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Condition-based maintenance for a multi-component system subject to heterogeneous failure dependences

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Title: Condition-based maintenance for a multi-component system subject to heterogeneous failure dependences

Many industrial facilities consisting of multiple components are prone to failure dependences that may accelerate the degradation of components. Due to system layout and functional interactions, not all components have the same failure dependence. In the dependent multi-component systems, heterogeneous failure dependences further complicate the maintenance activities.

Motivation

Main work

In the present study, a framework to evaluate the heterogeneous failure dependences and develop a maintenance optimization model for multi-component systems by Markov processes is developed. The proposed method is applied to a practical case consisting in a parallel subsea transmission system to illustrate the effects of heterogeneous failure dependences.

Key words: Multi-component system, maintenance optimization, heterogeneous failure dependence, Markov processes.

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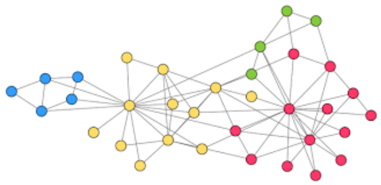
❑ Introduction



System with failure dependence

Type I failure dependence: A triggering event results in a direct damage.

In such context, a component could fail due to its normally inherent degradation, and the shock from the failures of the other components.

Type II failure dependence: A triggering event redistributes the total working load on the overall system.

In such a context, a component could fail due to its normally inherent degradation, and due to the accelerated degradation caused by the failures or malfunctions of other components.

Condition-based maintenance (CBM) is applied to many technical systems to keep system reliability while reducing maintenance cost.

We intend to build the CBM model to present the normal degradation process and accelerated degradation process of the complex system with failure dependence.

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❑ Introduction

- **Current contributions**

  Most of current studies consider a system with two components or two kinds of components with identical failure dependence.

- **Our goal**

  Develop a CBM model for multi-component systems, considering the heterogeneous failure dependences.

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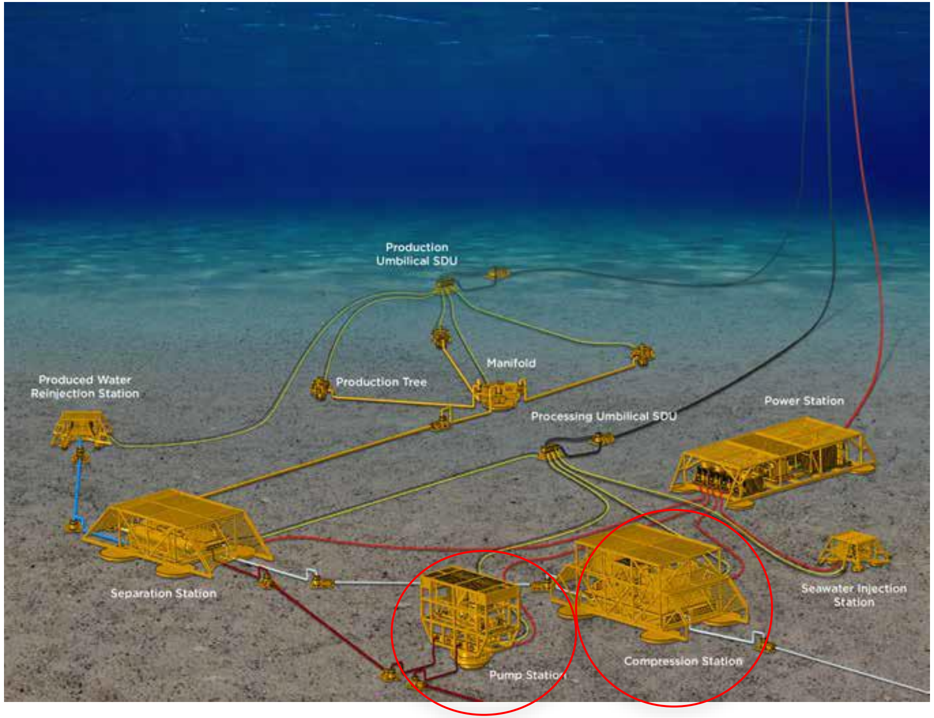
❑ Example description



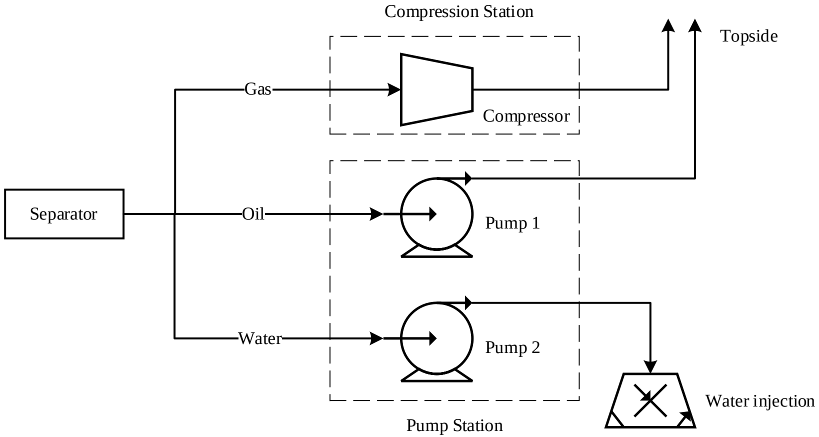


Fig. 1. The transmission system considered in the motivating case.

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- Motivating example and problem description
- Degradation models for a dependent multi-component system
- Modeling and formulation of condition-based maintenances
- Case-Study: assessment of the motivating example
- Conclusions

3.1 Independent general degradation model

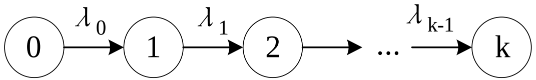
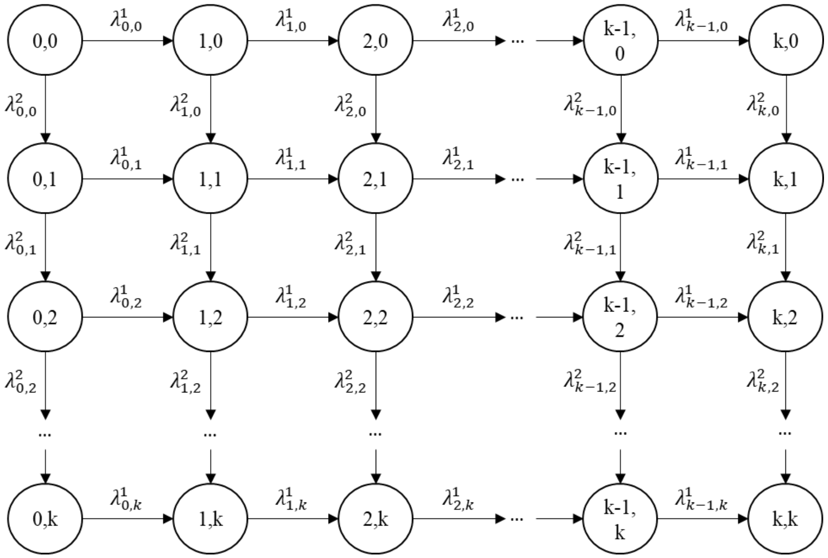


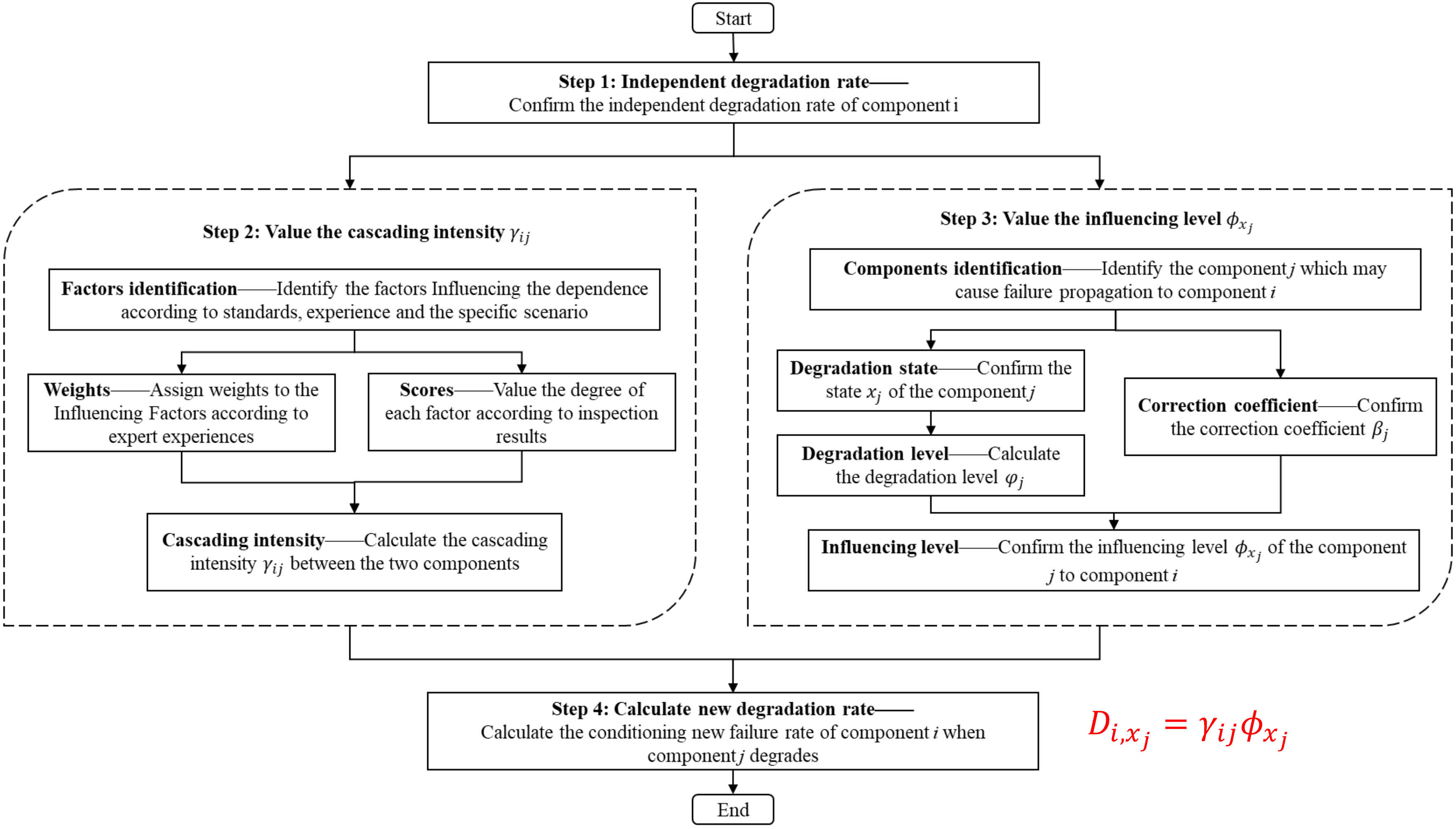
Fig. 2. State transition diagram of individual component.

3.2 Failure dependence model



$$\lambda^{i}_{x_i,x_j} = \left(1 + D_{i,x_j}\right) \lambda_{x_i}, \qquad \forall i \neq j$$

Fig. 3. State transition diagram of a two-component system with failure dependence.



$$D_{i,x_j} = \gamma_{ij}\phi_{x_j}$$

Fig. 4. Flowchart of new degradation rate identification considering failure dependence.

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❑ 3.3 Dependent multi-component degradation model

- **Failure dependences matrix $\mathbb{D}$**

$$\mathbb{D} = \begin{pmatrix} \mathbf{0} & \boldsymbol{D}_{1,2} & \cdots & \boldsymbol{D}_{1,n-1} & \boldsymbol{D}_{1,n} \\ \boldsymbol{D}_{2,1} & \mathbf{0} & \cdots & \boldsymbol{D}_{2,n-1} & \boldsymbol{D}_{2,n} \\ \vdots & & \ddots & & \vdots \\ \boldsymbol{D}_{n,1} & \boldsymbol{D}_{n,2} & \cdots & \boldsymbol{D}_{n,n-1} & \mathbf{0} \end{pmatrix}$$

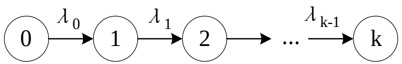
$\boldsymbol{D}_{i,j} = \left(D_{i,x_j=0}, D_{i,x_j=1}, \dots, D_{i,x_j=k}\right)$, denoting the failure dependence from component j on component i.

$\mathbf{0}$ is the null matrix whose order is corresponding by the dimensions of blocks $\boldsymbol{D}_{i,j}$

$D_{i,x_j} = \gamma_{ij}\phi_{x_j}$ is vector to represent the failure dependence from component j on component i when component j is in state x_j, then the new transition rates of the component is

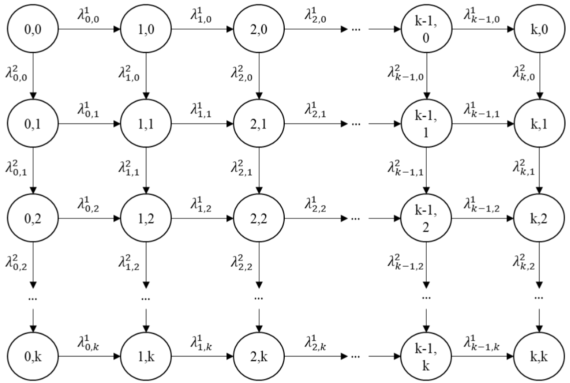
γ_{ij} Cascading intensity between component *j* to component *i*

ϕ_{x_j} Influencing level from component *j*



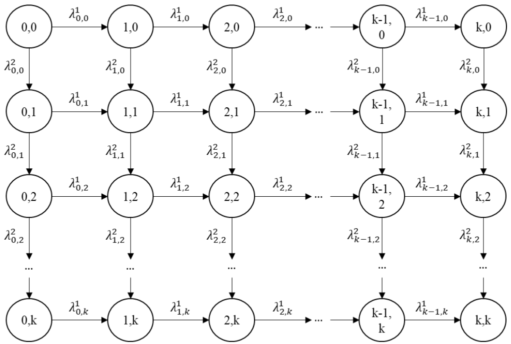
$$\lambda^i_{x_i,x_j} = \left(1 + D_{i,x_j}\right)\lambda_{x_i}$$

$$\lambda^j_{x_i,x_j} = \left(1 + D_{j,x_i}\right)\lambda_{x_j}$$



❑ 3.3 Dependent multi-component degradation model

- **Degradation matrix $\mathbb{A}$ of n-component system**



$$\lambda^{i}_{x_1,\ldots,x_n} = \lambda_{x_i} \cdot \prod_{j=1}^{n} \left(1 + D_{i,x_j}\right)$$

Dependent multi-component degradation process

Use the matrix $\mathbb{A}$ to denote the transition rates

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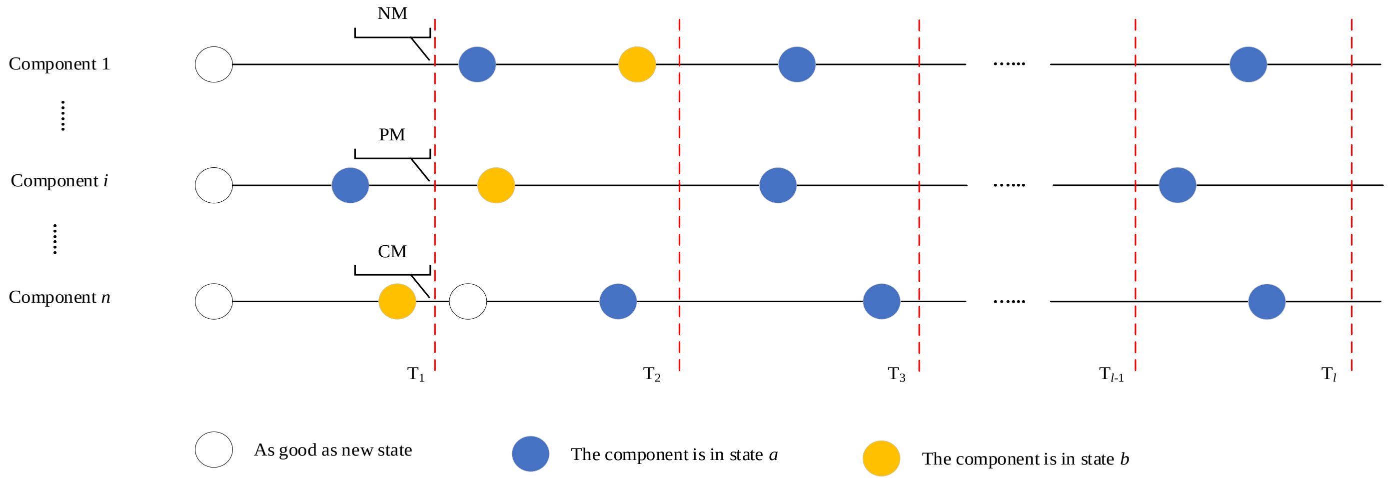

lifetime.

Fig. 5. Illustration of maintenance policies.

- **Inspections and maintenances policies**

❖ In phase Ⅰ ($x \leq a$), the component is in an acceptable state, and no maintenance activities (NM) are required.

❖ In phase Ⅱ ($a+1 \leq x \leq b$), the component is operating in a degrading state, and Minor preventive maintenance (PM) will be performed to take the component back to last state.

❖ In phase Ⅲ ($b+1 \leq x \leq k$), if the component degrades to a bad state or fail, do the Major corrective maintenance (CM) to restore the component to an as good as new state.

❑ 4.1 Inspections and maintenances

The probability that the system is in state X_j after inspections, maintenances and repairs (IMRs), given that it was in state X_i before inspection:

$$\Pr\left(X(T_s^+) = X_j \middle| X(T_s^-) = X_i\right) = b_{X_i,X_j}$$

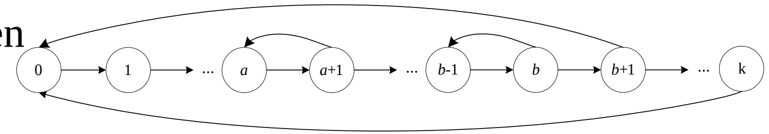

Fig. 6. Markov model of an individual component.

Let $\mathbb{B}$ describes the corresponding maintenance transition matrix of the system, then

$$\boldsymbol{P}(T_s^+) = \boldsymbol{P}(T_s^-) \cdot \mathbb{B}$$

$$\mathbb{B} = \begin{pmatrix} \mathbb{B}_{\boldsymbol{n}}^{\mathrm{I}} & 0 & 0 & 0 \\ 0 & \mathbb{B}_{\boldsymbol{n}}^{\mathbf{a}} & 0 & 0 \\ 0 & \mathbb{B}_{\boldsymbol{n}}^{\mathbf{a+1}} & 0 & 0 \\ 0 & 0 & \mathbb{B}_{\boldsymbol{n}}^{\mathrm{II}} & 0 \\ \mathbb{B}_{\boldsymbol{n}}^{\mathrm{III}} & 0 & 0 & 0 \end{pmatrix} \qquad \mathbb{B}_{\boldsymbol{i}}^{x_i} = \begin{pmatrix} \mathbb{B}_{\boldsymbol{i-1}}^{\mathrm{I}} & 0 & 0 & 0 \\ 0 & \mathbb{B}_{\boldsymbol{i-1}}^{\mathbf{a}} & 0 & 0 \\ 0 & \mathbb{B}_{\boldsymbol{i-1}}^{\mathbf{a+1}} & 0 & 0 \\ 0 & 0 & \mathbb{B}_{\boldsymbol{i-1}}^{\mathrm{II}} & 0 \\ \mathbb{B}_{\boldsymbol{i-1}}^{\mathrm{III}} & 0 & 0 & 0 \end{pmatrix} \quad \cdots$$

$$\mathbb{B}_2^{x_2} = \begin{pmatrix} 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & 1 & & & & & & & \\ & & & 1 & 0 & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & 0 & & & & \\ & & & & & & 1 & 0 & & & \\ 1 & & & & & & & & 0 & & \\ \vdots & & & & & & & & & \ddots & \\ 1 & & & & & & & & & & 0 \end{pmatrix}$$

The time-dependent state probability vector $\boldsymbol{P}(t)$ at time t

$$\boldsymbol{P}(t) = \boldsymbol{P}(0) \cdot \left(\prod_{s=1}^{s=N_{in}} \exp(\mathbb{A}(T_s - T_{s-1})) \cdot \mathbb{B} \right) \cdot \exp(\mathbb{A}(t - T_{N_{in}}))$$

❑ 4.2 System availability analysis

Suppose that the system is not available only when it fails, the mean value of the system failure probability over a period of time could then be used to represent the unavailability of the system

$$\overline{A_s} = \frac{1}{T}\int_0^T P_{X_F}(t)dt$$

X_F denotes that the component or the entire system is in the failed state at time t.

The availability is given by

$$A_S = 1 - \overline{A_s}$$

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❑ 4.3 Maintenance cost

The cumulative maintenance cost between two inspections in $(T_{s-1}, T_s]$ is

Minor preventive maintenance activity

Major corrective maintenance activity

$$C\left((T_{s-1}, T_s]\right) = \sum_{i=1}^{n} \left[c_{m1,i} \Pr(a+1 \le x_i(T_s) \le b) + c_{m2,i} \Pr(b+1 \le x_i(T_s) \le k)\right] = \sum_{i=1}^{n} \left[c_{m1,i} P_{a+1 \le x_i \le b}(T_s) + c_{m2,i} P_{b+1 \le x_i \le k}(T_s)\right]$$

The average life-time cost during the period T could be given by

$$C_S = \left[c_{in} N_{in} + c_p N_{in} + \sum_{s=1}^{N_{in}} C\left((T_{s-1}, T_s]\right)\right] \Big/ T + c_u \overline{A_s}$$

N_{in}	The total number of inspections
c_{in}	The inspection cost of the system for each time
$c_{m1,i}$	The cost of each minor preventive maintenance activity on component i
$c_{m2,i}$	The cost of each major corrective maintenance activity on component i
c_p	The planned downtime cost per inspection
c_u	The unplanned downtime cost of the system

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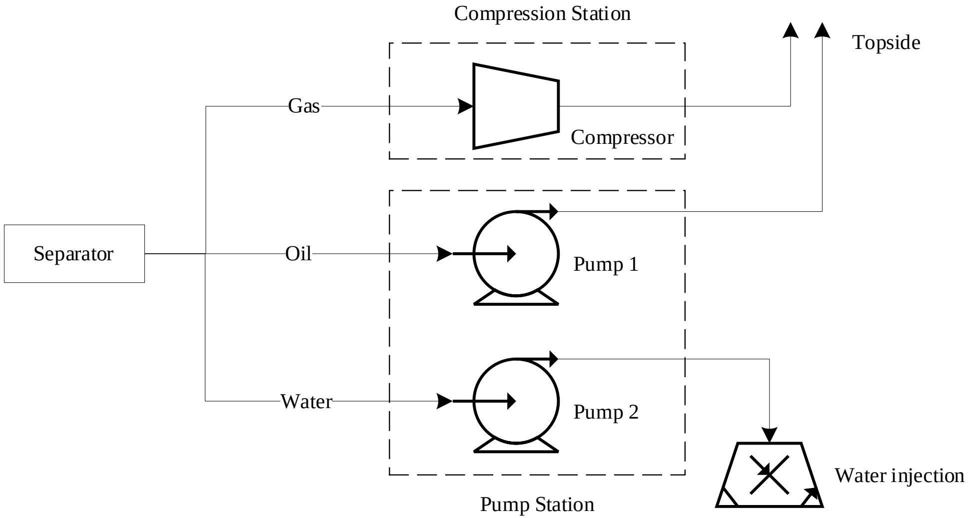
The states of the system $X = (x_1, x_2, x_3)$ from (0, 0, 0) to (3, 3, 3), are divided into 4^2 subsets (64 states in total)



0,0,0	0,0,1	0,0,2	0,0,3
1,0,0	1,0,1	1,0,2	1,0,3
2,0,0	2,0,1	2,0,2	2,0,3
3,0,0	3,0,1	3,0,2	3,0,3
0,1,0	0,1,1	0,1,2	0,1,3
1,1,0	1,1,1	1,1,2	1,1,3
2,1,0	2,1,1	2,1,2	2,1,3
3,1,0	3,1,1	3,1,2	3,1,3
0,2,0	0,2,1	0,2,2	0,2,3
1,2,0	1,2,1	1,2,2	1,2,3
2,2,0	2,2,1	2,2,2	2,2,3
3,2,0	3,2,1	3,2,2	3,2,3
0,3,0	0,3,1	0,3,2	0,3,3
1,3,0	1,3,1	1,3,2	1,3,3
2,3,0	2,3,1	2,3,2	2,3,3
3,3,0	3,3,1	3,3,2	3,3,3

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Parameter setting

Degradation rate	Value (/*year*)		Repair cost	Value (€)		Parameter	Value(€)
	Compressor	Pumps		Compressor	Pumps		
λ_0	0.046	0.104	c_{m1}	1.93×10^6	2.41×10^6	c_{in}	1.21×10^6
λ_1	0.021	0.105	c_{m2}	2.89×10^6	3.86×10^6	c_{p}	7.23×10^5
λ_2	0.041	0.056				c_{u}	6.51×10^7

Parameter	Value	Parameter	Value
γ_{12}	0.34	ϕ_0	0
γ_{13}	0.24	ϕ_1	1/3
γ_{23}	0.66	ϕ_2	2/3
γ_{21}	0.44	ϕ_3	1
γ_{31}	0.34		
γ_{32}	0.56		

❑ 5.1 Failure probabilities

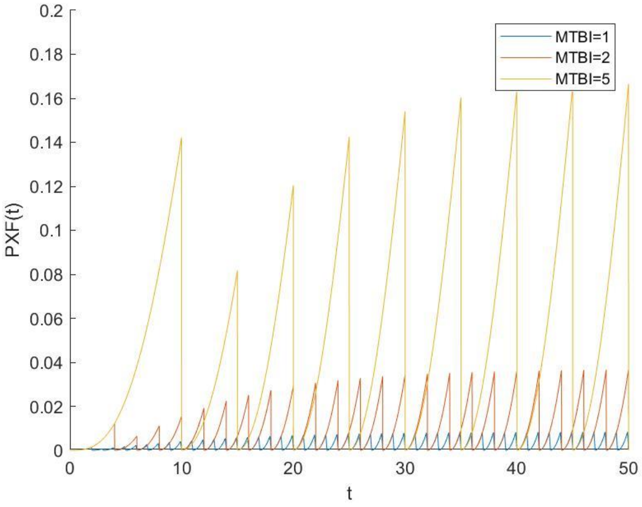
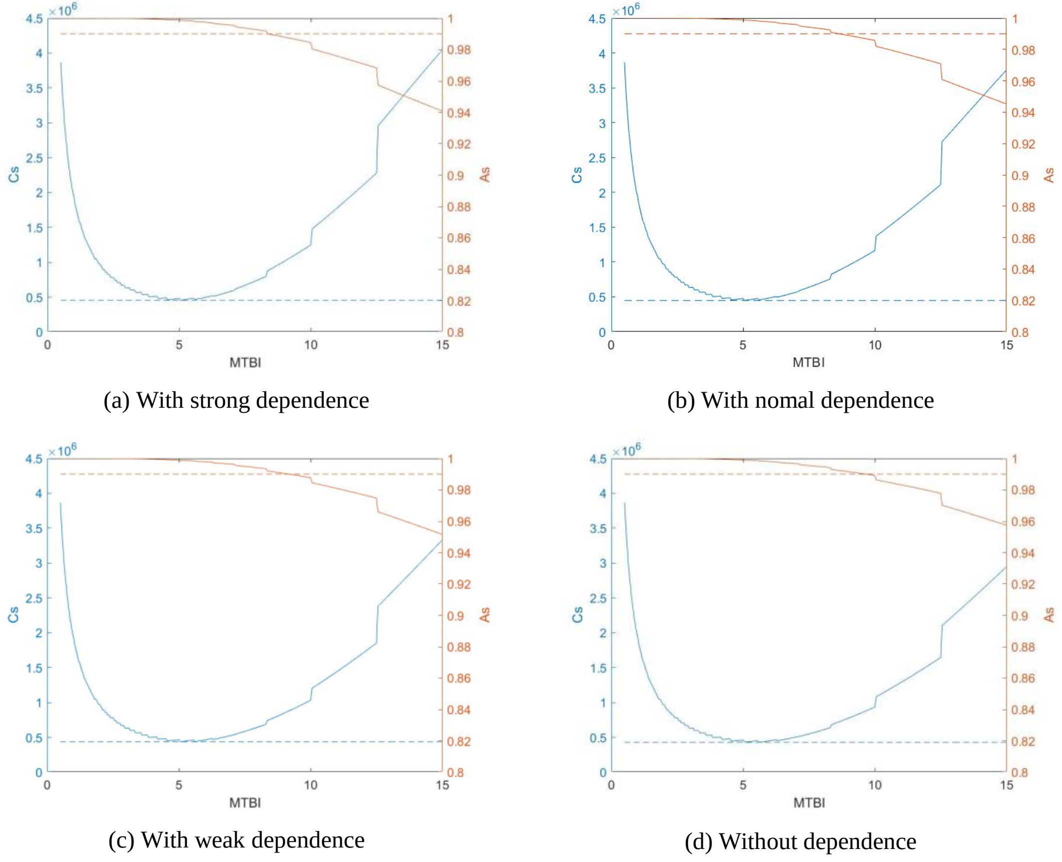


Fig. 7. Markov model of an individual component.

- The failure probability increases with time and decreases suddenly at the IMRs timepoints under varying MTBI (mean time between inspections).
- With smaller MTBI, the maximum values of failure probabilities are expected to be lower, which means that the system tends to be more reliable.

❑ 5.2 Maintenance strategies with various failure dependences



(a) With strong dependence

(b) With nomal dependence

(c) With weak dependence

(d) Without dependence

Fig. 8. Availability and average life-time cost of the transition system under different MTBI.

- The availability of the system decreases with the increase of MTBI. The average life-time cost falls initially and subsequently climbs as MTBI grows.
- These curves are not smooth, but rather contain distinct breaking lines.

- The availability of the system with stronger failure dependence is generally lower than that of the system with weaker failure dependence.
- It requires a higher investment when stronger failure dependence is considered.

5.3 Maintenance strategies for various initial costs input

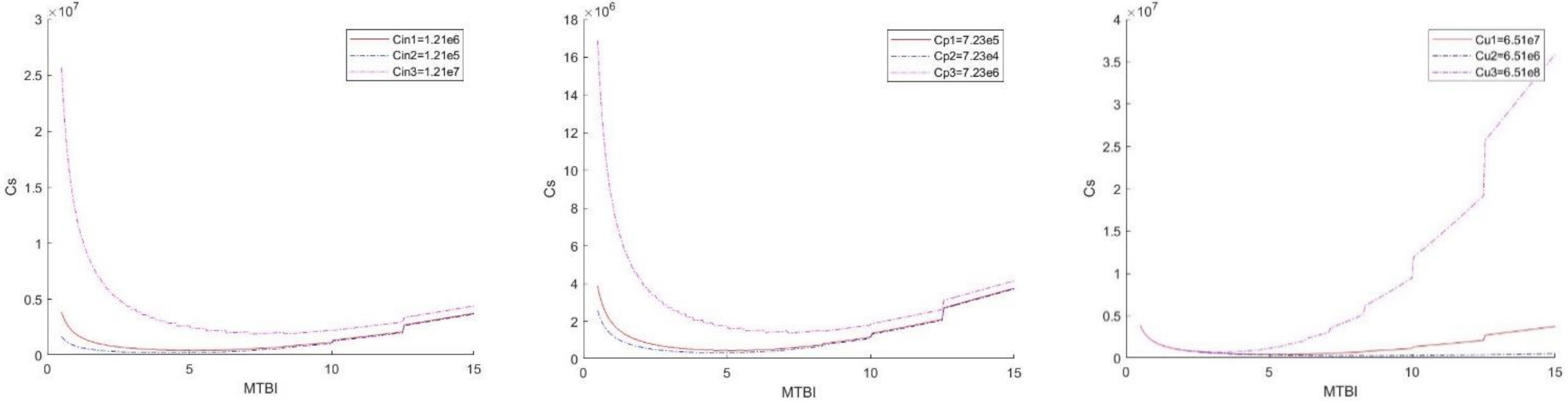


Fig. 9. Maintenance cost for different initial costs input.

- The average life-time cost basically increase as the three kinds of cost increase.
- The impact of inspection cost and the planned downtime cost are most prominent when the MTBI value is small, whereas the impact of unplanned downtime costs is most pronounced when the MTBI value is high.

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❑ Conclusions

- Proposed a framework to quantify the failure dependences between components.
- Developed a general CBM model to optimize the policy of condition-based maintenance.
- The impact of the heterogeneous failure dependences on the system maintenance strategies were discussed examining a practical subsea transmission system.

- Some other perspectives may be worth to investigate in future work.
- The applicability of the given method may be further verified applying the proposed model to the maintenance strategies of systems in other configurations.
- The comparison with other maintenance models, such as Age-based Maintenance or Opportunistic Maintenance, could be investigated to seek for the optimal maintenance policies for such complex systems.

Thanks for listening!

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