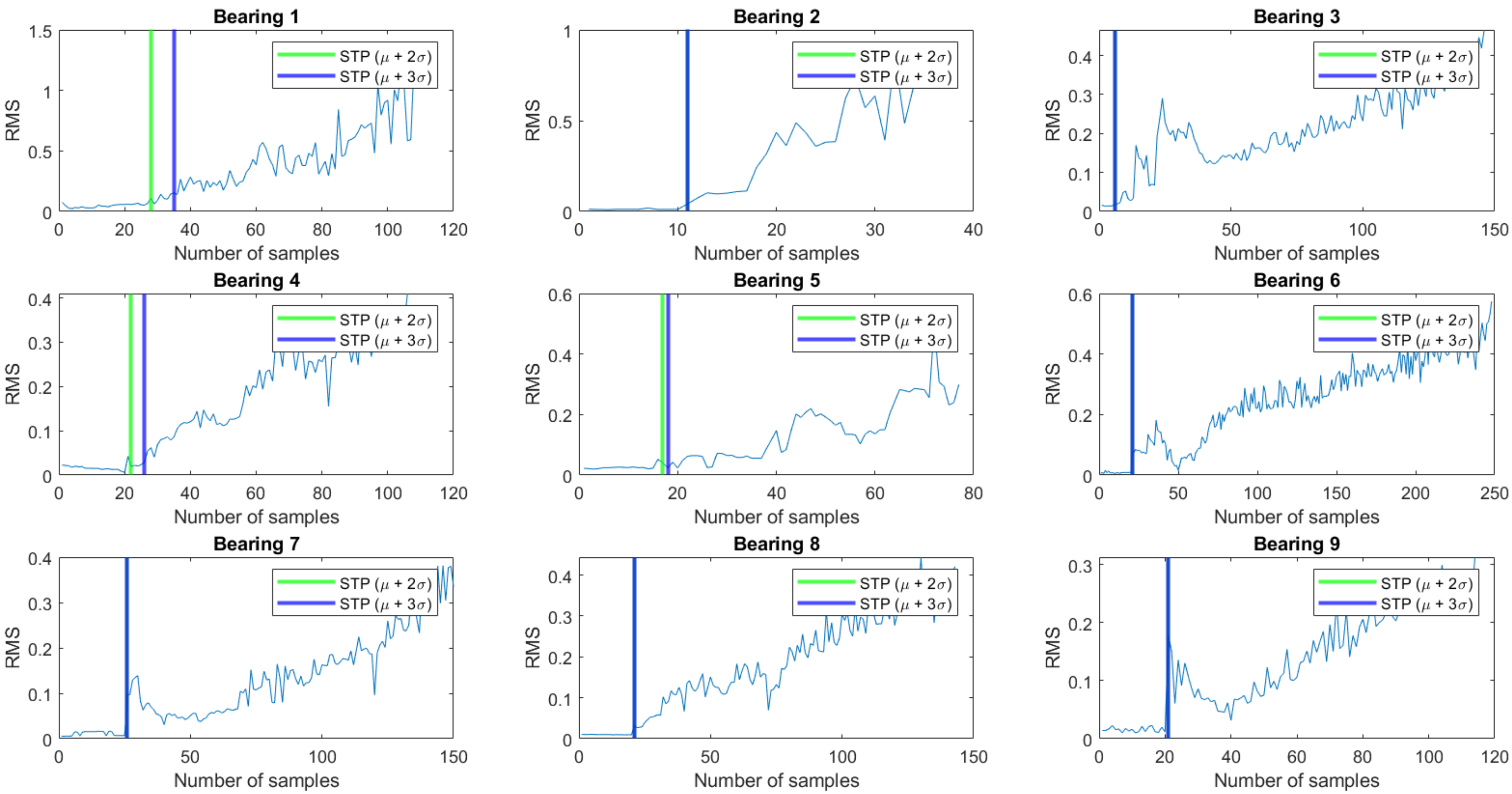


Fig. 11. Starting degradation point of RMS degradation paths

General Framework

Collecting vibration data → Preprocessing, Feature extraction and feature selection → Feature trending and feature selection → RUL prediction

Collecting vibration data

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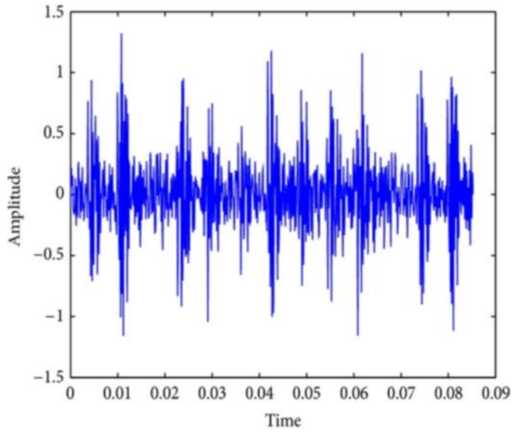
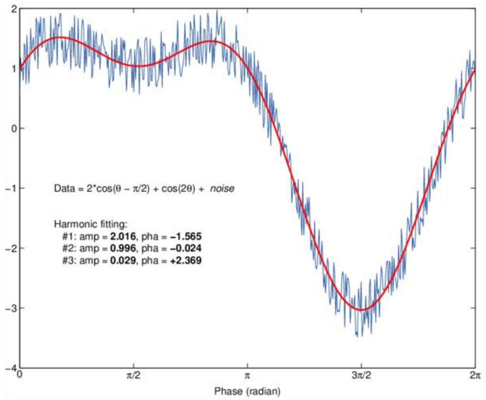
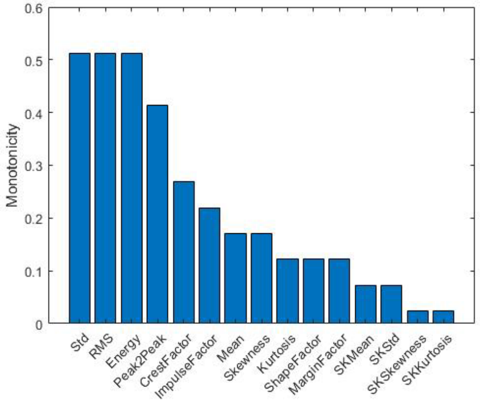
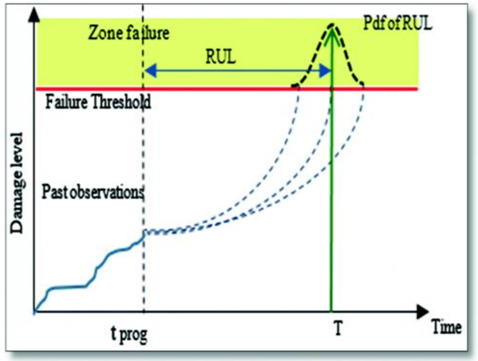
RUL prediction

Finding how much time is left until a failure occurs









Modeling

- Wiener process with linear drift

$$X_t = X_0 + \eta t + \sigma_B B(t)$$

  - $X_0 = 0$
  - X has random independent increments
  - X_t is continuous-time non-monotonic stochastic process following Normal distribution with mean ηt and variance $\sigma_B^2 t$.
  - “First Passage Time (FPT)” distribution: Inverse Gaussian Distribution

$$\mu = \frac{\omega - y_t}{\eta}$$

$$\lambda = \frac{(\omega - y_t)^2}{\sigma_B^2}$$

Bayesian Approach for RUL prediction

Prior belief from analysis of similar systems → Getting evidence from the actual system under consideration → Updating prior belief to obtain posterior belief

- Bearing 1 → $\mu_1, \delta_1, \eta_1 = \frac{1}{{\delta_1}^2}$
- Bearing 2 → $\mu_2, \delta_2, \eta_2 = \frac{1}{{\delta_2}^2}$
- .
- .
- .
- Bearing 8 → $\mu_8, \delta_8, \eta_8 = \frac{1}{{\delta_8}^2}$

$$\mu_0 = \frac{1}{m}\sum \mu_i$$

$$p_0 = \frac{m}{\sum(\mu_i - \mu_0)^2}$$

$$\kappa_0 = \frac{p_0\, \beta_0}{(\alpha_0 - 1)}$$

η_i → Standard MLE → $\sim Gamma\ distribution\ (\alpha_0, \beta_0)$

- Bearing 9 → Posterior distribution is Normal-Gamma distribution

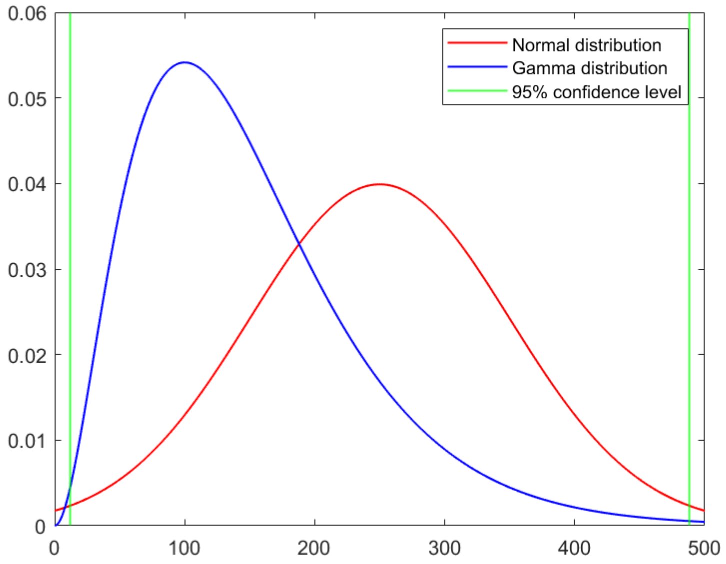
$$\kappa_n = \kappa_0 + n \qquad \alpha_n = \alpha_0 + \frac{n}{2} \qquad \beta_n = \beta_0 + \frac{1}{2}\sum_{i=1}^{n}(x_i - \bar{x})^2 + \frac{\kappa_0 n(\bar{x} - \mu_0)^2}{2(\kappa_0 + n)} \qquad \mu_n = \frac{\kappa_0\, \mu_0 + n\bar{x}}{k_0 + n}$$

$$\mu, \eta | y \sim G\left(\alpha_0 + \frac{n}{2}, \beta_0 + \frac{1}{2}\sum_{i=1}^{n}(x_i - \bar{x})^2 + \frac{\kappa_0 n(\bar{x} - \mu_0)^2}{2(\kappa_0 + n)}\right) N\left(\frac{\kappa_0\mu_0 + n\bar{x}}{\kappa_0 + n}, \frac{1}{(\kappa_0 + n)\eta}\right)$$

Numerical Solution

$$\Pr(\mathrm{RUL}(t|y_t,\mu_n,\kappa_n,\alpha_n,\beta_n)\le\tau)=\int_0^{\infty}\int_{-\infty}^{\infty}F_{IG}(\tau|\nu,\lambda)\ f_N\left(\mu\middle|\mu_n,\frac{1}{\kappa_n\boldsymbol{\eta}}\right)\ f_{GA}(\boldsymbol{\eta}|\alpha_n,\beta_n)\ d\mu\ d\eta$$

- ❑ $F_{IG}(\tau|\nu,\lambda)$: CDF of Inverse − Gauss distribution
  - $\mu=\frac{\omega-y_t}{\eta}$
  - $\lambda=\frac{(\omega-y_t)^2}{\sigma_B^2}=(\omega-y_t)^2\ \eta$
- ❑ $f_N\left(\mu\middle|\mu_n,\frac{1}{\kappa_n\eta}\right)$: PDF of Normal distribution
- ❑ $f_{GA}(\eta|\alpha_n,\beta_n)$: PDF of Gamma distribution
- ❑ Calculating PDF and CDF of RUL for every sample



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Numerical Solution

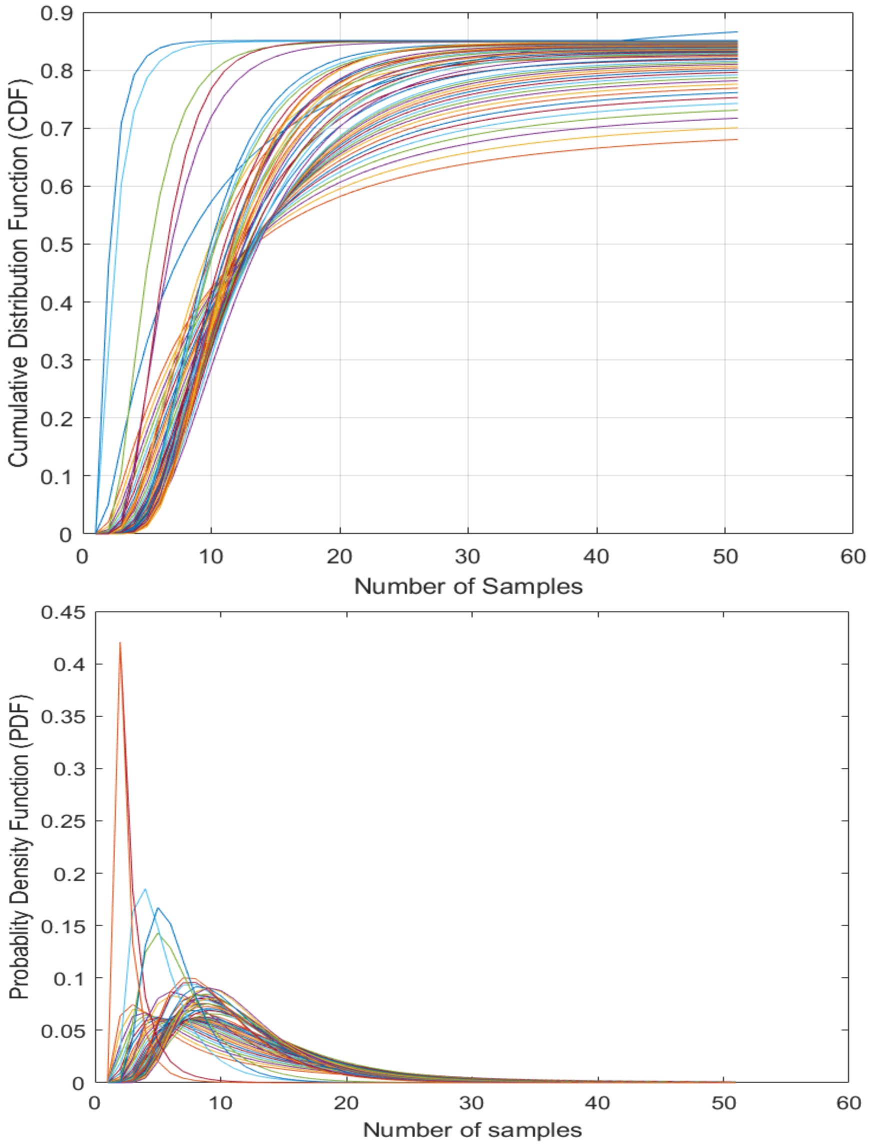

Fig. 11. Unconditional PDF and CDF of RUL

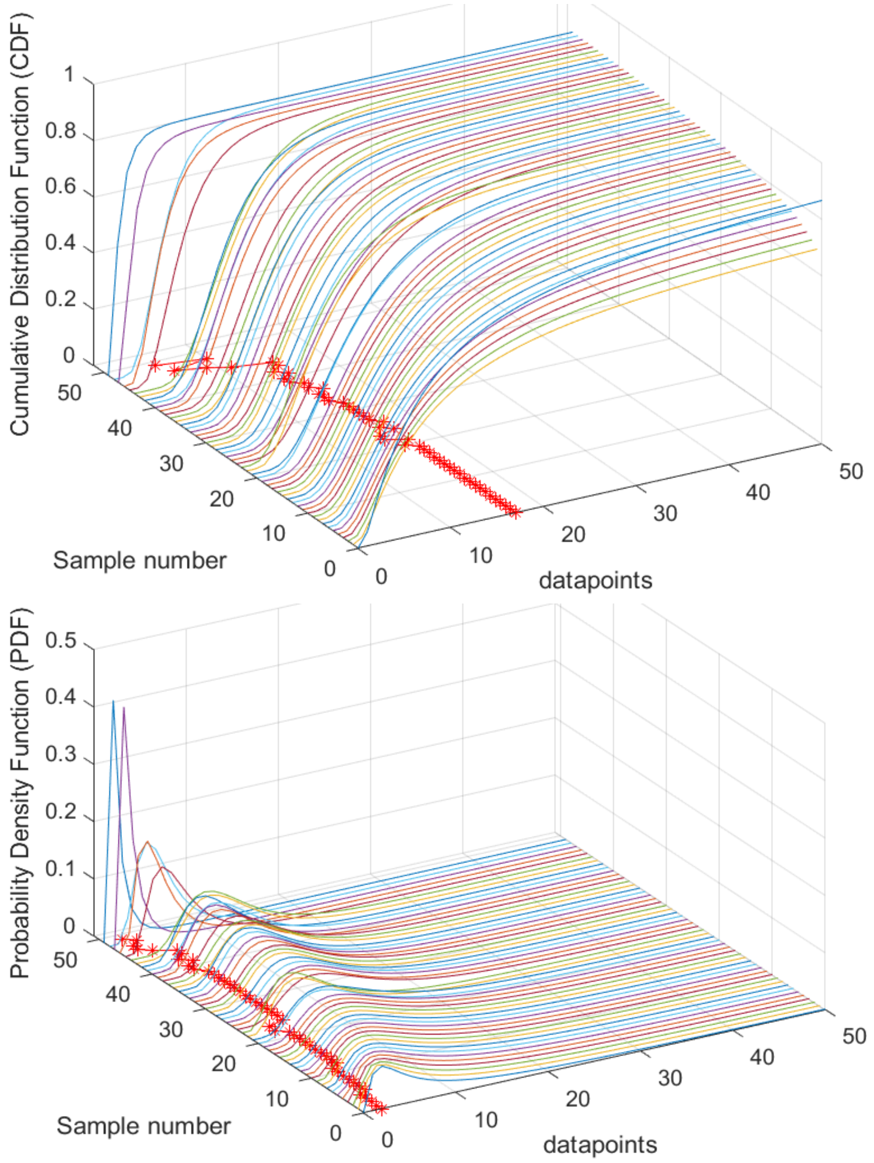

Fig. 12. 3D line plot of CDF and PDF of RUL at different samples. The mean values are connected by red dots

Further work

- Prognostics performance metrics
  - Accuracy

$$score = \frac{1}{n}\sum_{i=1}^{n} A_i$$

$$A_i = \begin{cases} \exp\left(-\ln(0.5) \times \frac{Er_i}{5}\right) & Er_i \leq 0 \\ \exp\left(+\ln(0.5) \times \frac{Er_i}{20}\right) & Er_i > 0 \end{cases}$$

$$Er_i = \frac{ActRUL_i - \widehat{RUL_i}}{ActRUL_i}$$

  - Prediction error: The root mean square relative error (RMSRE) of predicted and real lifetime

$$RMSE = (\frac{1}{n}\sum_{i=1}^{n}(\frac{L_i - \hat{L}_i}{L_i})^2)^{1/2}$$

  - Cumulative relative accuracy
  - Prognostics horizon
  - Convergence
  - $\alpha - \lambda$ accuracy

Further work

- Monotonicity, trendability and prognosability

$$Monotonicity = mean(\left|\frac{positive\ diff\ (x_i) - negative\ diff(x_i)}{n-1}\right|)$$

$$Trendability = \min\left(\left|corrcoef\ \left(x_i, x_j\right)\right|\right)\quad i,j = 1,2,\ldots,m$$

$$Prognosability = \exp(-\frac{std(failure\ values)}{mean(|failure\ value\ - starting\ value|)})$$

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Thank you!