Prognostics and health management of safety-instrumented systems

— Approaches of degradation modeling and decision-making

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Outlines

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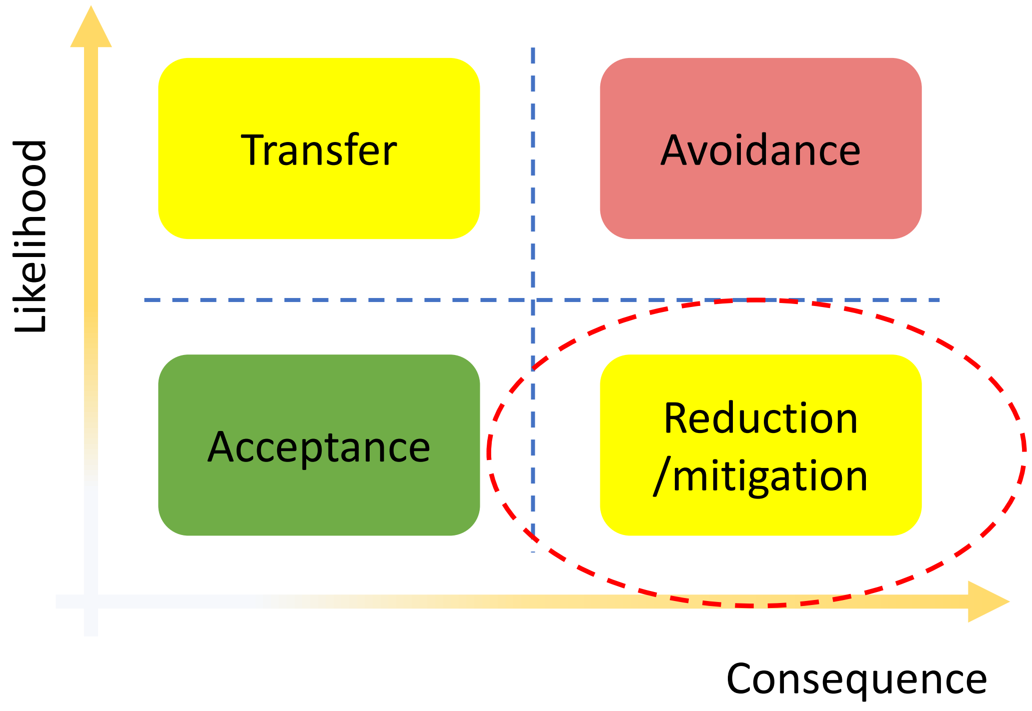
Research motivation

Risk?



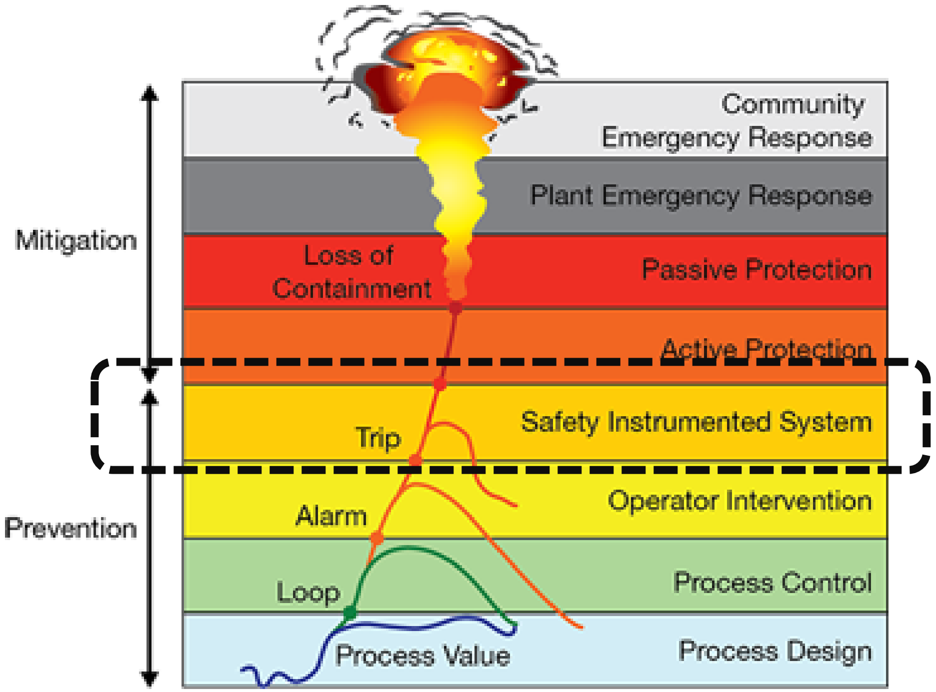
Deepwater Horizon oil spill,2010

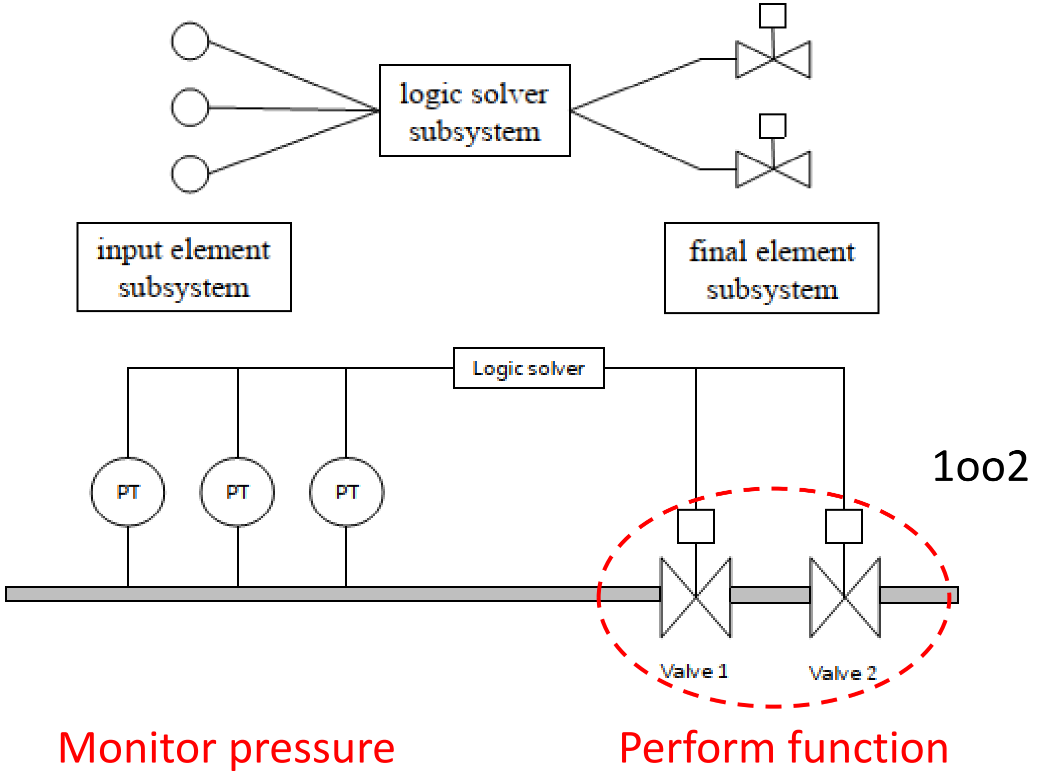
Fukushima Daiichi Accident,2011

* https://www.google.com

Layer of protection

Example of safety barriers in process industry





High-Integrity Pressure Protection System

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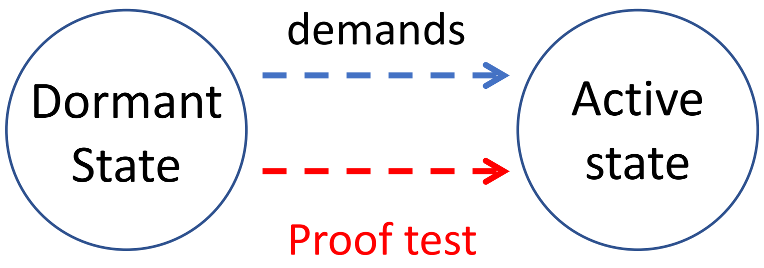
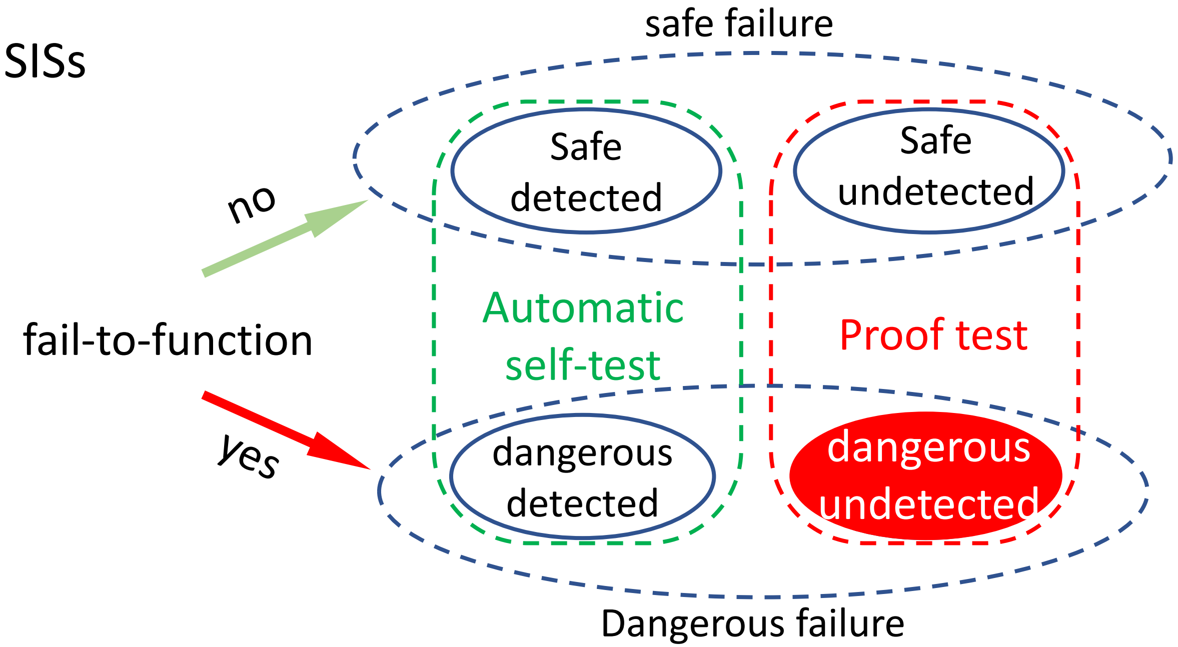
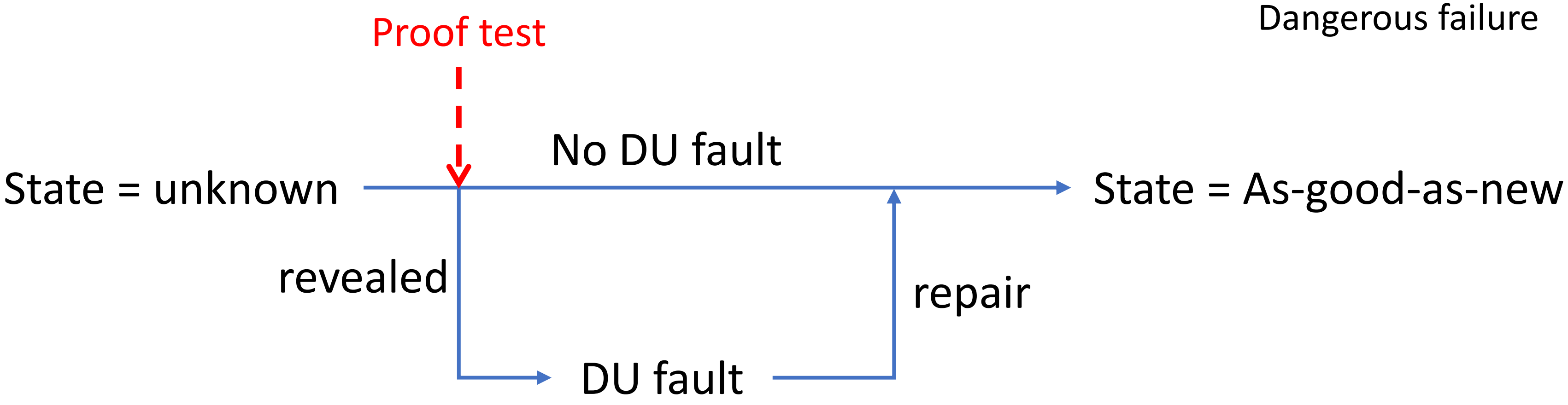
Safety-instrumented systems (SISs)

Operational modes and failure modes of SISs

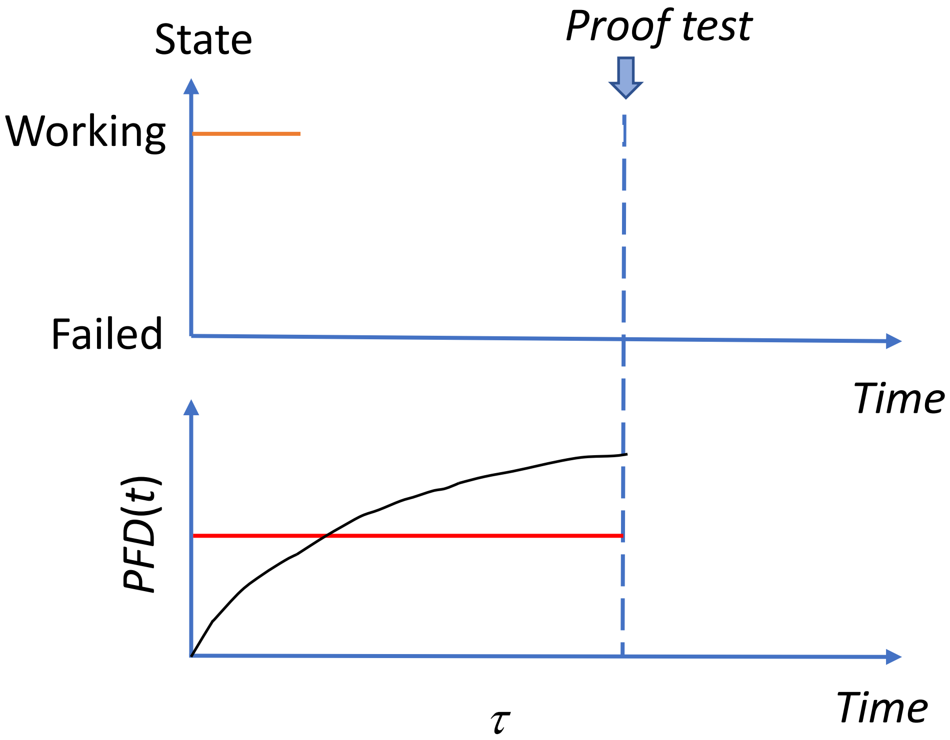






Performance measurement – Binary states

PFD_{avg}: average probability failure on demand in each test interval is used to quantify the reliability of SIS.



$$\text{PFDavg} = \frac{1}{\tau}\int_0^{\tau} PFD(t)dt$$

Test interval

Failure probability at time *t*

$$\text{PFDavg} = \frac{E[D(0,\tau)]}{\tau}$$

Research questions

Testing and maintenance policy

Time-based*

- The predefined proof tests
- Binary system state
- Failure rate- based assessment methods

Gap analysis

Key steps to bridge gap

1. System degradation modeling
2. Decision-making for the upcoming tests with collected information.

Performance-based

- Performance-based updating tests
- Time-dependent performance
- Degradation modeling

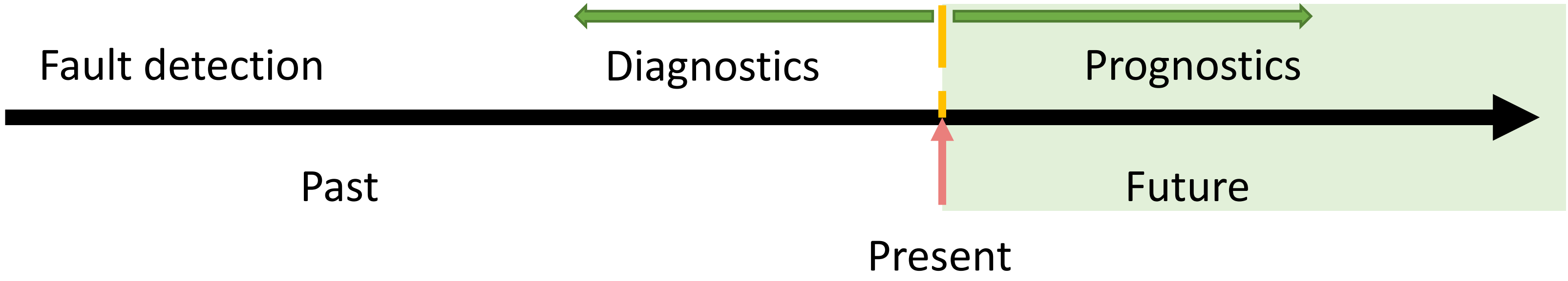
* IEC61508 Functional Safety of Electrical/Electronic/Programmable Electronic Safety-related Systems (E/E/PE, or E/E/PES)
IEC61511 Functional safety - Safety instrumented systems for the process industry sector

Prognostics and health management (PHM)

PHM can used for*:

1. evaluating the reliability of systems of their life cycle;
2. determining the possible occurrence of failures and risk reduction;
3. highlighting the residual useful lifetime(RUL) estimation.



PHM architectures**

*Haddad, Gilbert , et al. "An options approach for decision support of systems with prognostic capabilities." IEEE Transactions on reliability 61.4 (2012): 872-883.
**Ibrahim, Mesfin Seid, et al. "Machine Learning and Digital Twin Driven Diagnostics and Prognostics of Light-Emitting Diodes." *Laser & Photonics Reviews* 14.12 (2020): 2000254.

Benefits and challenges

Benefits

1. Advance warning of failures and maintain the required function;
2. Aviod unnecessary tests;
3. Optimized maintenance;
4. Logistic support and cost reduction.

Challenges

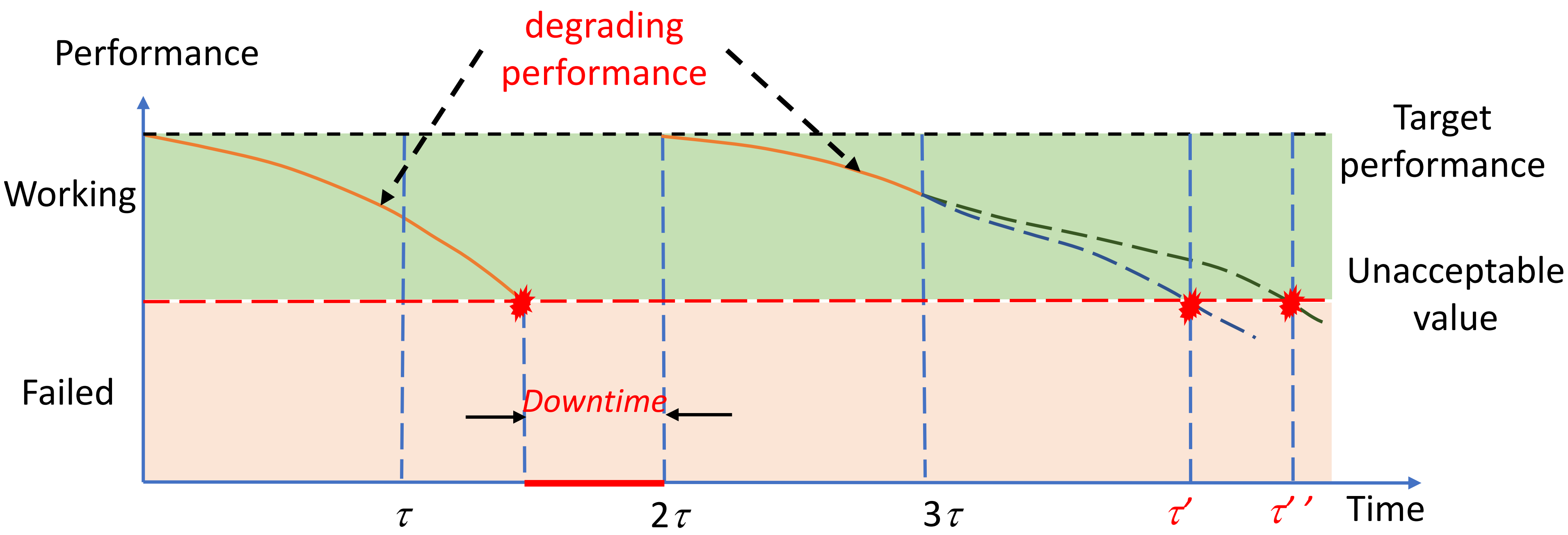
1. Degradation modeling;
2. Redundancy structure in degradation modeling;
3. Time dependent measurement of SISs;
4. Decision-making within the required SIL.

* Article Ⅰ:
Zhang, Aibo, et al. "Prognostic and health management for safety barriers in infrastructures: Opportunities and challenges." *ESREL 2018*.

Influencing factor of SIS

Working condition	Random demands	Structure	Testing and maintenance
Aging	Demand rate	Redundancy	Testing strategy
Required function	Damage size	Heterogeneity	Maintenance strategy
			Imperfectness

Time-dependent Performance



Binary state VS time-dependent performance

Research questions and objectives

Degrading performance

- Continuous aging
- The required performance

Redundancy structure

- Same damage
- Only the activated ones

Evaluation criteria

- Condition-based maintenance
- Economics

Modeling approach

- Continuous aging on time-dependent SISs degrading performance;
- Hybrid effects of continuous aging and random demands;

Decision-making

- Assessment method considering the effectiveness of collected information in tests
- Balancing SIS performance and economic targets in decision-making

Contribution

Contribution

— Degrading performance

Objective: Continuous aging on time-dependent SISs degrading performance

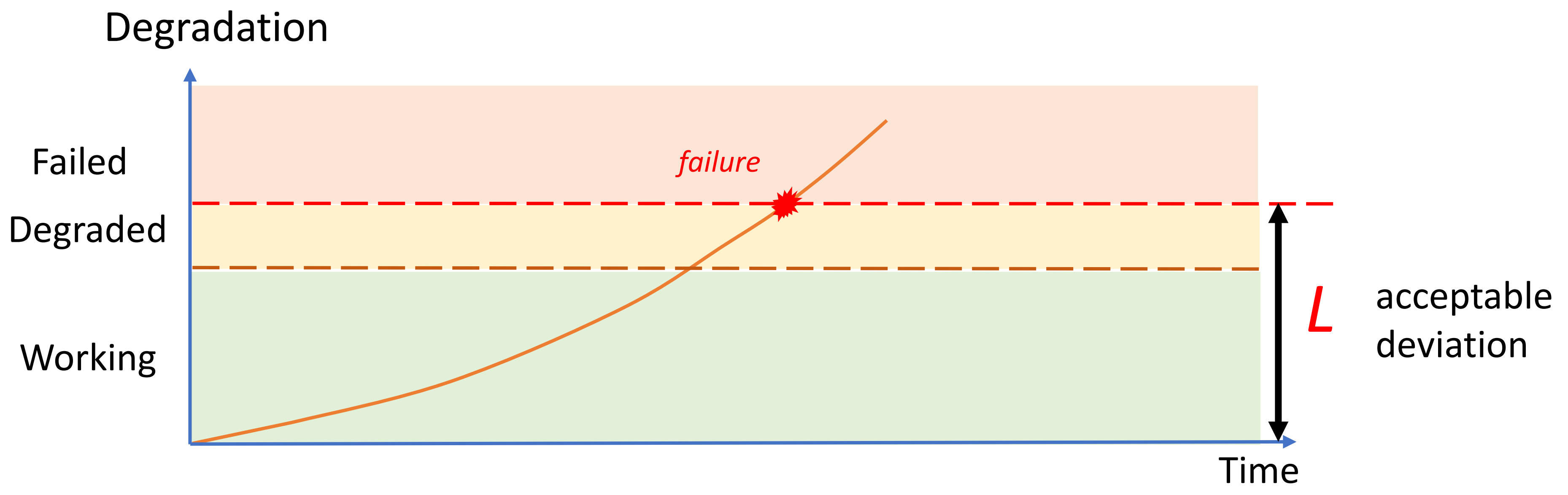
Method: Stochastic process

Output:

Article III:
Zhang, Aibo, et al. "A degrading element of safety-instrumented systems with combined maintenance strategy" *ESREL* 2019.
Article IV:
Zhang, Aibo, et al. “Optimization of maintenances following proof tests for the final element of a safety-instrumented system.” *Reliability Engineering & System Safety* (2020).

Degrading performance

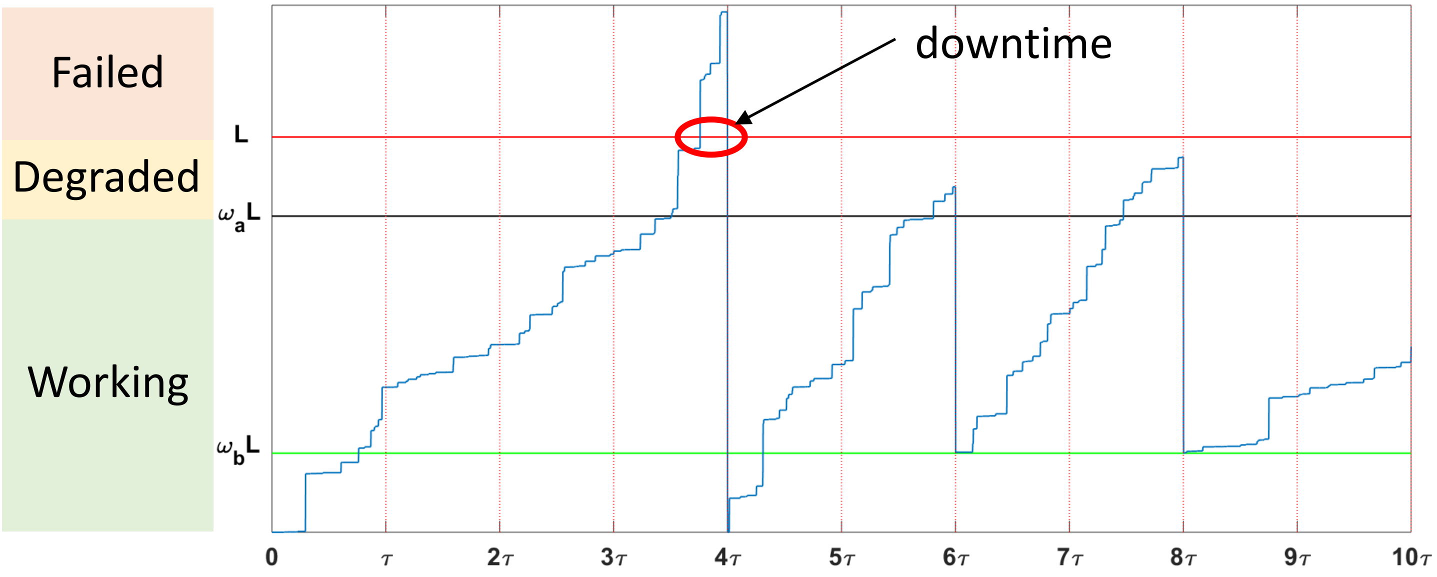
1. Time dependent state: working, degraded and failed;



State	Status	State description
0	Working	The system is functioning as specified
1	Degraded	The system has a degraded performance but still functioning
2	Failed	The system has a failed fault

Degrading performance

2. Periodic proof test with interval τ;
3. Different maintenance strategies are taken based on the state of component
   - Failed state: corrective maintenance (AGAN)
   - Degraded state: imperfect preventive maintenance ($\omega_b L$)
   - Working state: no maintenance



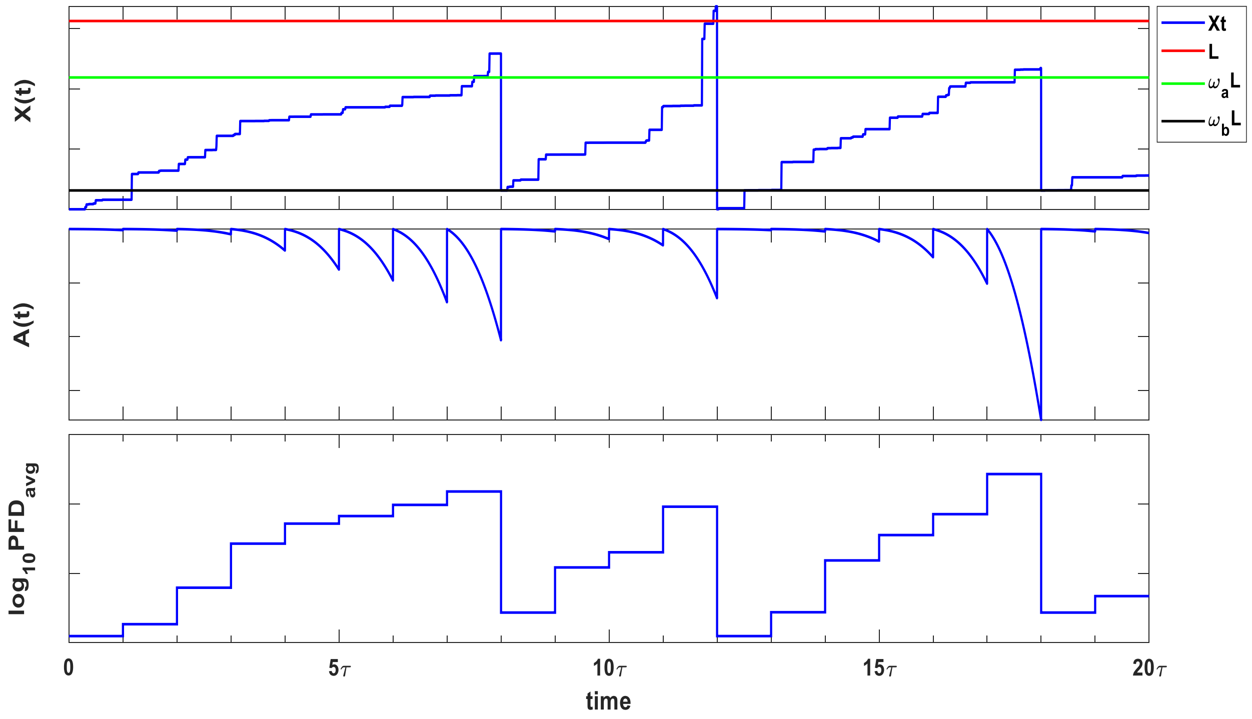
Research questions:
- System performance ?
- Conditional PFD(t)
- $\omega_a L$, $\omega_b L$?

Degrading performance

Degradation process : homogenous Gamma degradation process
Maintenance : only at proof test date

- A(t)=Pr ($X(t)<L$)
- The conditional A(t) : A(t)=Pr ($X(t)<L|X(\tau)=\mu$)

$$\mathrm{PFD_{avg}} = \frac{1}{\tau}\int_0^\tau [1 - A(t)]dt$$



- Degradation level X(t) accumulates with time;
- A(t) reduces ;
- PFDavg increases with time.

PHM of SISs

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PhD defense

Research motivation

Research questions

Contribution

Concluding remarks

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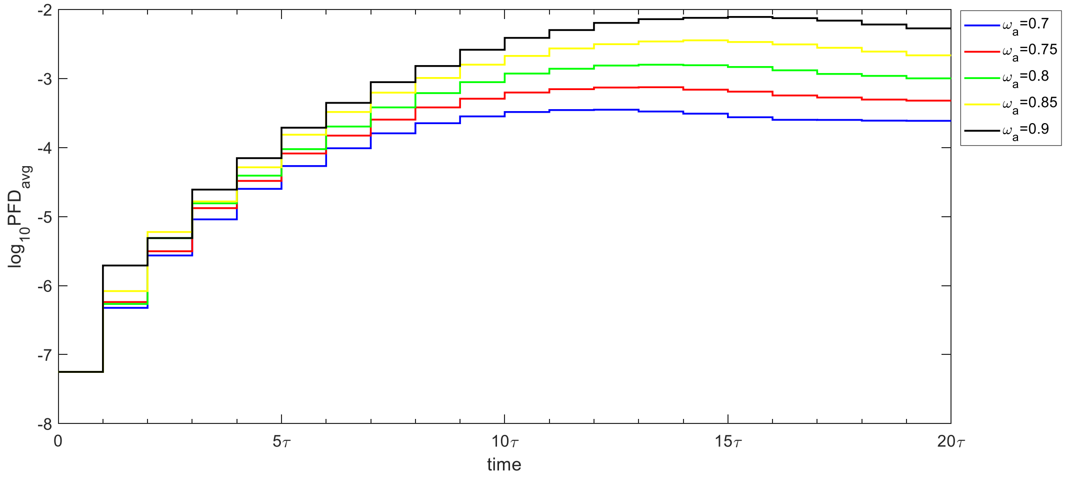
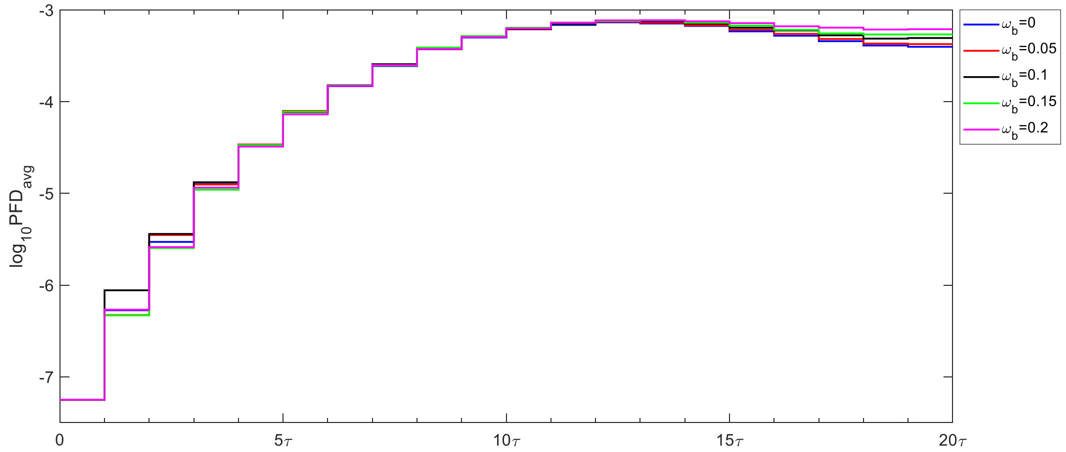
Degrading performance



(a) Parameter ω_a effect on PFD_{avg}



(b) Parameter ω_b effect on PFD_{avg}

Conclusion:

- System PFD_{avg} increases with time even working at tests.
- PFD_{avg} is more susceptible to the degree of degradation initiating a PM;
- The theoretical basis for the updating testing interval given SIL.

Contribution

— Redundant structure

Objective: Hybrid effects of continuous aging and random demands

Method: Stochastic process + Poisson process

Output:

Article II:
Zhang, Aibo, et al. "Performance analysis of redundant safety-instrumented systems subject to degradation and external demands". *Journal of Loss Prevention in the Process Industries* (2019).
Article VI:
Zhang, Aibo, et al. "Optimal activation strategies for heterogeneous channels of safety instrumented systems subject to aging and demands." *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability* (Under revision).

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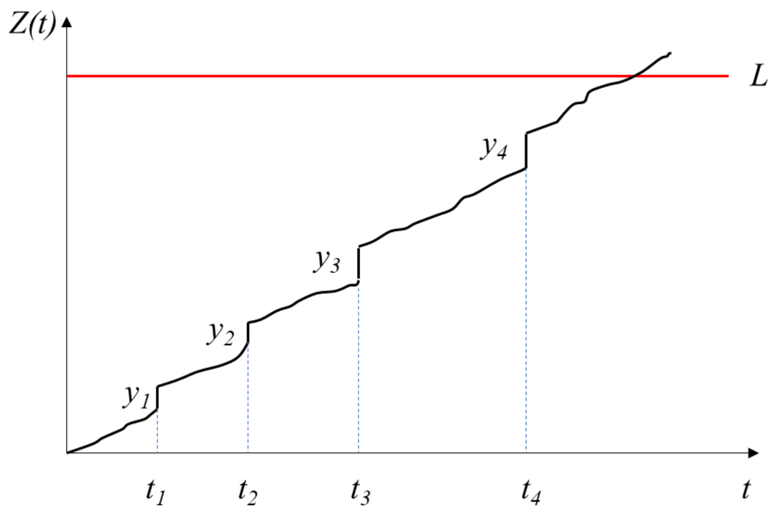
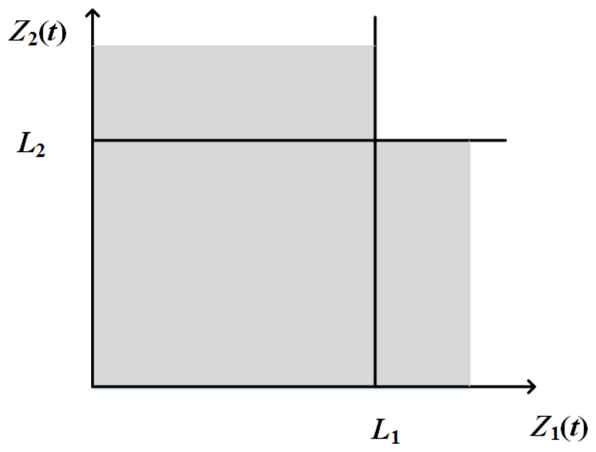
Redundancy structure — 1oo2 configuration

Degradation process of single unit:

- Continuous aging;
- Random demands

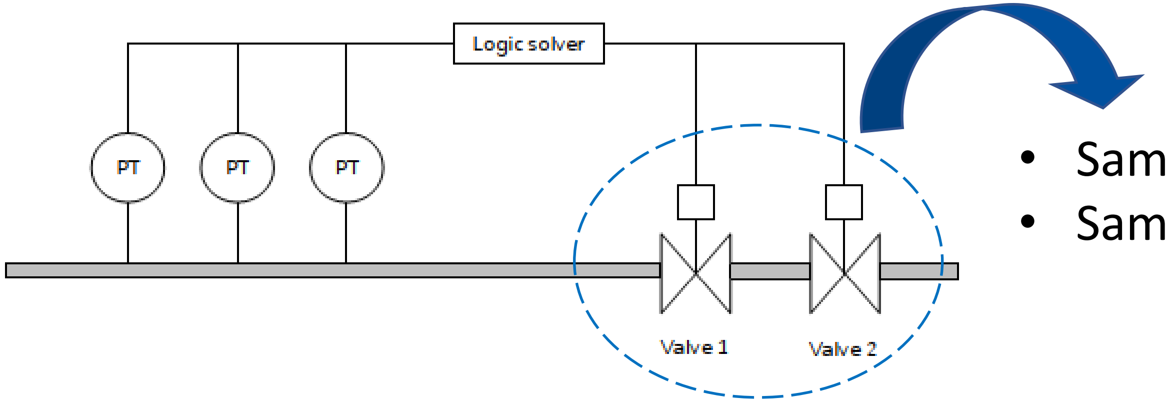




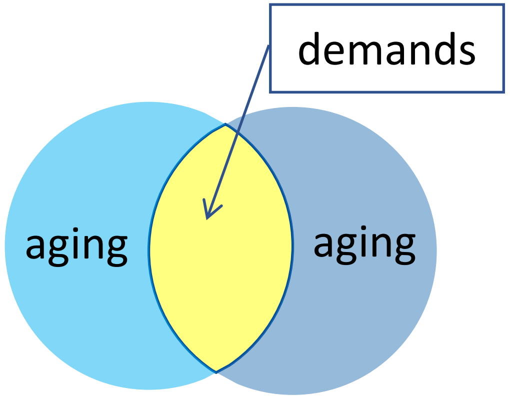
System reliability of such a 1oo2 by time t is the probability that total degradation of at least one unit is less than the threshold L, as,

$$R_S(t) = \Pr(\{Z_1(t) < L_1\} \cup \{Z_2(t) < L_2\})$$

Performance analysis



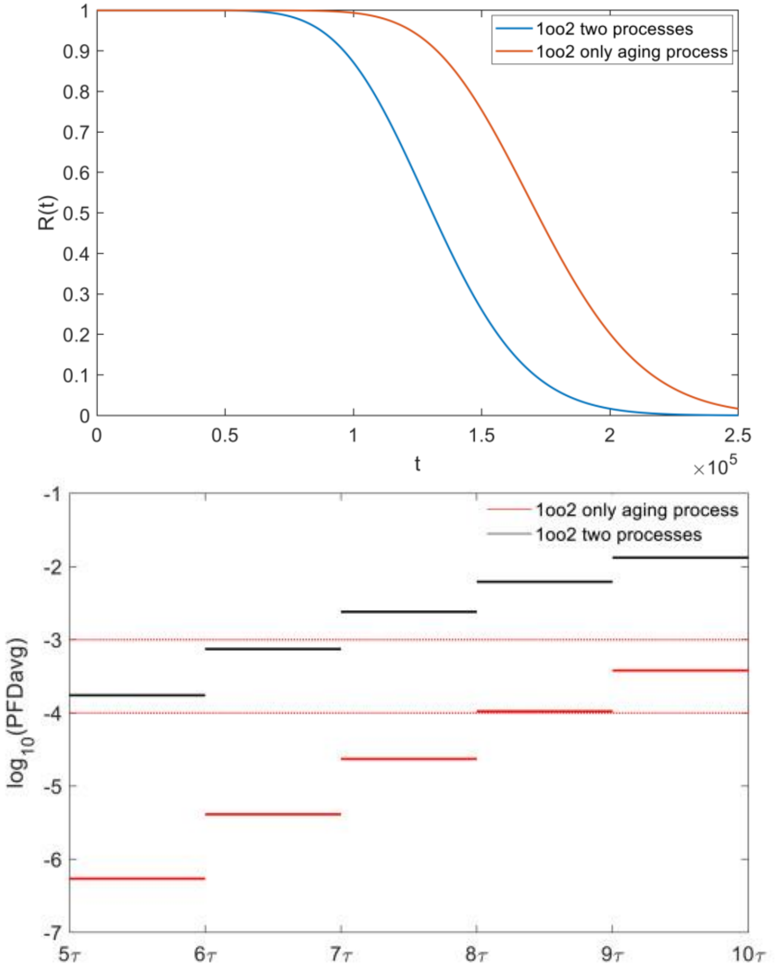
- Same working conditions
- Same demands



1. Degradation processes of one unit:
   - Continuous aging process: homogeneous gamma process
   - Random demands: Poisson process with rate λ_{de}
   - Demand damage: Gamma distribution
2. For 1oo2, random demands will have same damage effects on two units.
3. Two components are dependent due to the same damage caused by random demands.

Performance analysis

1oo2 System performance: R(t) and conditional PFDavg



- System is quite reliable at beginning;
- System reliability will be overestimated if only the aging process is considered.
- System conditional PFD_{avg} increases with time.
- Considering aging and damage caused by random demands can make the system reliability and PFD_{avg} stricter than only aging process.

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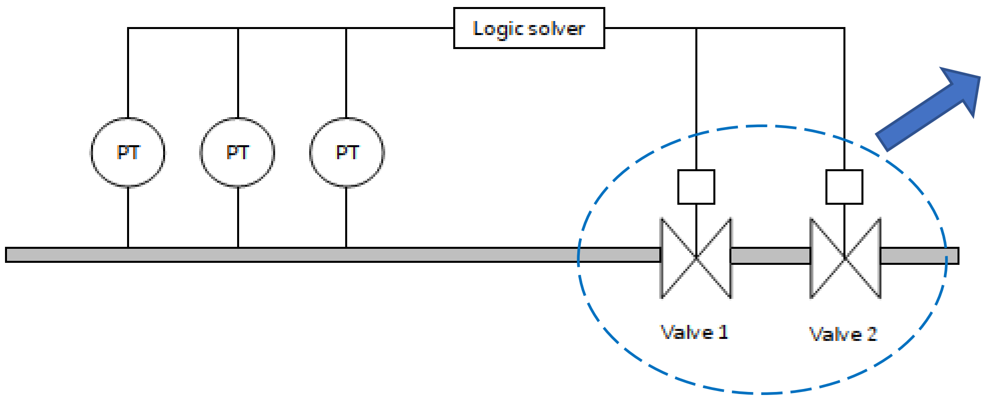
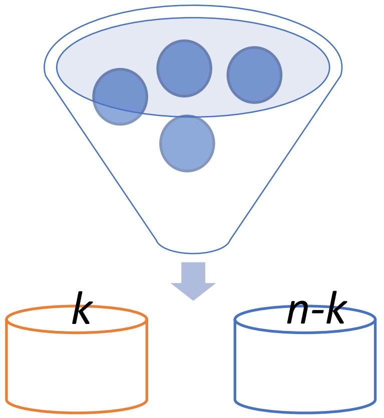
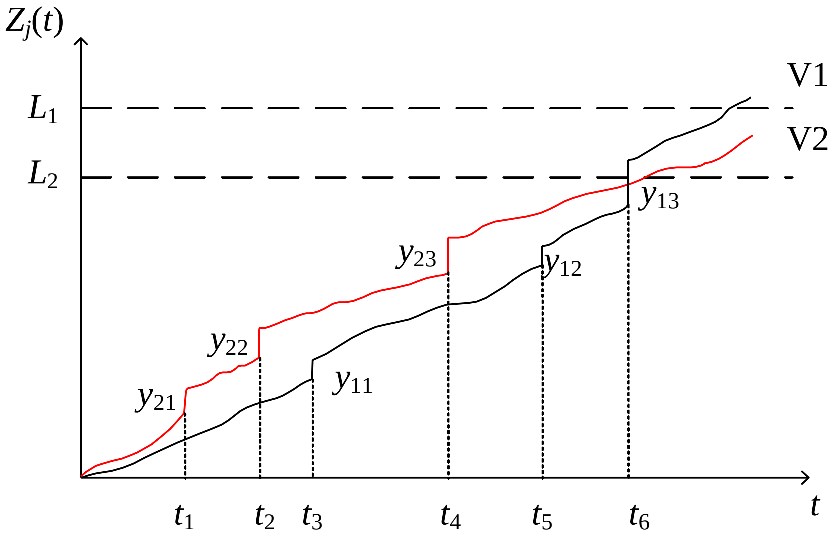
Performance analysis



- Same working conditions
- Demand only on the 'effective activated one'.

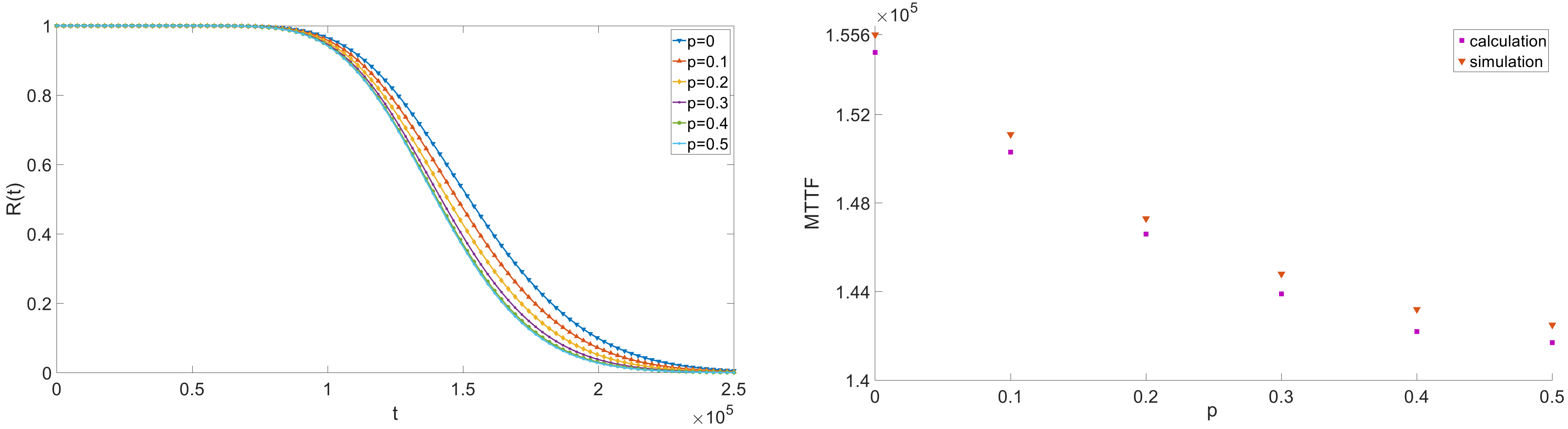




Binomial distribution of getting k demands in total n demands on unit 1

$$f(k, n, p) = \Pr(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

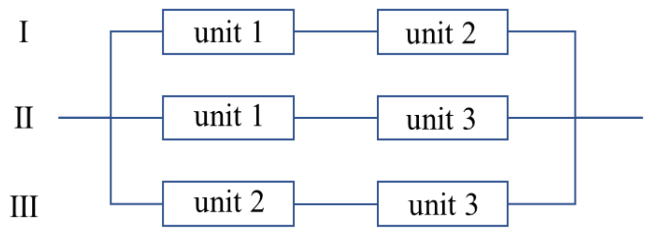
Performance analysis



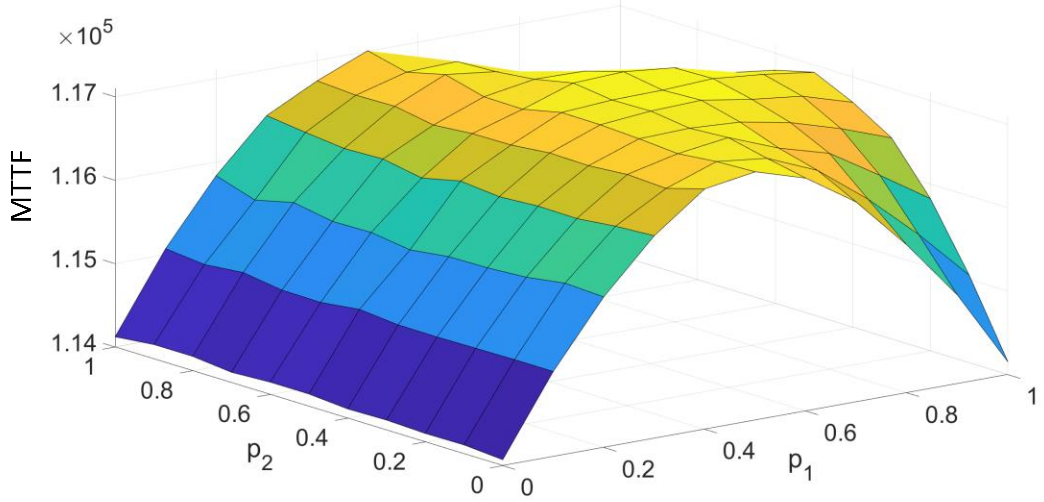
R(t) and MTTF of 1oo2 system with activation probability *p* of unit 1

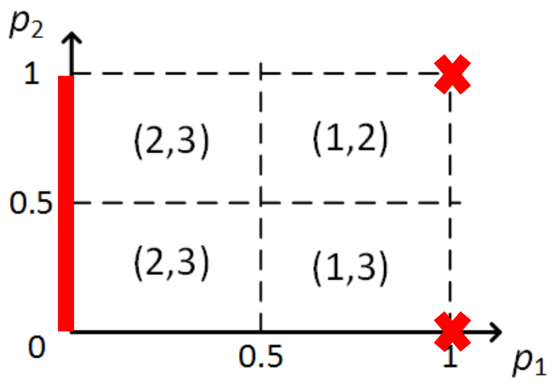
1. R(t) is quite high at beginning;
2. System R(t) and MTTF reach a minimum value when *p*=0.5 , a maximum value with *p*=0.
3. The optimal strategy: one unit as standby until the primary one failed.

2oo3 configuration peformance



$$p_1 = \Pr(\text{Activating unit 1})$$
$$p_2 = \Pr(\text{Activating unit 2}|\, p_1)$$





Dominant activation paths with (p_1, p_2)

1. System MTTF reaches the minimum when $p_1 = 0$, also when $(p_1, p_2) = (1,0)$ and $(p_1, p_2) = (1, 1)$;
2. System performance reaches the worst state while keeping the fixed combinations for all demands;
3. For 2oo3 configuration, demands should be arranged equally to each unit.

Contribution

—— Decision-making approach

Objective: Assessment method considering the effectiveness of collected information in tests
Balancing SIS performance and economic targets in decision-making

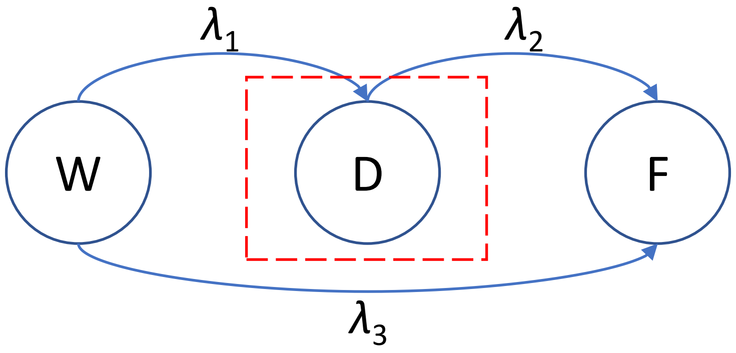
Method: Markov process

Output:

Article Ⅴ:
Zhang, Aibo, et al. “Study of testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state”. *Reliability Engineering & System Safety* (2020).

Discrete degradation — 1oo1 configuration

Notation	Status
W	Working
D	Degraded
F	Failed



Imperfect degraded state revealing

- Degradation is not observed directly
- Inaccurate threshold setting for the state
- Subjective errors

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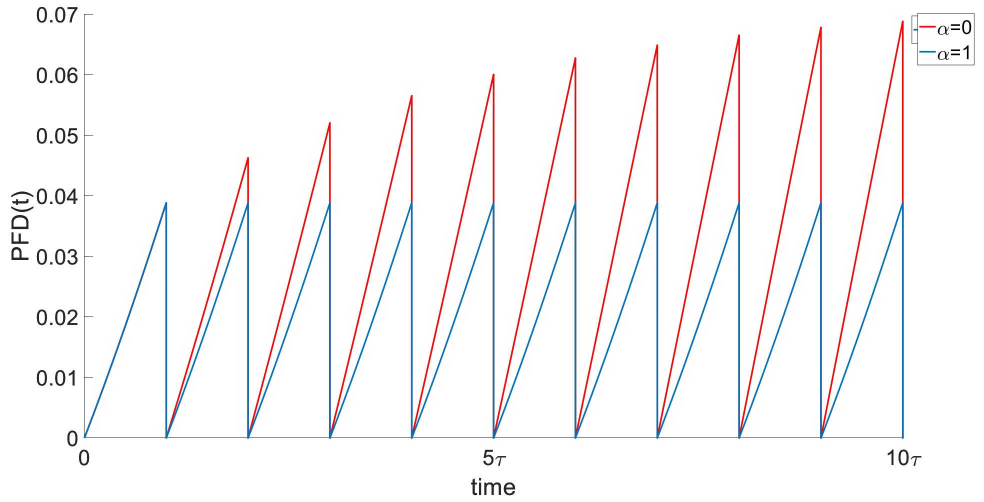
Discrete degradation — 1oo1 configuration

α = Pr(Degradation is detected in a proof test | Degradation has occurred)

Testing and maintenance matrix:

- PM for the degraded state
- CM for the failed state

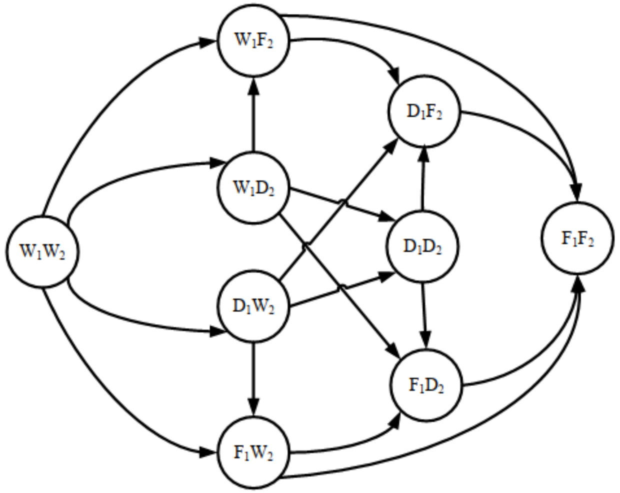
$$A = \begin{pmatrix} 1 & 0 & 0 \\ \alpha & 1-\alpha & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

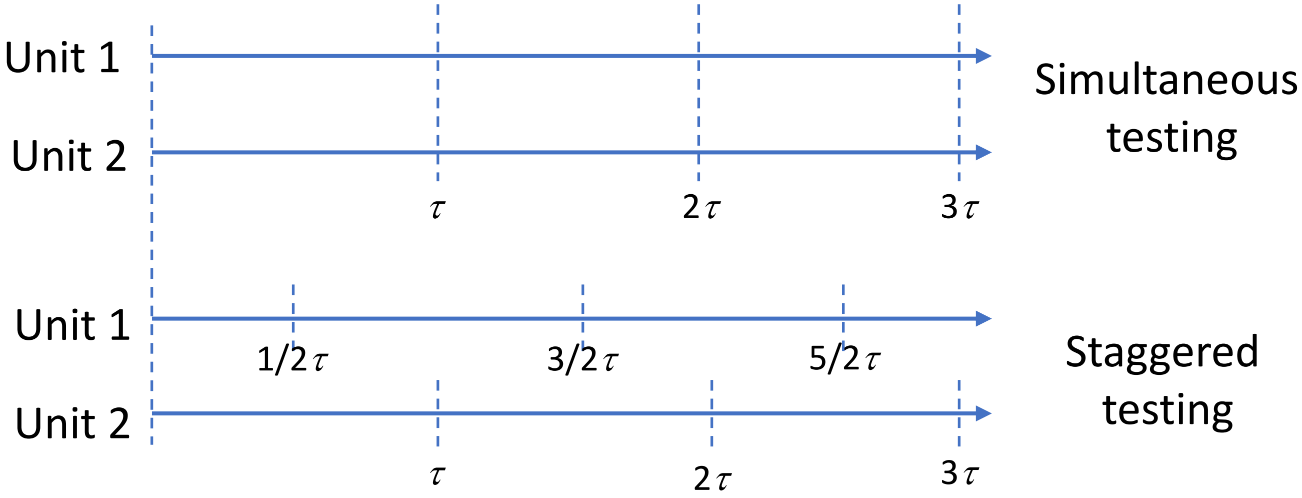


1. When α=1, the degraded state will be repair, PFD(t) keeps the same in each test interval.
2. When α=0, no degraded state will be repair, PFD(t) increases each test interval.

Discrete degradation — 1oo2 configuration

Testing and maintenance strategy for 1oo2 configuration





Strategy I	Simultaneous testing	PM and CM for the degraded and failed state, respectively.
Strategy II	Staggered testing	•PM and CM for the tested unit •No action on the other
Strategy III	Staggered testing	•PM and CM for the tested unit •When CM, perform a replacement on the other

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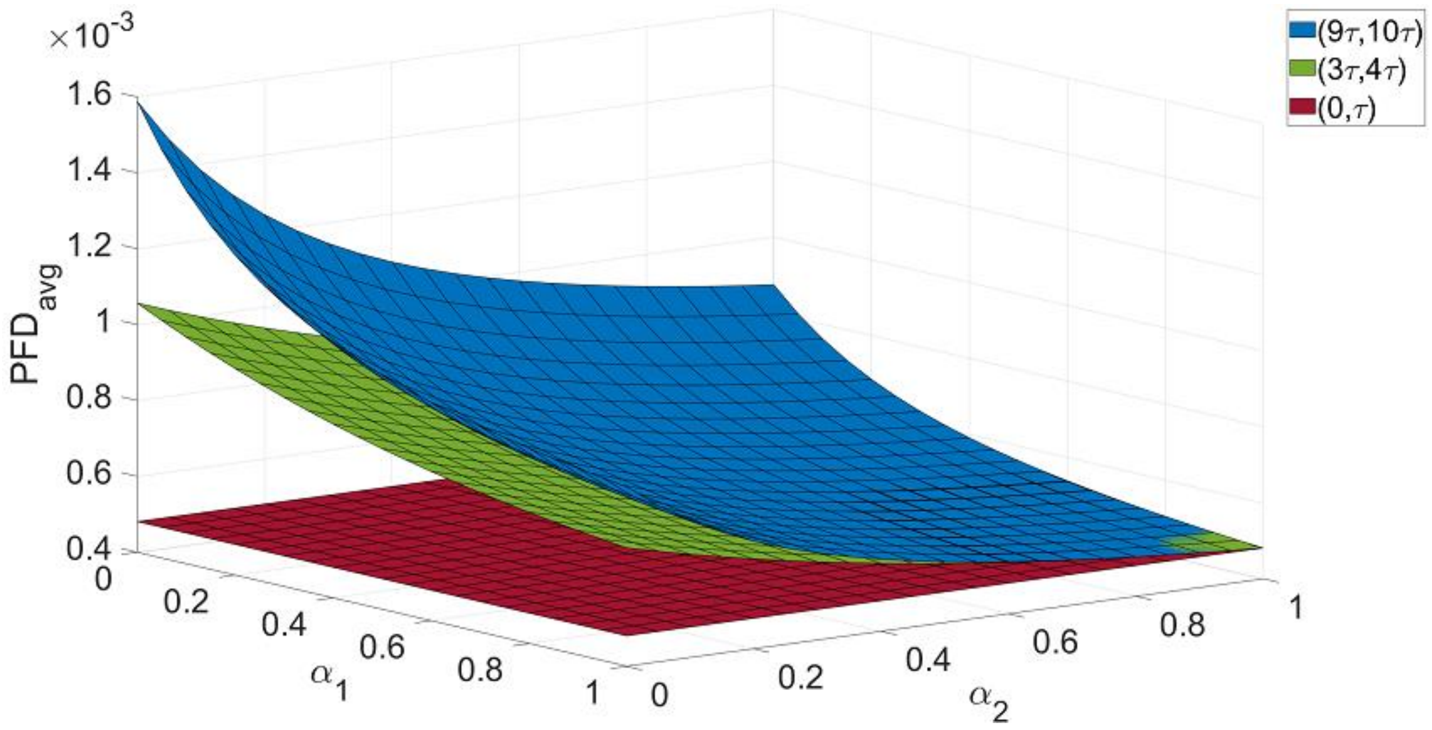
Discrete degradation — 1oo2 configuration



1. System PFDavg independents with (α_1, α_2) in first test interval (0, τ).
2. System PFDavg keeps a constant value with α_1= α_2 =1.
3. System PFDavg increases with time when $\alpha_1 \neq 1$, $\alpha_2 \neq 1$.

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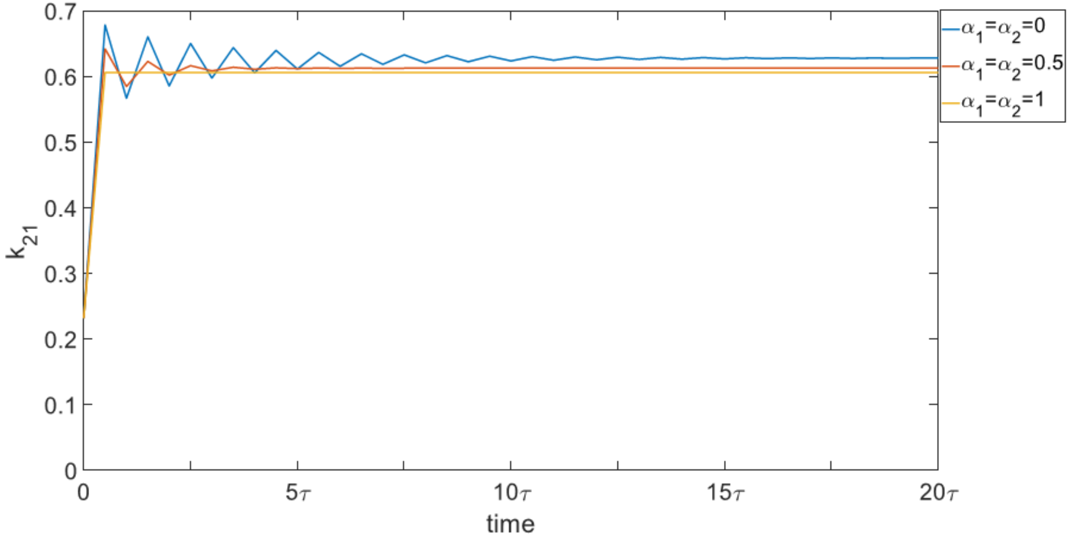
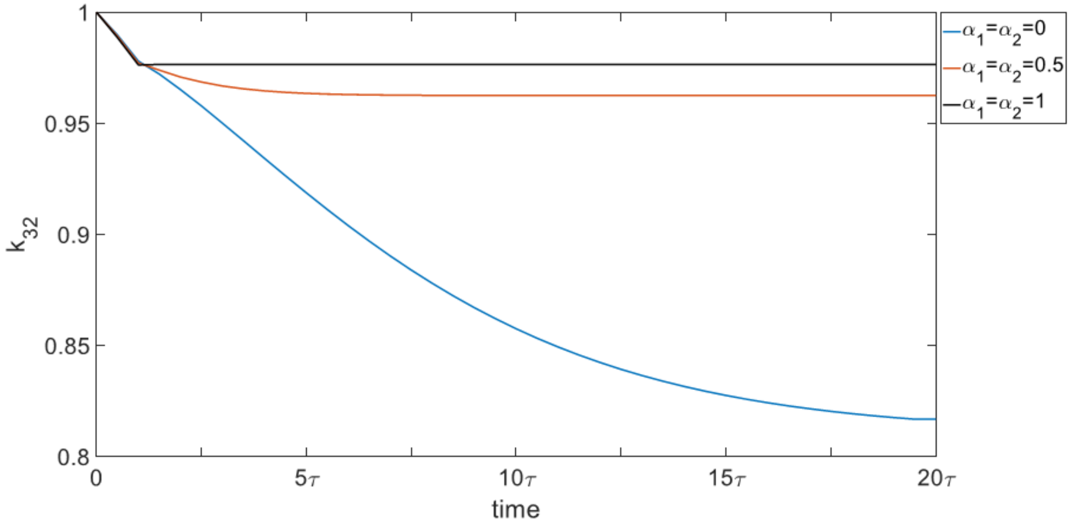
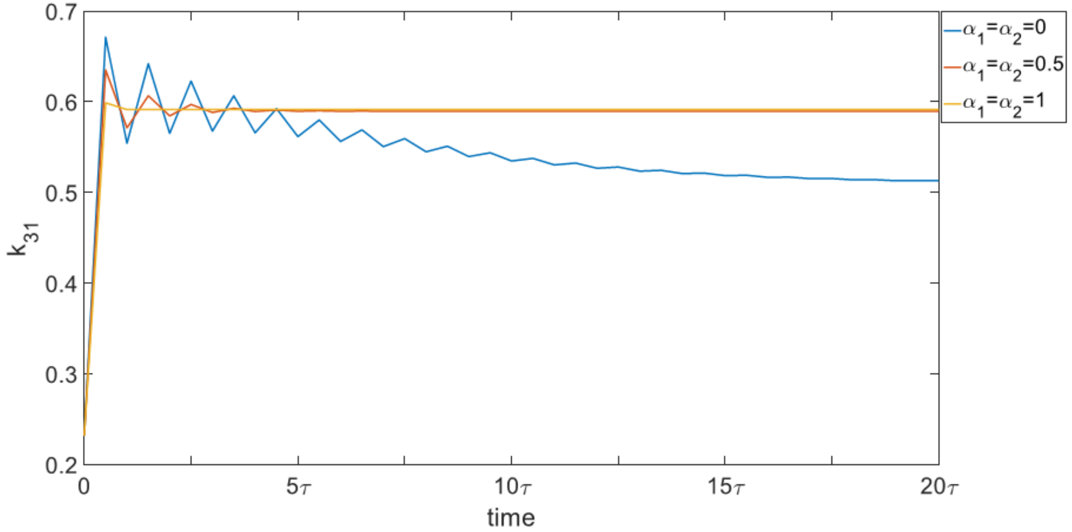
Discrete degradation — 1oo2 configuration







$$k_{ji} = \frac{\text{PFDavg with strategy } j}{\text{PFDavg with strategy } i}$$

Strategy Ⅲ> Strategy Ⅱ> Strategy Ⅰ

PHM of SISs

Aibo Zhang
PhD defense

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Discrete degradation — 1oo2 configuration

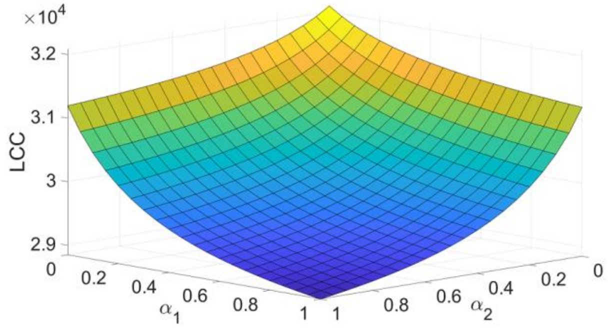
Expected cost in single test interval

$EC_i = EC_{PT} + EC_{PM} + EC_{CM}$

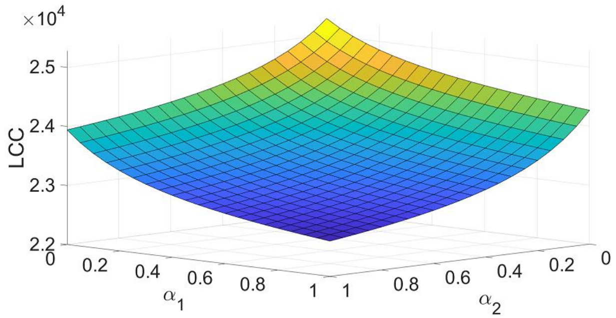
Linked with revealing coverage α

$$LCC = C_0 + \sum_{i=1}^{n} EC_i$$

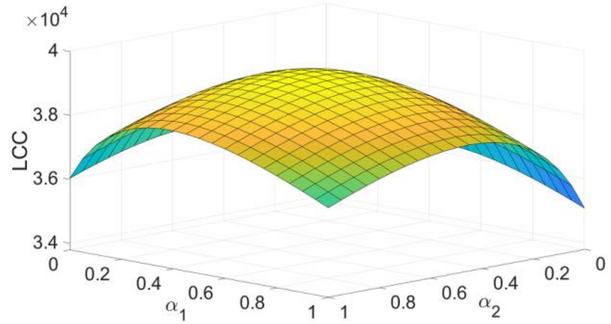
One-time installation cost per unit



Strategy Ⅰ



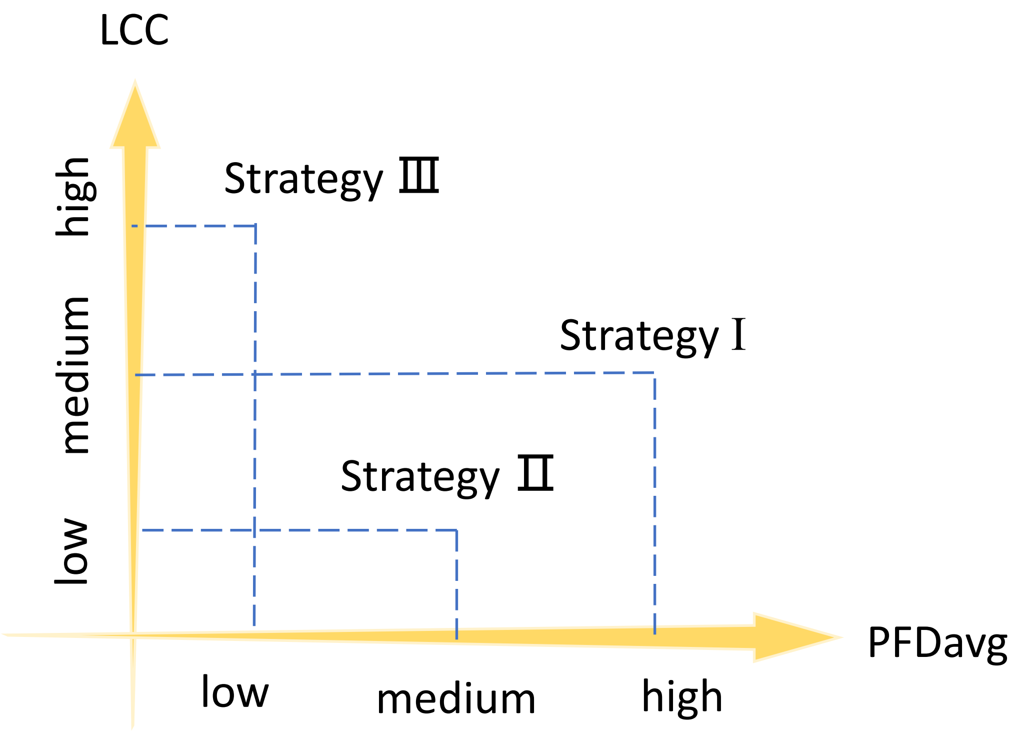
Strategy Ⅱ



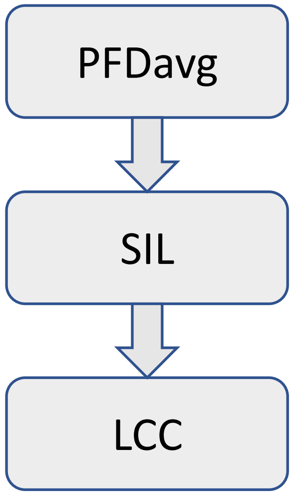
Strategy Ⅲ

Strategy Ⅲ > Strategy Ⅰ > Strategy Ⅱ

Discrete degradation — 1oo2 configuration



Selection procedure for optimal testing and maintenance strategy



Concluding remarks

Conclusions

1. The proposed stochastic process-based degradation model provide an advantage of calculating the conditional system performance based on the collected information in tests;
2. Quantitative degradation models are proposed for single-unit and redundant structure systems, to address several factors, as aging, operational history and configuration;
3. A performance-based maintenance framework is proposed to evolve the maintenance scheme.
4. Algorithms are proposed to coordinate system performance and maintenance cost, which provides the quantitative references in the decision-making step of PHM on SISs.

Acknowledgements

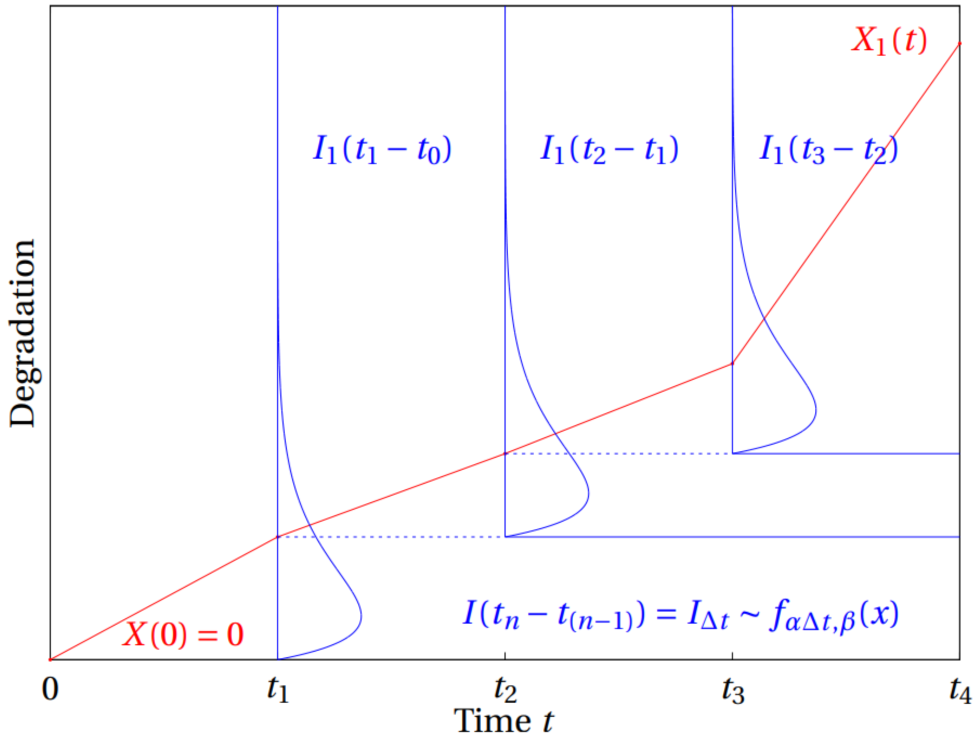
- Supervision team: Yiliu, Anne, Elias, and Tieling(UOW);
- Co-authors in these publications;
- RAMS colleagues and friends;
- Families.

Thank you!

Gamma process

Properties of homogeneous gamma process *Γ*(*t*; *α*; *β*):
An homogeneous gamma process with shape parameter *α* and scale parameter *β*, is a stochastic process *X*(*t*); *t* > 0, *α*; *β* > 0

1. $X(0) = 0$;
2. $X(t)$; $t > 0$ is a stochastic process with independent increments;
3. for $s < t$, the distribution of the random variable $X(t) - X(s)$ is the gamma distribution



1. the increment degradation X for $t-s$, $X(t-s)$ follows a Gamma PDF

$$\Delta X(t-s) \sim \Gamma(\alpha(t-s), \beta) = f_{\alpha(t-s),\beta}(x)$$
$$= \frac{\beta^{\alpha(t-s)}}{\Gamma(\alpha(t-s), 0)} x^{\alpha(t-s)-1} e^{-\beta x}, \alpha, \beta > 0$$

2. total degradation $X(t)$ at time t is less than x, $F_X(x,t)$, can be derived as:

$$F_X(x,t)(t) = P\{X(t) < x\} = \int_0^x f_{\alpha t,\beta}(z) dz = \frac{\gamma(\alpha t, x\beta)}{\Gamma(\alpha t)}$$

3. the mean and variance of $X(t)$ are $\frac{\alpha}{\beta} t$ and $\frac{\alpha}{\beta^2} t$,respectively.

System reliability and conditional PFDavg– 1oo2 (Article II)

$$R_S(t) = \Pr(\{Z_1(t) < L_1\} \cup \{Z_2(t) < L_2\})$$

In this example, such a 1oo2 SIS needs to meet SIL3. Here, we take different thresholds L in Fig9 as an example. Values of the two variables are at first set as $\lambda_{de} = 2.5 \times 10^{-5}$, and $\xi = 4$ respectively. Similar to Eq. 18, we can connect reliability and average PFD in a test interval

$$\mathrm{PFD_{avg}} = 1 - \frac{1}{t - t_0} \int_{t_0}^{t} \frac{R(u)}{R(t_0)} du \tag{21}$$

The average value of $\mathrm{PFD}_1(t)$ in the first proof test interval$(0,\tau)$ can be obtained then

$$\mathrm{PFD_{avg}} = \frac{1}{\tau} \int_0^\tau \mathrm{PFD}_1(\mathrm{t})dt = 1 - \frac{1}{\tau} \int_0^\tau R(t)dt \tag{13}$$

Using the survivor function of the system $\mathrm{R}(t)$ in (11), we can get

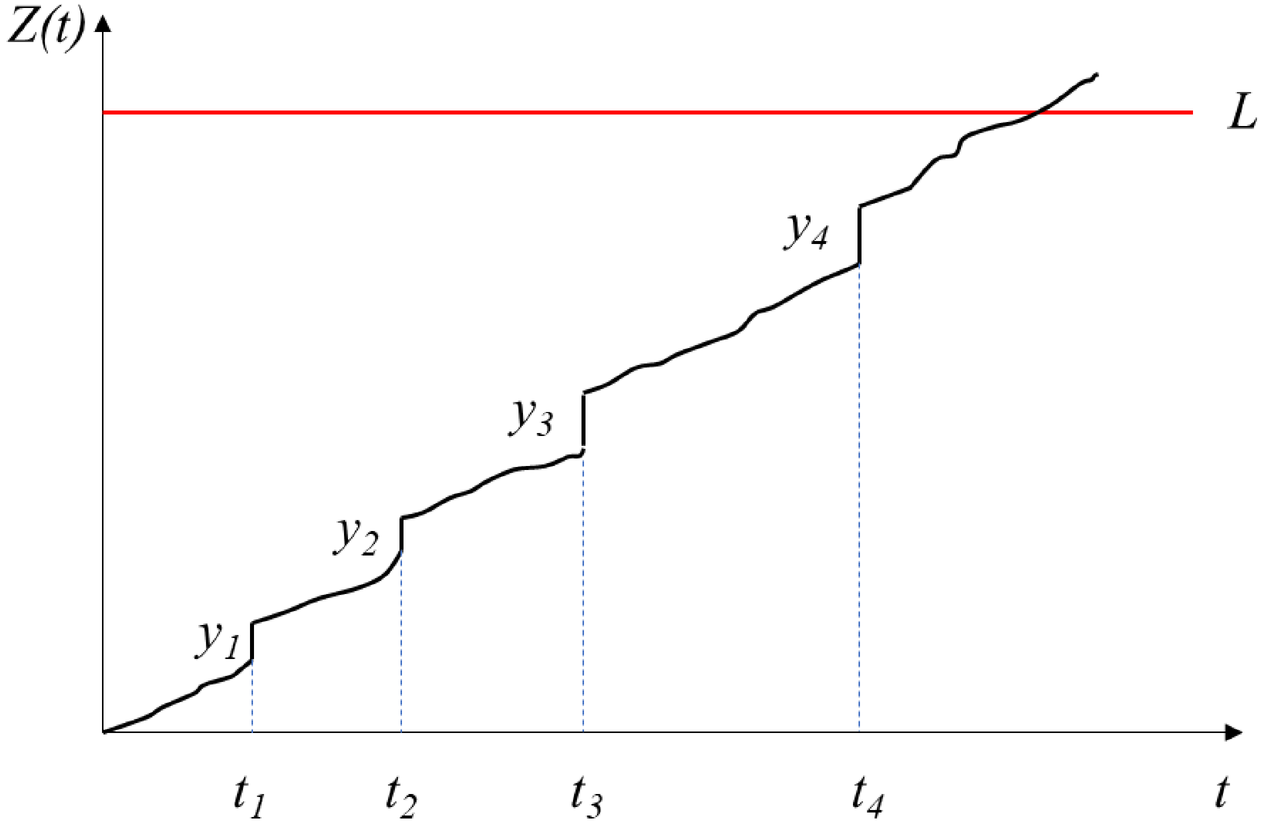
$$\begin{aligned}\mathrm{PFD_{avg}} &= 1 - \frac{1}{\tau} \int_0^\tau R(t)dt \\ &= 1 - \frac{1}{\tau} \int_0^\tau \{[1 - (1 - \frac{\gamma(\alpha t, L\beta)}{\Gamma(\alpha t)})^2] \cdot e^{-\lambda_{de} t} + \\ &\sum_{k=1}^{\infty} \int_0^L [1 - (1 - \frac{\gamma(\alpha t, (L-y)\beta)}{\Gamma(\alpha t)})^2] \frac{\rho^{k\xi} \cdot y^{k\xi - 1} \cdot e^{-\rho y}}{\Gamma(k\xi)} dy \cdot \frac{e^{-\lambda_{de} t}(\lambda_{de} t)^k}{k!}\}dt\end{aligned} \tag{14}$$

A proof-test will be executed at time τ. If the subsystem is functioning at τ with unknown degradation level, $\mathrm{PFD}_2(t)$ becomes the conditional probability of failure with $t > \tau$ given functioning by τ

$$\begin{aligned}\mathrm{PFD}_2(t) &= \Pr[T < t | T > \tau, t > \tau] = 1 - \Pr[T > t | T > \tau, t > \tau] \\ &= 1 - \frac{\Pr[T > t \bigcap T > \tau, t > \tau]}{\Pr[T > \tau]} = 1 - \frac{R(t)}{R(\tau)}\end{aligned} \tag{15}$$

The $\mathrm{PFD_{avg}}$ in the second test interval$(\tau,2\tau)$ is then:

$$\begin{aligned}\mathrm{PFD_{avg}} &= \frac{1}{\tau} \int_\tau^{2\tau} PFD_2(t)dt \\ &= \frac{1}{\tau} \int_\tau^{2\tau} [1 - \frac{R(t)}{R(\tau)}]dt \\ &= 1 - \frac{1}{\tau} \int_\tau^{2\tau} \frac{R(t)}{R(\tau)} dt\end{aligned} \tag{16}$$



Z(t)
L
y1
y2
y3
y4
t1
t2
t3
t4
t