A New Availability Allocation Method

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- In an early design phase, availability targets are often set and will be the reference point for the selection of design solution and operational strategy.
- In situations where the target is not met, it is important to improve the reliability and restoration time of individual components such that the target is met.
- However, almost always availability improvements involve increasing in cost, complexity and size.
- Hence, an optimization method that takes into account such constraints should be used.

- Most availability allocation methods are complex and resource demanding such as heuristic, meta-heuristic, dynamic programming and so on.
- Such methods may be too complex to be applied, because
  - in the early design phase, only little knowledge about the system is available,
  - results from the allocation process are not definite, and
  - regular reliability engineers may not be familiar with such mainstream optimization methods.
- Therefore, the main objective is to develop an alternative method that is more simple and user-friendly.

- Simple and user-friendly methods for reliability allocation for non-repairable system have long been used, such as
  - Equal apportionment
  - ARINC
  - AGREE
  - Minimum Effort Algorithm
- The proposed approach here is to adopt the **Minimum Effort Algorithm** for availability allocation.
- Three allocation options are proposed depending on the objective of the allocation and availability of cost data:
  1. Allocation with identical cost/complexity function
  2. Allocation with varying cost function
  3. Allocation based on complexity of components for improvement

- Only unplanned downtime due to critical item failure is considered
- Exponentially distributed time to failure with parameter λ and time to restoration with parameter μ are assumed. And MTTF and MTTR are the respective means. Thus, for item i

$$A_i = \frac{\mathrm{MTTF}_i}{\mathrm{MTTF}_i + \mathrm{MTTR}_i} = \frac{\mu_i}{\lambda_i + \mu_i} \qquad (1)$$

- The system is assumed to be expressed by n critical components connected in series. Thus, the system availability is

$$A = \prod_{i=1}^{i=n} \frac{\mu_i}{\lambda_i + \mu_i} \approx \left(1 + \sum_{i=1}^{n} \sigma_i\right)^{-1} \qquad (2)$$

where $\sigma_i = \lambda_i M_i$ is the current unavailability contribution of component i. For brevity purpose we use M instead of MTTR.

Let C_{λ_i} and C_{M_i} be respectively the cost to increase the mean time to failure and to increase the restoration rate of component i per hour, for $i = 1, 2, \cdots, n$. The cost functions will then be

$$h_i(\lambda_i, \lambda_i^a) = C_{\lambda_i} \left(\mathrm{MTTF}_i^a - \mathrm{MTTF}_i\right) \quad (3)$$

$$g_i(M_i, M_i^a) = C_{M_i} \left(\mu_i^a - \mu_i\right) \quad (4)$$

The allocation is to minimize the total improvement cost (TC) subject to the constraint of achieving the availability target, i.e.

$$\begin{aligned} &\text{Min } TC = \sum_{i=1}^{n} h_i(\lambda_i, \lambda_i^a) + \sum_{i=1}^{n} g_i(M_i, M_i^a) \\ &\text{Subject to } A_i^a = A^t \end{aligned} \quad (5)$$

where a and t stand for allocated and target respectively.

1. Arrange the current unavailability contributions in descending order, i.e. $\sigma_1 \geq \sigma_2 \geq \sigma_3 \cdots \geq \sigma_n$.
2. Determine k weakest components that need to be improved, and that can be done by finding the maximum value of i such that

$$\sigma_i > \frac{U_t - \sum_{i=i+1}^{n+1} \sigma_i}{i} = \psi_i \tag{6}$$

where $\sigma_{n+1} = 0$ and $U_t = 1/A^t - 1$.
3. The allocated unavailability contributions are

$$\sigma_i^a = \begin{cases} \psi_i & i \leq k \\ \sigma_i & i > k \end{cases} \tag{7}$$

Assumptions:
- The effort to improve low-availability components is less than that of the effort to improve high-availability components.
- It is economically infeasible to allocate an availability for a component higher than the availability of any other component.

4. Calculate the unavailability contribution reduction factor for component i, i.e. $\beta_i = \sigma_i^a / \sigma_i$.
5. Calculate the relative reliability and restoration time improvement costs as

$$r_{\lambda_i} = \frac{C_{\lambda_i}^2}{(C_{\lambda_i} + C_{M_i}) \sum_{i=1}^{k} C_{\lambda_i}}, \quad r_{M_i} = \frac{C_{M_i}^2}{(C_{\lambda_i} + C_{M_i}) \sum_{i=1}^{k} C_{M_i}} \qquad (8)$$

In situations where

- there is no room (or desire) for improvement for the failure rate *or* the restoration time of a component, the value of the associated relative cost (r) should be one.
- improvement is known to be impossible (or undesired) for *both* the failure rate and restoration time of a component, the following modification is necessary. Suppose no improvement is allowed for component l, the unavailability contribution reduction factors should be adjusted as

$$\beta_i^* = \frac{k \cdot \sigma_k^a - \sigma_l}{\sigma_i}, i \neq l \qquad (9)$$

6. Determine θ_i such that

$$\beta_i = (r_{\lambda_i} \cdot r_{M_i})^{\theta_i} \Rightarrow \theta_i = \frac{\ln(\beta_i)}{\ln(r_{\lambda_i} r_{M_i})} \tag{10}$$

7. The allocated λ and M will then be

$$\lambda_i^a = r_{\lambda_i}^{\theta_i} \lambda_i, \qquad M_i^a = r_{M_i}^{\theta_i} M_i \tag{11}$$

and the rest $n - k$ remain unchanged.

- *Identical cost function:* If no improvement cost data is available or the cost is not relevant to be considered, allocation with *identical cost function* can be used.
  - From step four above, the allocated reliability and restoration time can be calculated as

$$\lambda_i^a = \beta_i^{0.5}\lambda_i, \qquad M_i^a = \beta_i^{0.5} M_i \tag{12}$$

- *Complexity as a cost function:* If accurate cost data is unavailable or estimating the absolute total improvement cost is of less relevance, it may be important to estimate costs relative to each other (i.e. Z), rather than in absolute values. Then r becomes

$$r_{\lambda_i} = \frac{Z_{\lambda_i}^2}{(Z_{\lambda_i} + Z_{M_i})\sum_{i=1}^{k} Z_{\lambda_i}}, \quad r_{M_i} = \frac{Z_{M_i}^2}{(Z_{\lambda_i} + Z_{M_i})\sum_{i=1}^{k} Z_{M_i}} \tag{13}$$

- The method is suitable in the early design phases of product development to identify the most economically and technically feasible alternative to carry out availability improvement.
- The improvement effort function can be
  - equal for all components
  - varying and expressed in absolute values, or
  - varying but expressed based on the level of complexity/difficulty to make an improvement
- Qualitative aspects of components are crucial to decide on the prioritization of components for improvement. The method is capable of accommodating such aspects through the complexity score (Z) that can be assigned by expert judgment.
- The most important advantage of the proposed approach is that the minimum effort algorithm has been used by regular engineers for reliability allocation.

- The paper utilized complexity scores (Z) without discussing in detail what factors to be considered and how.
- The focus has only been on production systems. It is thus important to adopt the method for safety systems.
- Availability allocation for complex systems with complex operational philosophies require advanced algorithms. It would be beneficial to evaluate the efficiency of the proposed method by comparing it with advanced algorithms using a more realistic and complex system.

Thank you for your attention!