NTNU

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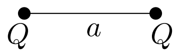
TFE4120 Electromagnetics - Crash course

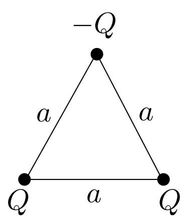
Exercise 2

Problem 1

a) Two point charges Q have a distance a from each other (see figure below). Use Coulomb's law to find the force between them.



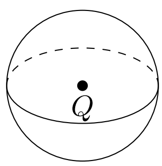
b) We add another point charge, $-Q$, so that they form an equilateral triangle with side length a (see figure below). Find the force (magnitude and direction) acting on the charge $-Q$.



c) Imagine now that we remove the charge $-Q$ at the top of the triangle and replace it with an infinitely small charge q. What is the electric field $\mathbf{E}$ perceived by this charge? Why is it important to assume that q is infinitely small when we want to find the field from the two charges Q in task (a)?

Problem 2

We have a charge Q and draw a spherical surface with radius r around it (see figure below).



a) We want to find the **E**-field flux through the surface of the sphere. Show that the flux can be expressed by

$$\oint \mathbf{E} \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^{\pi} \frac{Q}{4\pi\epsilon_0 r^2} r^2 \sin\phi d\phi d\theta,$$

and solve this to find the flux.

b) The field from the charge Q is constant along the surface of the spherical cap, and is always pointing radially outwards (we can see this from the Coulomb force); thus it is parallel with the surface elements $d\mathbf{S}$. Use this to solve the equation above in a simplified way.

c) Find the flux using the divergence theorem, and by that prove Gauss' law.

Tip: Look at the deduction of Gauss' law from the notes, compendium or book!

d) Now, let's assume that the charge Q is evenly distributed inside the volume of the sphere. Thus we have the charge density

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3},$$

inside the sphere, where a is the radius. Find the spatial electric field **E** inside and outside the sphere.

Problem 3

Given two infinitely large planes with the surface charge densities ρ_s and $-\rho_\mathrm{s}$. Find the spatial electric field.

Problem 4

a) Imagine that you have a disc with radius a that has a constant surface charge density ρ_s (see the left figure below). Show that the potential V at a height z above the center of the disc is given by

$$V(z) = \frac{1}{4\pi\epsilon_0} \int_{\text{disk}} \frac{\rho_s dS}{R},$$

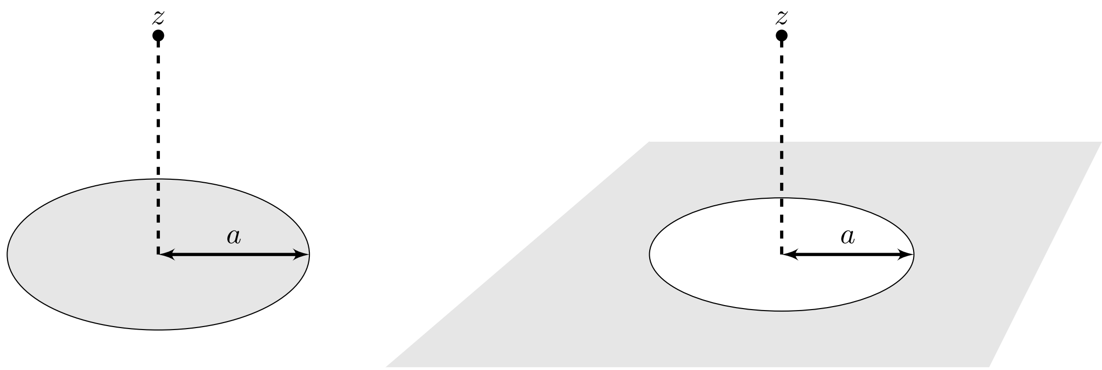
by first finding the potential from a sum of point charges using superposition. Let your point of reference be infinity and assume $z > 0$.

b) R represents the distance between a point on the disc and an observation point on the z-axis. Find an expression for R given by r (the distance between the origin and the point on the disc) and z (the height of the observation point). Find an expression for dS given by r and use these expressions to solve the integral above so that you find:

$$V(z) = \frac{\rho_s}{2\epsilon_0}\left(\sqrt{z^2 + a^2} - z\right).$$

c) Use your results from the previous task to find the electric field **E** for the same point. *Hint:* $\mathbf{E} = (0, 0, E_z)$ due to symmetry.

d) Find the electric field **E** in the limits $z \ll a$ and $z \gg a$. Interpret the results physically.



Problem 5

By using the results from the previous task, find **E** at a height z above an infinitely large plane with a hole with a radius a (see the right figure above). The plane has a constant surface charge density ρ_s.

Hint: Superposition!