Lecture 3: stationy electric field

1) Energy in electric field
2) Boundary conditon in electro-statics
3) perfect conductor
4) Ohm’s law

Electric energy in capacitor

The energy stored on a capacitor can be expressed in terms of the work done by the battery.

Voltage represents energy per unit charge

$$W_e = \int_a^b F d\boldsymbol{l} = \int_a^b qE d\boldsymbol{l} = qV$$

The work to move a charge element dq from the negative plate to the positive plate is equal to V dq

$$\mathrm{d}W_e = V dq$$

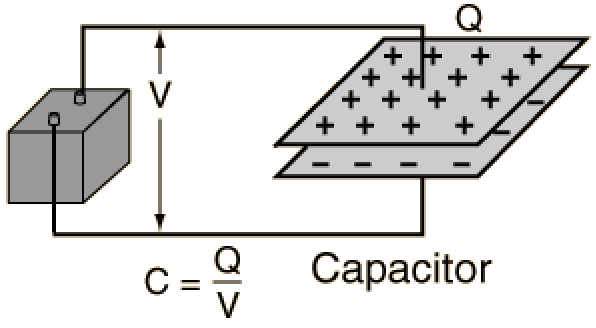
$$Q = CV$$

$$W_e = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

$$W_e = \int IU dt = \int_0^V C \frac{dU}{dt} U dt = \int_0^V CU d\boldsymbol{U} = \frac{1}{2} CV^2$$

$$\mathrm{I} = C \frac{dU}{dt}$$

$$W_e = \frac{1}{2} CV^2$$

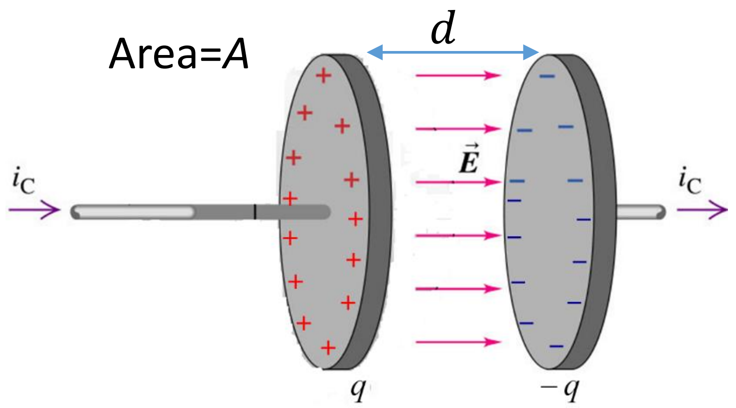


Energy in electric field

$C = \frac{\varepsilon A}{d}$ Volume=Ad

$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon A}{d}V^2 = \frac{1}{2}\varepsilon Ad\frac{V^2}{d^2} = \frac{1}{2}\varepsilon V_{vol}E^2 = \frac{1}{2}V_{vol}\boldsymbol{DE}$$

$\frac{V^2}{d^2} = E^2$ $\boldsymbol{D} = \varepsilon\boldsymbol{E}$



Electric Energy density $\eta_e = \frac{W_e}{V_{vol}} = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\boldsymbol{DE}$

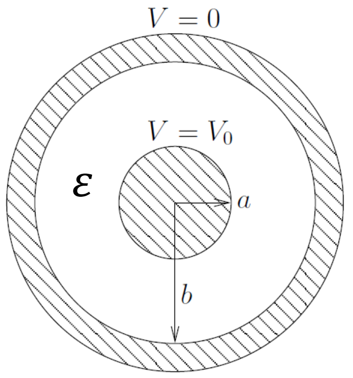
Electric Energy density

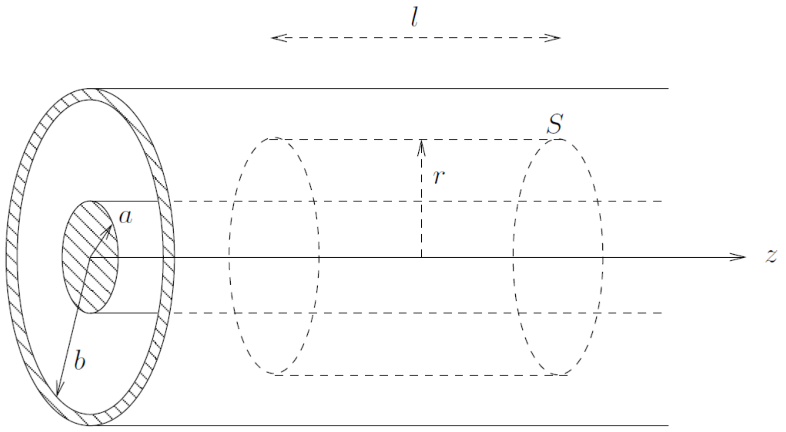
$$\eta_e = \frac{1}{2}ED = \frac{1}{2}\varepsilon E^2$$

Example:

Assumming : infinite long cable and the permittivity of the dielectric in between is ε

1) calculate the capacitance per unit length.
2) calculate the electric energy stored in unit length





$$C' = \frac{Q'}{V_0}$$

$$V_0 = \frac{Q'}{2\pi\epsilon}\ln\frac{b}{a}$$

$$C' = \frac{Q'}{V_0} = \frac{2\pi\epsilon}{\ln\frac{b}{a}}.$$

$$W'_{\mathrm{e}} = \frac{1}{2}C'V_0^2 = \frac{\pi\epsilon}{\ln\frac{b}{a}}V_0^2.$$

$$W_{\mathrm{e}} = \frac{1}{2}\int_{\text{mellom lederne}}\epsilon E^2 \mathrm{d}v = \frac{\epsilon V_0^2}{2\left(\ln\frac{b}{a}\right)^2}\int_a^b \frac{2\pi r \mathrm{d}r l}{r^2} = \frac{2\pi\epsilon V_0^2}{2\ln\frac{b}{a}}l,$$

$$W'_{\mathrm{e}} = \frac{W_{\mathrm{e}}}{l} = \frac{\pi\epsilon}{\ln\frac{b}{a}}V_0^2.$$

Boundary conditions in electrostatics

Electric field invloves more than one materials:

$$D_{n1}\,\Delta S - D_{n2}\,\Delta S = \rho_s\,\Delta S$$

Gauss' law: $\epsilon_0 \oint_S \mathbf{E}\cdot \mathrm{d}\mathbf{S} = Q_{\text{total i } S}$

$$D_{n1} - D_{n2} = \rho_s$$

Conservative field : electrostatic electric field

$$\oint \mathbf{E}\cdot \mathbf{dl} = E_{t1}\,\Delta l - E_{t2}\,\Delta l = 0$$

$$E_{t1} = E_{t2}$$

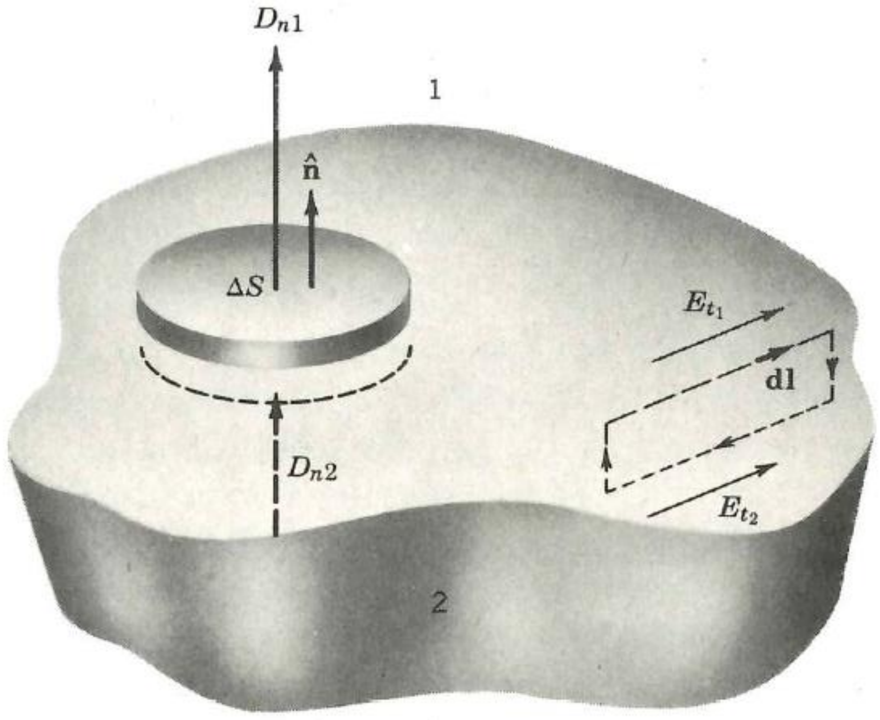
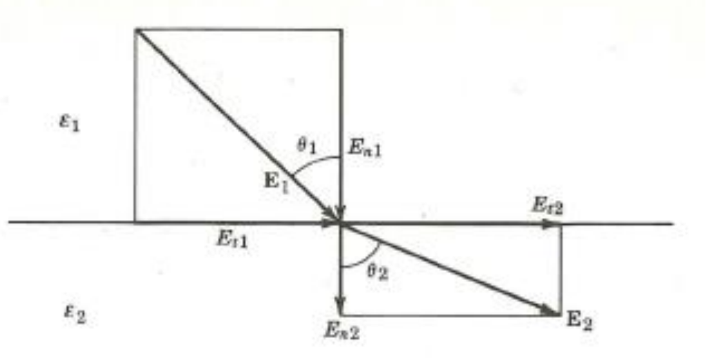


Fig. 1.14*a* Boundary between two different media.

Electric field change dirction accross two different dielectric materials



Considering, no charge on the surface between two dielectric

$$D_{n1} - D_{n2} = \rho_s = 0;$$

$$\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$$

$$E_{n2} = \frac{\varepsilon_1 E_{n1}}{\varepsilon_2}$$

$$E_{t1} = E_{t2}$$

$$E_{t2} = E_{t1} = E_{n1}\tan\theta_1$$

$$\tan\theta_1 = \frac{E_{t1}}{E_{n1}}$$

$$\theta_2 = \tan^{-1}\frac{E_{t2}}{E_{n2}} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1\right)$$

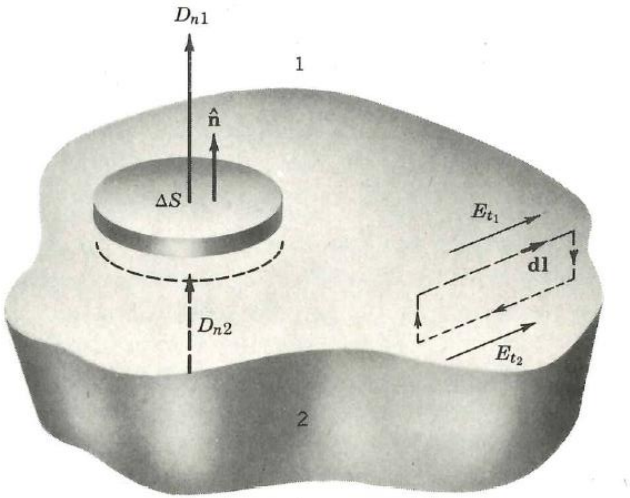


Fig. 1.14*a* Boundary between two different media.

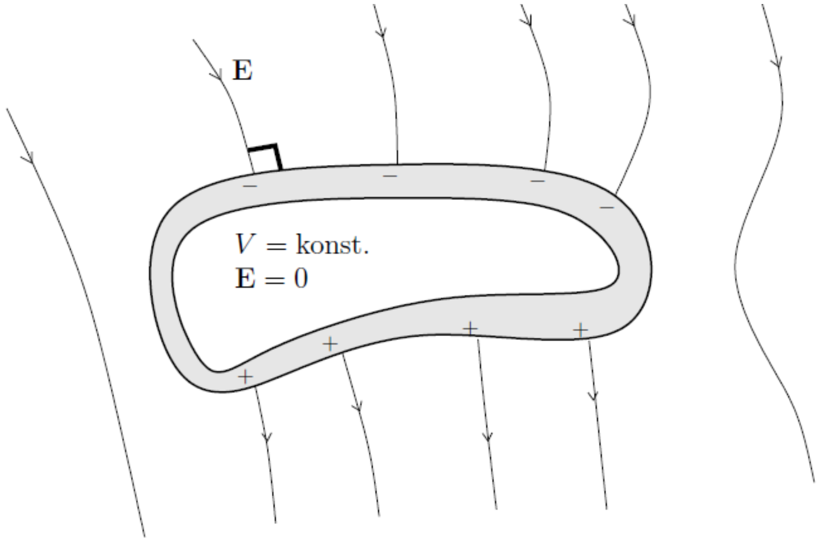
Perfect conductor in electric field

1) E=0 inside the conductor. Both $E_t = E_n$=0.
2) $\rho_{in} = 0$, no charge inside the conductor.
3) $\rho_s \neq 0$, there is surface charge .
4) The electric field outside the boundary of the perfect conduct is

$$E_n = \frac{\rho_s}{\varepsilon},\ E_t = 0,$$

5) The conductor is an equipotential surface.

$$V_{AB} = -\int_A^B Edl = 0.$$



$$D_{n1}\,\Delta S - D_{n2}\,\Delta S = \rho_s\,\Delta S$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = E_{t1}\,\Delta l - E_{t2}\,\Delta l = 0$$

$$D_{n1} - D_{n2} = \rho_s$$

$$E_{t1} = E_{t2}$$

Faraday cage:

https://www.youtube.com/watch?v=t23iXhEiQUc

Material conductivity and ohm’s law

Good conductive material:

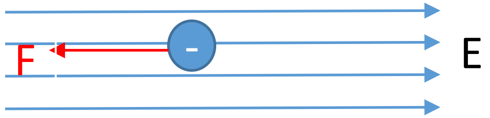
Silver: $6.2 \times 10^7 S/m$

Copper $5.8 \times 10^7 S/m$

Gold $4.1 \times 10^7 S/m$

Aluminium $3.5 \times 10^7 S/m$

Once there are free charges in an electric field, the charges can move along the electric field direction.



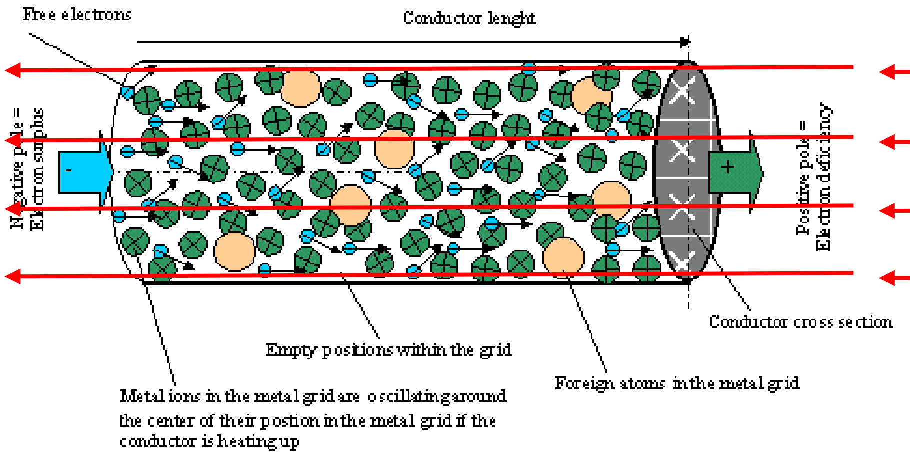
That how many charges can move is dependent of material and electric field
When electrons move, they collide atoms and lost energy.
The capability is represented by σ, conductivity.
J is called current density. The charges go through the material per unit area per second.

$$\mathbf{J} = \sigma \mathbf{E}$$

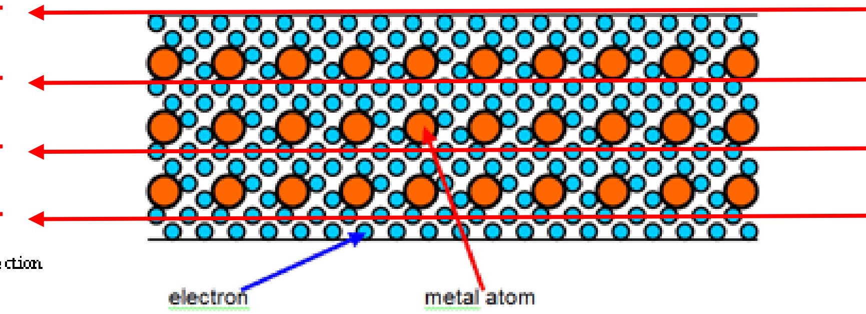
Non conductive material

Glass $\times 10^{-12} S/m$

Air $\times 10^{-14} S/m$



Low conductive material



High conductive material

Ohm’s law (II)

An electric current is a flow of electric charge.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Current density *J* is the current per unit area

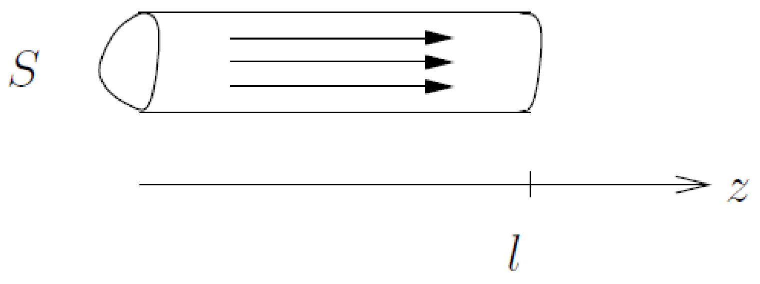
$$I = \int_S \mathbf{J} \cdot \mathrm{d}\mathbf{S}.$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$R = \frac{V}{I}$$

Example:

A conductor with constant conductivity σ, and cross − sectional area S , the length is l and the constant current is I



1) Calculating the resistance of the conductor and deriving $R = \frac{V}{I}$.
2) Calculating the power done by the current.

Area *S*, and I is also constant. $J = I/S$

Constant σ , the electric field is constant: $E = J/\sigma$

$$V = \int_0^l E\mathrm{d}z = \int_0^l (J/\sigma)\mathrm{d}z = Jl/\sigma$$

$$R = \frac{l}{S\sigma} = \frac{Jl}{\sigma}\frac{1}{JS} = \frac{V}{I}$$

Vector: $\vec{\boldsymbol{E}} = \frac{\boldsymbol{Q}}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}}$

Vector: $\vec{\boldsymbol{F}} = \frac{\boldsymbol{qQ}}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}}$: ***Coulomb's law***

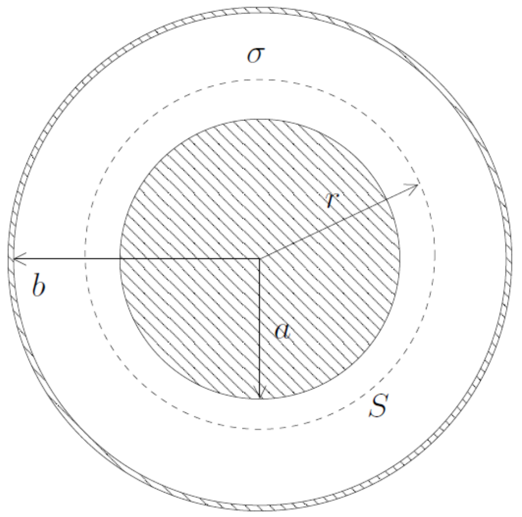
$\vec{\boldsymbol{E}} = \vec{\boldsymbol{F}}/\boldsymbol{q}$ $\quad V = -\int \boldsymbol{E}.\boldsymbol{dl}$

$$W_e = \int_0^l Fdl = \int_0^l q\boldsymbol{E}d\boldsymbol{l} = qV \Rightarrow W_e = \int VIdt$$

q= $\int Idt$

$$P = VI = I^2R$$

Example:



A solid conductive ball with a radius *a* is put into a hollow conductive ball with inner radius *b*, between is a type material with conductivity σ .

a) What is the resistance between the two balls?
b) If the solid ball is buried deeply in to earth, what is the earth resistance?

$$a = 0.5 \text{ m} \qquad \sigma = 10^{-2} \text{ m}^{-1}\Omega^{-1}$$

$$\mathbf{J} = \frac{I}{4\pi r^2}\hat{\mathbf{r}}, \quad \text{for } a < r < b.$$

$$\mathbf{E} = \mathbf{J}/\sigma = I\hat{\mathbf{r}}/(4\pi\sigma r^2)$$

$$V = -\int_a^b E\mathrm{d}r = -\int_a^b \frac{I}{4\pi\sigma r^2}\mathrm{d}r = \frac{I}{4\pi\sigma}\left(-\frac{1}{b} + \frac{1}{a}\right)$$

$$R = \frac{V}{I} = \frac{1}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{b}\right)$$

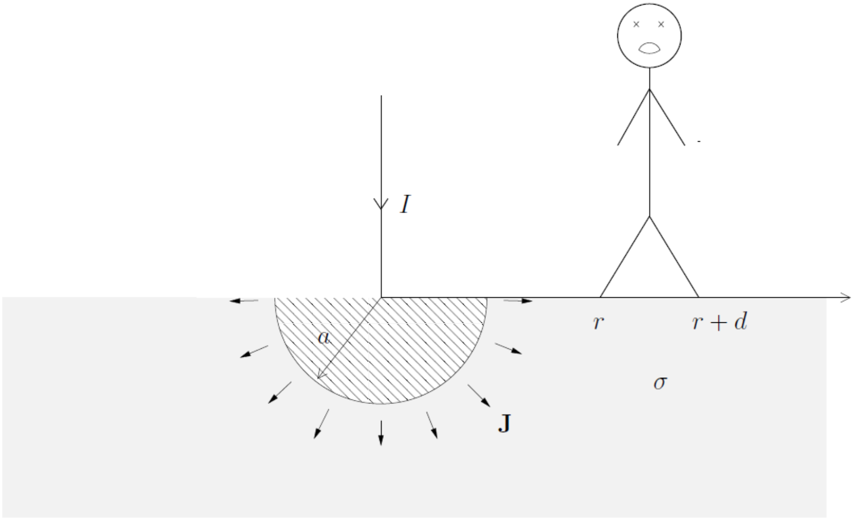
example

Now if a half solid ball is buried in earth as shown in picture, recalculate the resistance?
If the current is 1000A $\sigma = 10^{-2}\,\mathrm{m}^{-1}\Omega^{-1}$, $r = 1\,\mathrm{m}$ og $d = 0.75\,\mathrm{m}$
What is the voltage between the two legs of the people

$$\mathbf{J} = \frac{I}{2\pi r^2}\hat{\mathbf{r}}, \quad \mathbf{E} = \frac{I}{2\pi\sigma r^2}\hat{\mathbf{r}}, \quad \text{for } r > a.$$

$$R = \frac{1}{2\pi\sigma a}$$

$$V = -\int_r^{r+d} E(r)\mathrm{d}r = -\int_r^{r+d} \frac{I}{2\pi\sigma r^2}\mathrm{d}r = \frac{I}{2\pi\sigma}\left(\frac{1}{r} - \frac{1}{r+d}\right)$$



Kirchoff’s law: for constant

Current conservation:

Kirchoff’s law: at any point at electric circuit, the sum of the current is zero.

$$\oint_S \mathbf{J} \cdot \mathrm{d}\mathbf{S} = 0.$$