Lecture 6: Electromagnetic field and wave

- Electric potential in electro-dynamic
- Electromagnetic wave
- Energy stored in electromagnetic field
- Poynting’s theorem

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Electric potential in electric-dynamic field

In stationary electric field, conservative **E** field has:

$$V = \oint \mathbf{E} \cdot d\mathbf{l} = 0$$

Stationary electric field:

$$\nabla \times \mathbf{E} = 0$$

$$\mathbf{E} = -\nabla V$$

In dynamic field, vector/magnetic potential, **A**, can be introduced due to $\nabla \cdot \mathbf{B} = 0$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) = \nabla \times (-\frac{\partial \mathbf{A}}{\partial t})$$

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) = 0$$

Electric/scalar potential *V* can then be introduced

Electro-dynamic field:

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) = 0$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V$$

Maxwell's equations in vacuum

In vacuum, source free

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t} \nabla \times \mathbf{B} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$
$$\nabla \times (\nabla \times \mathbf{B}) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \nabla \times \mathbf{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Vector identity:

$$\nabla \times (\nabla \times \mathbf{V}) = \nabla (\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V}$$

In vacuum: $\nabla \cdot \mathbf{E} = 0$, $\nabla \cdot \mathbf{B} = 0$

$$\frac{1}{c_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = \mathbf{0}$$
$$\frac{1}{c_0^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = \mathbf{0}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}792{,}500 \text{ m/s}$$

Electromagnetic waves in time domain

$$\frac{1}{c_0^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = \mathbf{0}$$

$$\frac{1}{c_0^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = \mathbf{0}$$

3D

$$\frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} + \frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{1}{c_0{}^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

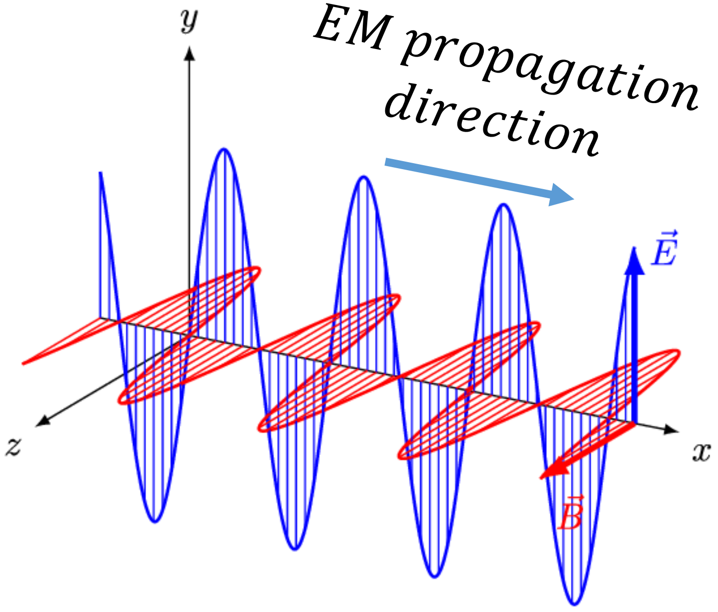
1D

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c_0{}^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

$$f = \frac{1}{T}$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{\omega}{2\pi}$$



Electromagnetic waves in frequency/Fourier domain

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

Helmholtz equations

$$k = \sqrt{\mu \varepsilon \omega^2}$$

Wave number

Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Time-harmonic electromagnetics

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = \mu \mathbf{J} + \frac{\mu\varepsilon \partial \mathbf{E}}{\partial t}$$

Electromagnetic fields

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V$$

Given an arbitrary function *f*, we define

$$\mathbf{A}' = \mathbf{A} + \nabla f,$$
$$V' = V - \frac{\partial f}{\partial t}.$$

Lorentz gauge

$$\nabla \cdot \mathbf{A} + \varepsilon\mu \frac{\partial V}{\partial t} = 0$$

Coulomb gauge $\nabla \cdot \mathbf{A} = 0$

Antenna equations

With **A** and V satisfying LG:

$$\nabla^2 V - \epsilon\mu \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$$
$$\nabla^2 \mathbf{A} - \epsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

Impedance

Impedance, Z, is a physical parameter relating the magnitudes of the electric and magnetic fields.

$$Z = \frac{E}{H}$$

E: Amplitude of electric field

H: Amplitude of magnetic field

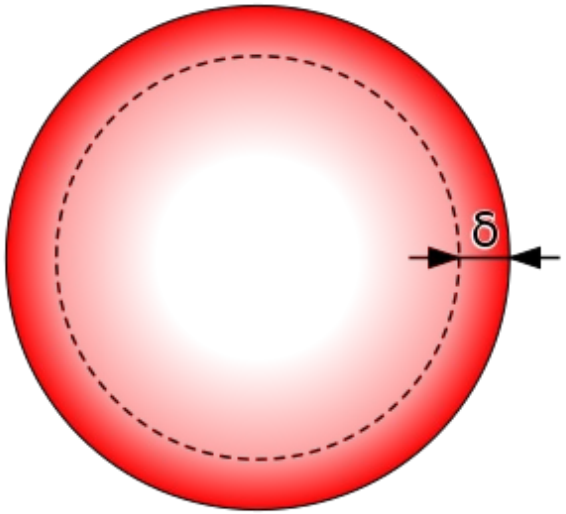
In vacuum,

$$Z_0 = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Skin effect

For alternating current (AC), current density/electric field in a conductor decreases exponentially from the surface towards the inside.

Skin depth, δ, is defined as the depth where the current density/electric field is reduced to 1/e.



$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

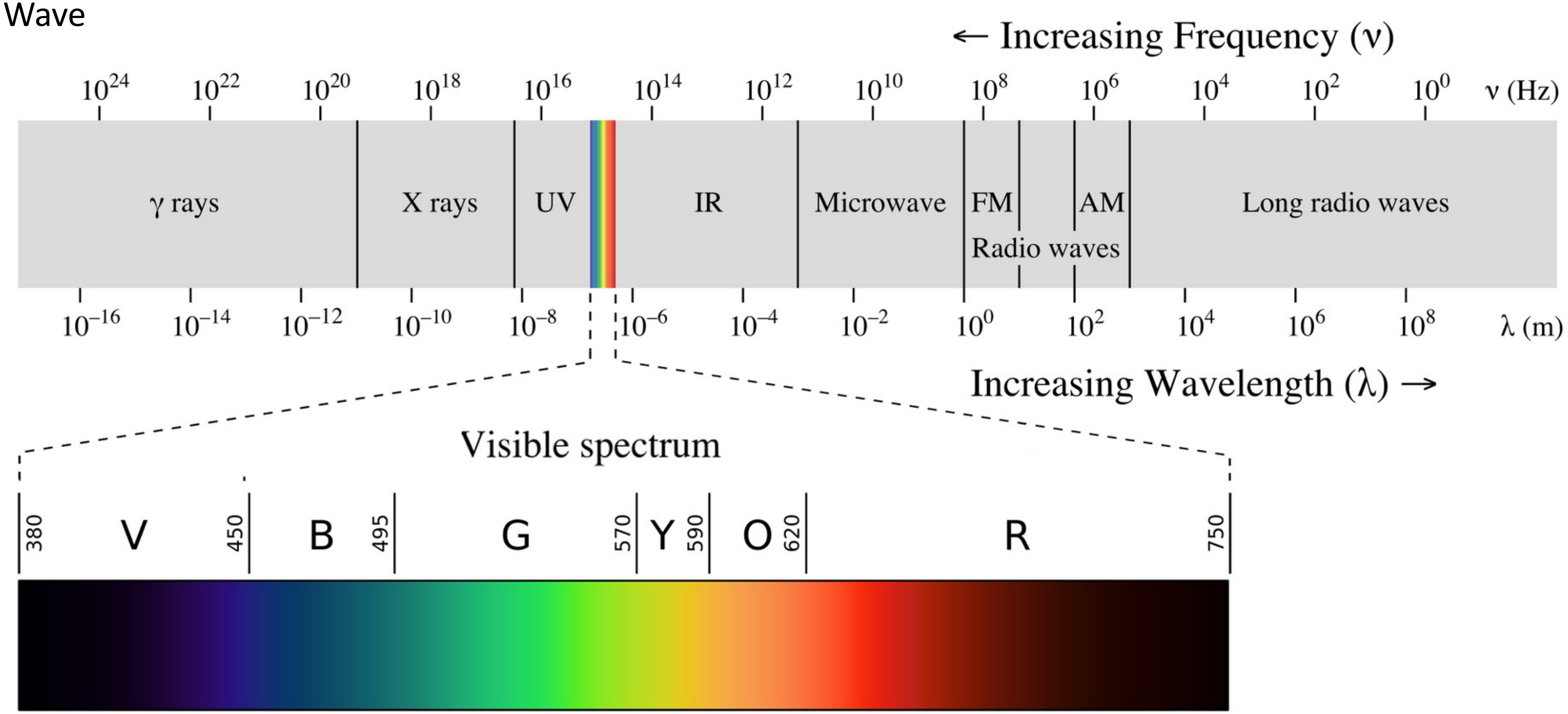
ρ: Resistivity

ω: Angular frequency

Electromagnetic spectrum

Diffusive field

Wave



Poynting's theorem: Power flow in electromagnetic fields

Poynting's theorem states that the rate of energy transfer per unit volume from a region of space equals the rate of work done on the charge distribution in the region, plus the energy flux leaving that region.

$$-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon\mathbf{E}^2+\frac{1}{2}\mu\mathbf{H}^2\right) = \mathbf{J}\cdot\mathbf{E}+\boldsymbol{\nabla}\cdot(\mathbf{E}\times\mathbf{H})$$

$$-\boldsymbol{\nabla}\cdot(\mathbf{E}\times\mathbf{H}) = \mathbf{E}\cdot\boxed{\boldsymbol{\nabla}\times\mathbf{H}}-\mathbf{H}\cdot\boxed{\boldsymbol{\nabla}\times\mathbf{E}} = \mathbf{J}\cdot\mathbf{E}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}$$

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\,dv$$

$$\int_v\left(\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\boxed{\mathbf{E}\cdot\mathbf{J}}\right)dv = -\oint_s(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{s}$$

$$W = -\oint(\mathbf{E}\times\mathbf{H})\cdot ds$$

$$\mathbf{E}\cdot\mathbf{J} = \mathbf{J}^2\rho$$

Ohmic power density

$$\frac{1}{2}\frac{\partial(\mathbf{B}\cdot\mathbf{H})}{\partial t} = \boxed{\frac{1}{2}\frac{\partial(\mu\mathbf{H}^2)}{\partial t}} = \mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}$$

Magnetic energy density

$$\frac{1}{2}\frac{\partial(\mathbf{D}\cdot\mathbf{E})}{\partial t} = \boxed{\frac{1}{2}\frac{\partial(\varepsilon\mathbf{E}^2)}{\partial t}} = \mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

Electric energy density

Poynting vector

The integral of **P** over a closed surface equals the power leaving the enclosed volume.

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \ (\mathrm{W/m^2})$$

Poynting's vector: Power density

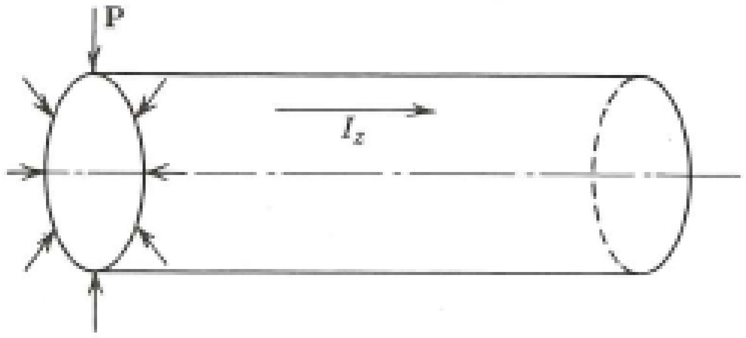
$$\mathbf{W} = -\oint_{s} (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{s} = \int_{v} \mathbf{P} dv$$

The energy flow direction is perpendicular to both **E** and **H** fields.

Example

One conductor with DC current I_z, if R is the resistane per unit length.

Calculating the loss over the conductor in unit length.



Poynting vector directed radially inward

$$E_z = I_z R$$

$$H_\phi = \frac{I_z}{2\pi r}$$

$$P_r = -E_z H_\phi = -\frac{R I_z^2}{2\pi r}$$

P has no component normal to the end surfaces

$$W = 2\pi r(-P_r) = I_z^2 R$$

Surface integral over the conductor

Summary

Divergence and Stokes' theorem

Gradient: fastest rate of increase in spatial

$$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

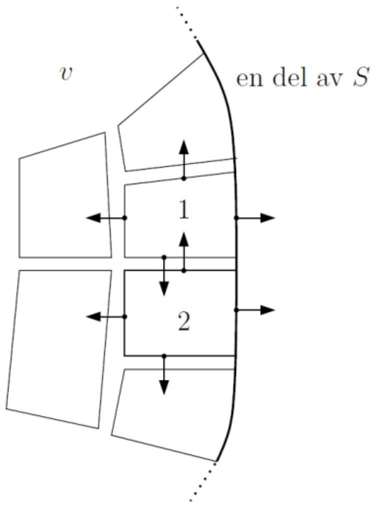
Divergence: Flux out of a point

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

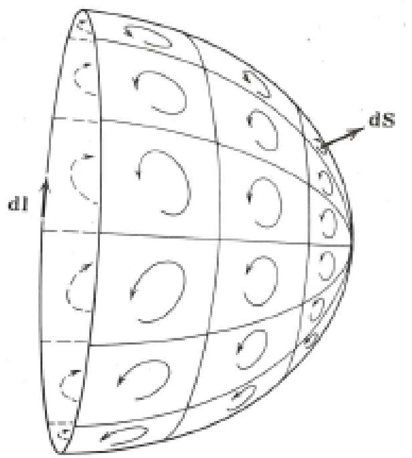
How much does a field circulate around a point

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{z}}$$

$$\oint_S \mathbf{A} \cdot \mathrm{d}\mathbf{S} = \int_v \nabla \cdot \mathbf{A}\,\mathrm{d}v$$



$$\oint_C \mathbf{A} \cdot \mathrm{d}\mathbf{l} = \int_S \nabla \times \mathbf{A} \cdot \mathrm{d}\mathbf{S}.$$



Electric field and electric displacement field

Electric field: $\vec{\boldsymbol{E}} = \frac{\boldsymbol{Q}}{\boldsymbol{4\pi\varepsilon_0 r^2}}\hat{\boldsymbol{r}}$ $\vec{\boldsymbol{E}} = \vec{\boldsymbol{F}}/\boldsymbol{q}$

Electric displacement field: $\mathrm{D} = \varepsilon_0 \mathbf{E}$ $\vec{\boldsymbol{D}} = \frac{Q}{4\pi r^2}\hat{\boldsymbol{r}}$

Electric field and potential

$$\mathbf{E} = -\nabla V,\ \mathrm{V/m}$$

The potential difference between two points A and B:

$$V_{AB} = -\int_{\boldsymbol{r}_A}^{\boldsymbol{r}_B} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 {r_i}^2}\, dr_i = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$V = -\oint_c \mathbf{E}\cdot d\boldsymbol{l} = 0$$ Conservative vector field

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Poisson's equation

Inserting $\mathbf{E} = -\nabla V$ into Maxwell's equation $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ gives

$$-\nabla \cdot (\nabla V) = \frac{\rho}{\epsilon_0},$$

which gives

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}.$$

Gauss's law

Electric flux flowing out of a closed surface = Enclosed total charges

divided by the permittivity ⇨ $\oint \mathbf{E} d\boldsymbol{s} = Q/\boldsymbol{\varepsilon}$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}; \qquad \nabla \cdot \mathbf{D} = \rho$$

Electric polarization and material permittivity

The influence of electric polarization: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E} = (1 + \chi_e)\varepsilon_0 \mathbf{E} = \varepsilon_r \varepsilon_0 \mathbf{E} = \varepsilon \mathbf{E}$$

- χ_e is ***electric susceptibility***
- $1 + \chi_e = \varepsilon_r$, ***relative permittivity***
- $\varepsilon = \varepsilon_r \varepsilon_0$, ***electric permittivity***

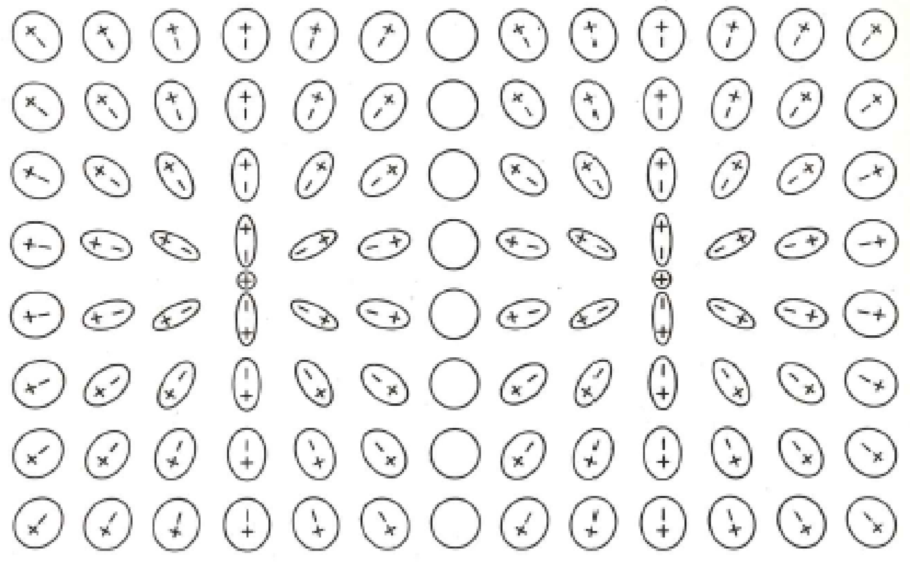


Fig. 1.3c Polarization of the atoms of a dielectric by a pair of equal positive charges.

Capacitor and electric energy

Capacitance is the ability of capacitor to store electrical charge.

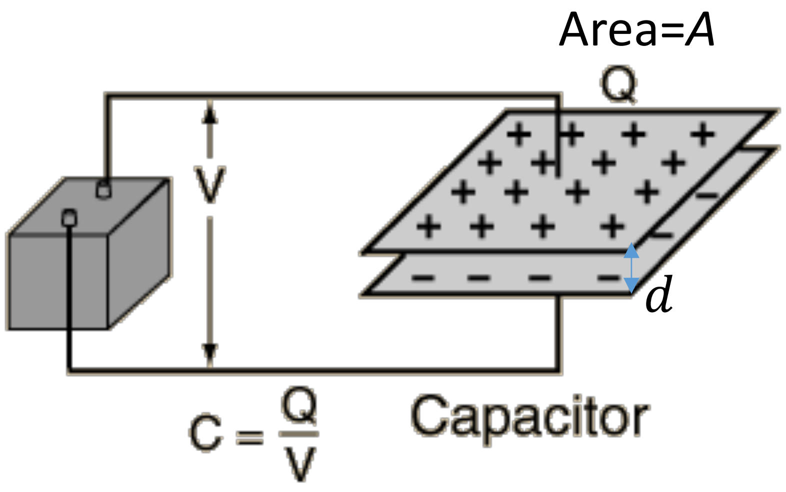
$$C = \frac{Q}{V} = \frac{\varepsilon A}{d}$$

Electric energy in capacitor

$$W_e = \frac{1}{2} C V^2$$

Electric energy density for electric field

$$\eta_e = \frac{1}{2} E D = \frac{1}{2} \varepsilon E^2$$



Conductivity and current

The capability of allowing current to flow is defined by conductivity: $\sigma = 1/\rho$.

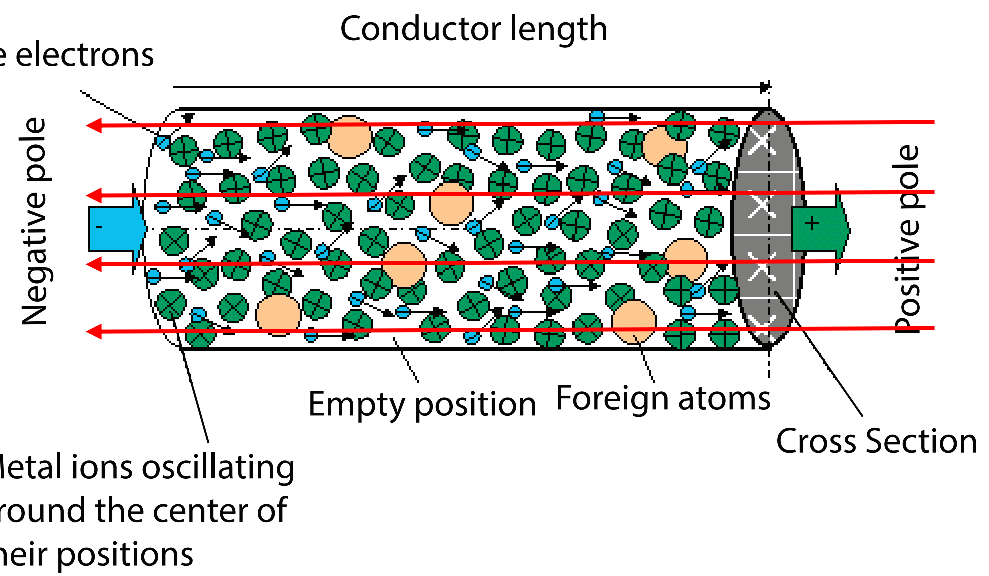
When electrons move, they collide atoms and lost energy. → Resistance: $R = \rho \frac{l}{S}$

An electric current is a flow of electric charge

$$I = \frac{dQ}{dt}$$

Current density **J** is the current per unit area

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}.$$



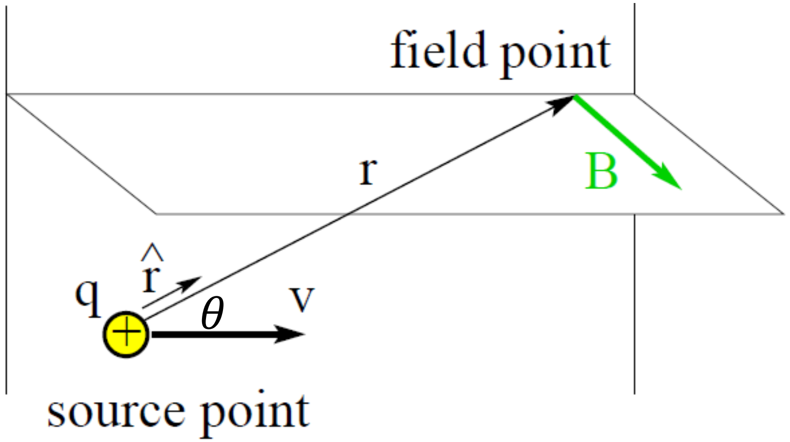
Ohm's law $I = \frac{V}{R}$ $\mathbf{J} = \frac{\mathbf{E}}{\rho}$

Kirchhoff's law $\oint_S \mathbf{J} \cdot d\mathbf{S} = 0.$

Magnetic field

Charges in motion generate magnetic field

Magnetic field H in vacuum generated by moving charge q: $\boldsymbol{H} = \frac{1}{4\pi}\frac{q\boldsymbol{v}\times\hat{\boldsymbol{r}}}{r^2}$



Magnetic flux density B in vacuum generated by moving charge q: $\mathbf{B} = \mu_0\,\mathbf{H}$

$\boldsymbol{v}\times\hat{\boldsymbol{r}} = |v|\sin(\theta)\,\boldsymbol{n}$ **n** is given by **the right-hand rule**

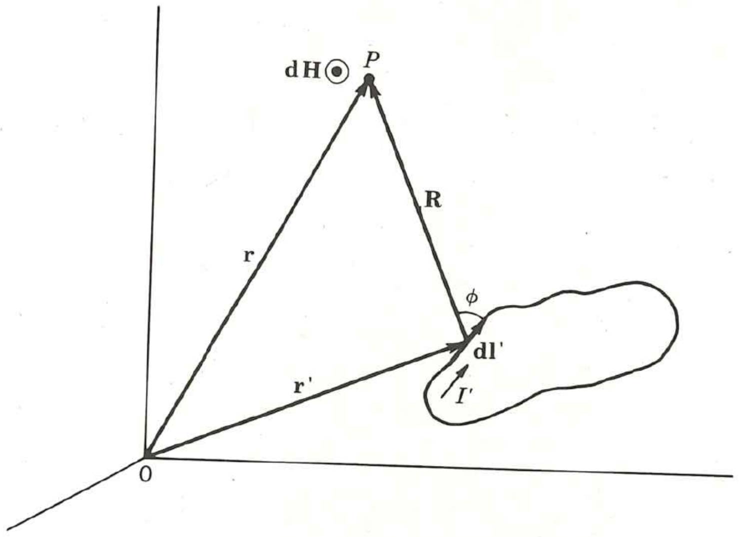
In free space is $\mu_0 = 4\pi\times10^{-7} H/m.$

A *steady current* *I* generates magnetic field, Biot–Savart law

$$\boldsymbol{H}(\boldsymbol{r}) = \int_c \frac{I'(r)d\boldsymbol{l}'\times\widehat{\boldsymbol{R}}}{4\pi R^2}$$

$$I'd\boldsymbol{l}'\times\widehat{\boldsymbol{R}} = |I'|dl'\sin(\phi)\,\boldsymbol{n}$$

n is given by **the right-hand rule**

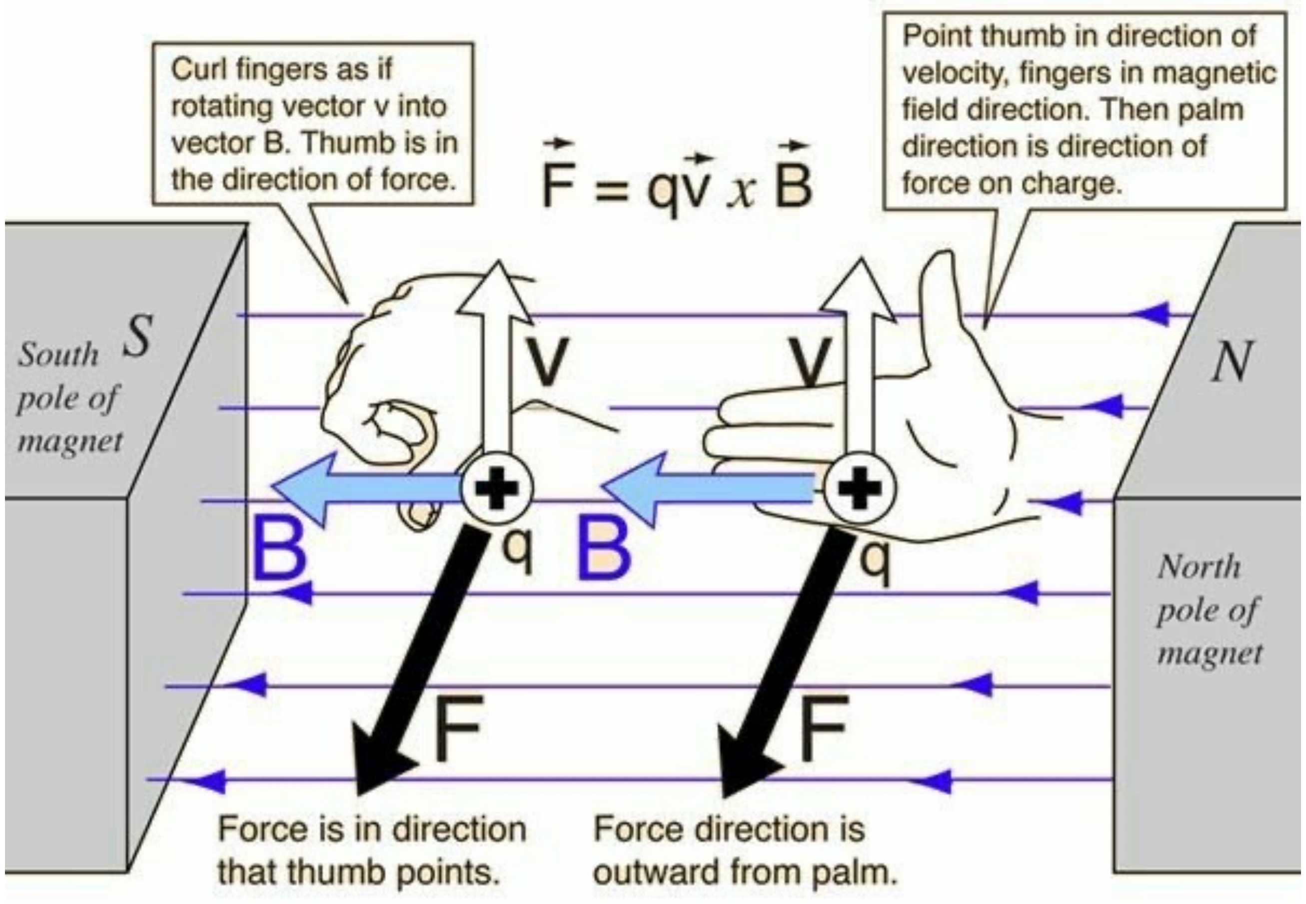


Lorentz law

Force exerted by magnetic field **B** on a moving point charge Q is:

$$\boldsymbol{F} = Q\,\boldsymbol{v}\times\boldsymbol{B}$$

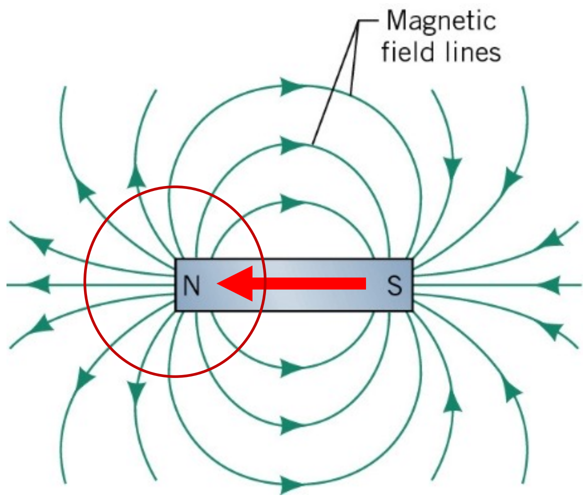
The direction is given by the right-hand rule



Lorentz law

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

Magnetic flux and Gauss's law



Magnetic flux ϕ is the integral of the flux density across surface

$$\phi = \int_s \mathbf{B} \cdot d\mathbf{S}$$

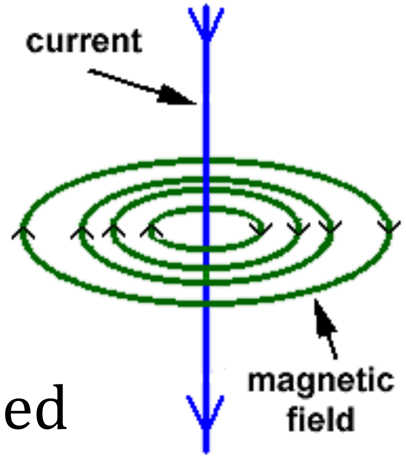
For an enclosed surface, the flux is zero

$$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0.$$

Gauss's law

$$\nabla \cdot \mathbf{B} = 0$$

Ampere's law



Ampere's law states that the line integral of the magnetic field around closed loop C is equal to the electric current enclosed in the loop.

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int_s \mathbf{J} \cdot d\mathbf{s} = I$$

Currents generate magnetic field

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

Magnetic field in material

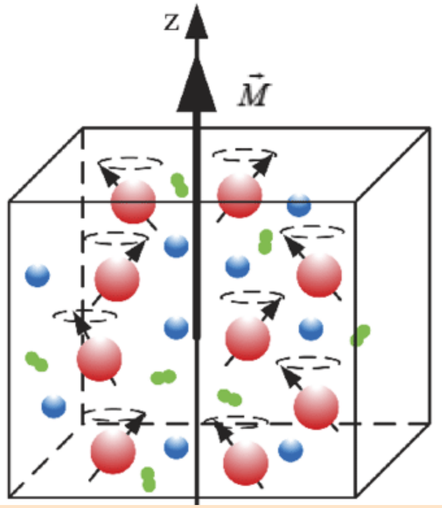
Once there is magnetic field applied to medium, magnetization, **M**, occurs.

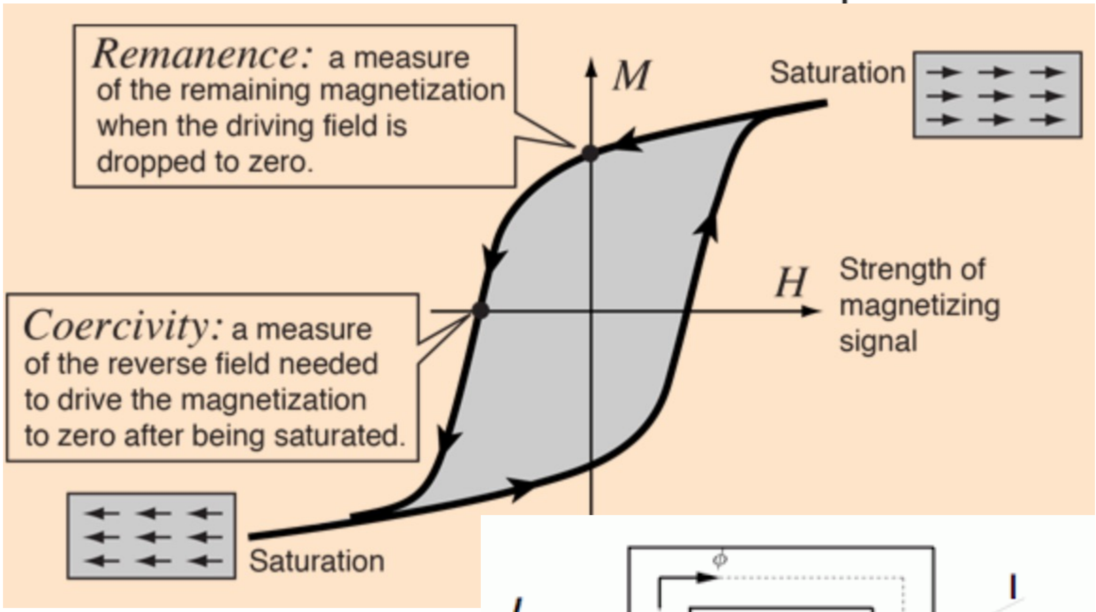
$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

$$\mathbf{M} = \chi_m \mathbf{H}$$

$$\mathbf{B} = \mu_0(1 + \chi_m)\boldsymbol{H} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H}$$

χ_m is magnetic susceptibility, used to quantify the additional field **M**.
μ_r *relative permeability*.



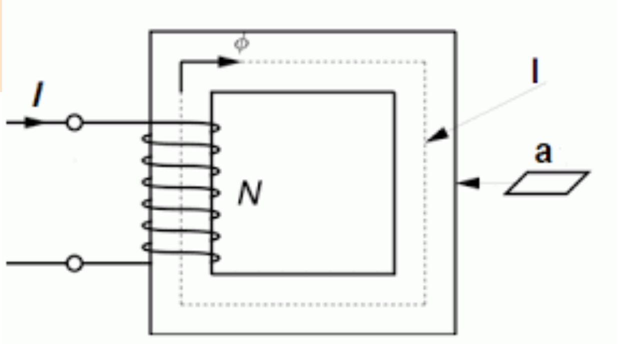


Magnetic circuit

Magnetomotive force (MMF): $F = NI = HL = \Phi R$

Φ, magnetic flux

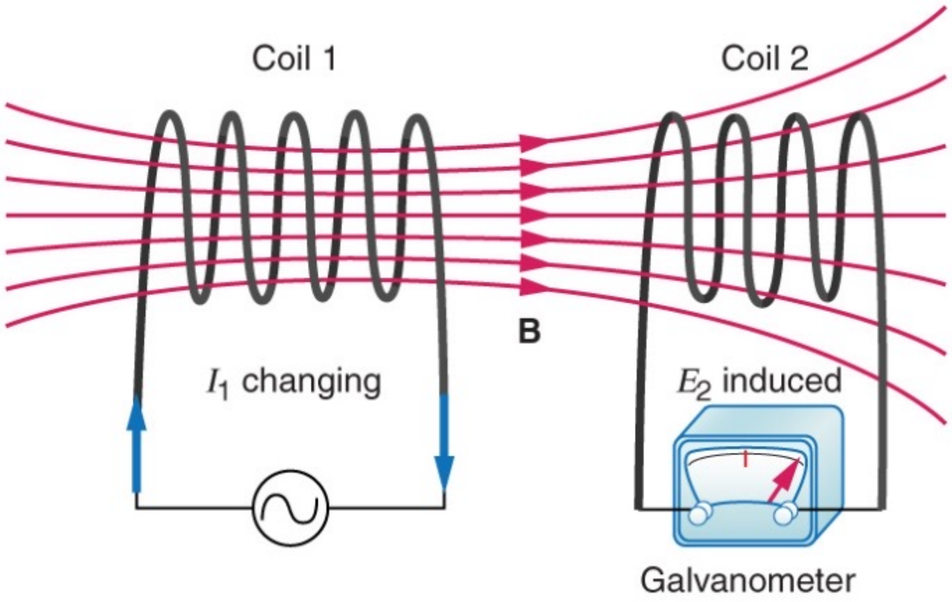
$R = \frac{l}{\mu S}$, magnetic reluctance



Faraday's law

$\boldsymbol{\varepsilon} = -\frac{d\phi}{dt}$ **Faraday's law,** $\phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$ **is the magnetic flux.**

$$\nabla \times \boldsymbol{E} = -\frac{\partial B}{\partial t}$$

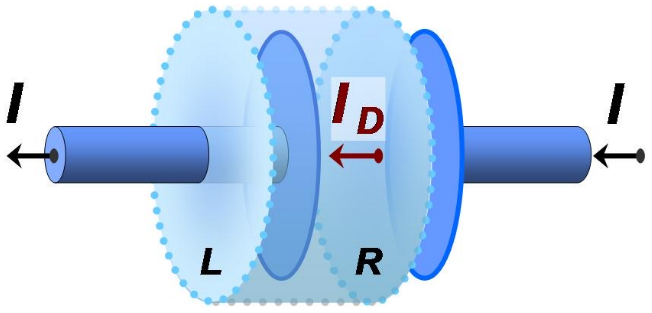


Time varying magnetic field generates electric field.

Displacement current

Displacement current is defined as the rate of change of electric displacement field.

$$\mathbf{J}_D = \frac{\partial \mathbf{D}}{\partial t}$$



Capacitor

Ampere's law

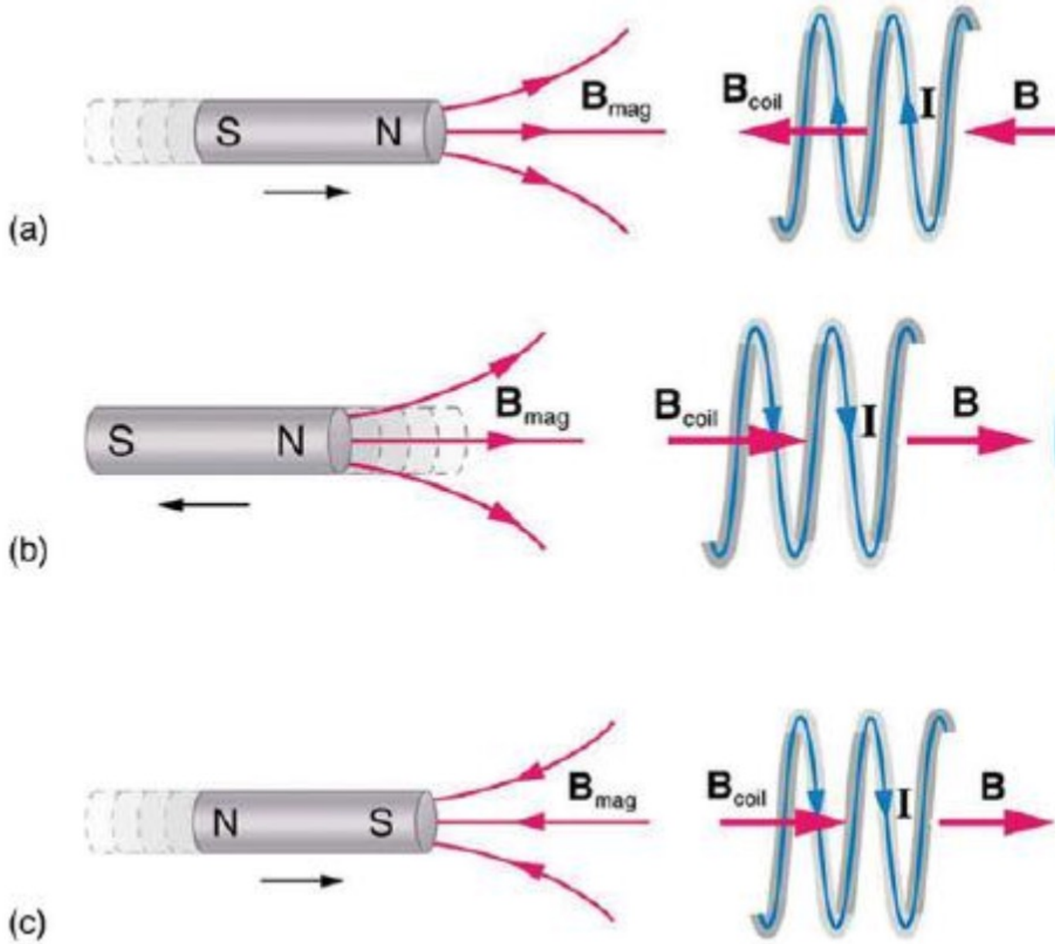
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$\mathbf{J} = \sigma \mathbf{E}$ is conduction current in materials

$\frac{\partial \mathbf{D}}{\partial t}$ is displacement current

Time varying electric field generates magnetic field.

Lenz’s law

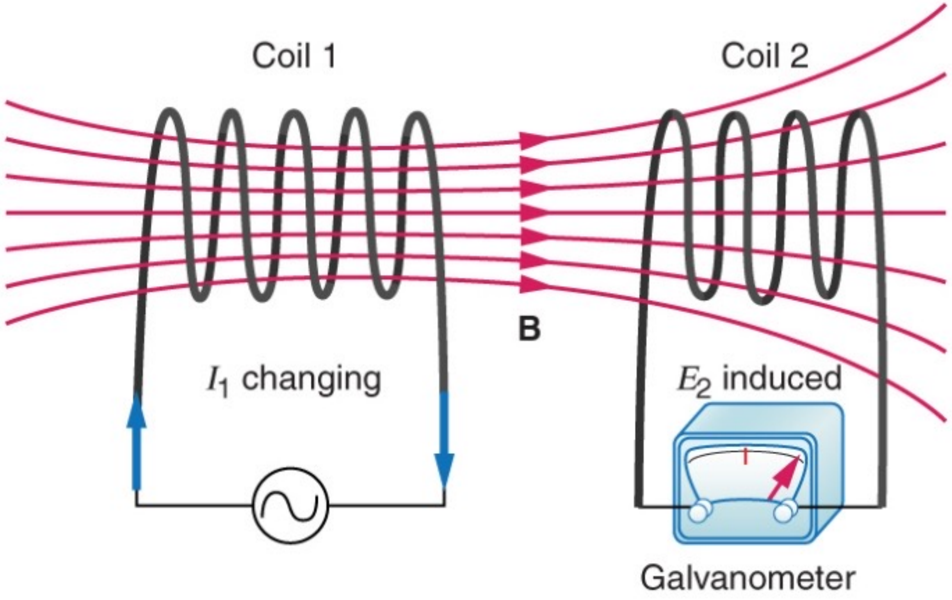


The magnetic field created by the induced current opposes changes in the initial magnetic field.

Inductor and magnetic energy

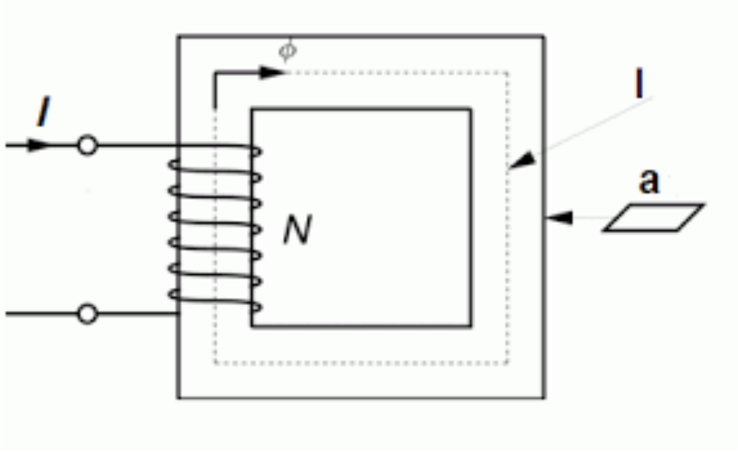
$$L = \frac{\int_S \boldsymbol{B} \cdot d\boldsymbol{S}}{I}$$

$$W = \int_0^I LI\frac{dI}{dt}dt = \int_0^I LIdI = \frac{1}{2}LI^2$$



Magnetic energy density

$$\eta_m = \frac{W}{V_{vol}} = \frac{B^2}{2\mu}$$



$$L = \frac{N^2 a\mu}{l} = \frac{N^2}{R_m}$$

Electro-magnetic waves in time domain

$$\frac{1}{c_0^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = \mathbf{0}$$

$$\frac{1}{c_0^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = \mathbf{0}$$

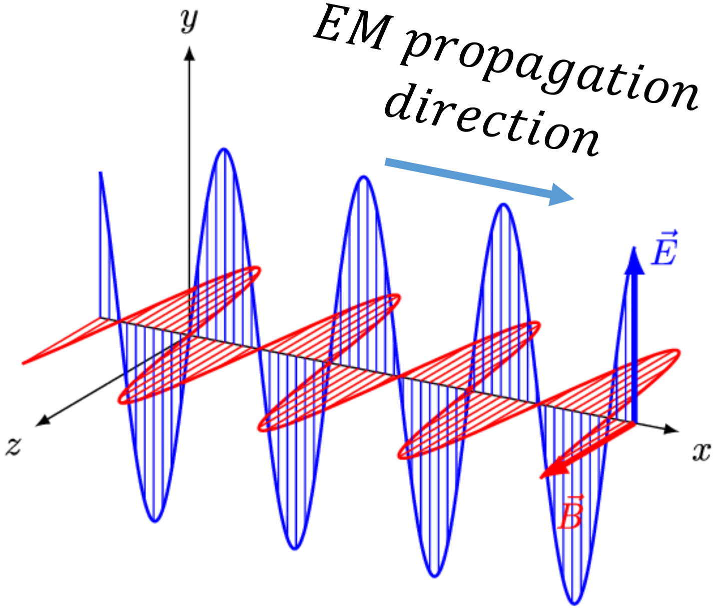
3D

$$\frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} + \frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{1}{c_0{}^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

1D

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c_0{}^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$



Electro-magnetic waves in frequency/Fourier domain

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

Helmholtz equations

$$k = \sqrt{\mu \varepsilon \omega^2}$$

Wave number

Boundary condition

$$D_{n1} - D_{n2} = \rho_s \qquad E_{t1} = E_{t2}$$

$$H_{t1} - H_{t2} = J_s \qquad B_{n1} = B_{n2}$$

Electromagnetic waves in media

Electromagnetic fields

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V$$

Antenna equations

Lorentz gauge

$$\nabla \cdot \mathbf{A} + \varepsilon\mu \frac{\partial V}{\partial t} = 0$$

With **A** and V satisfying LG:

$$\nabla^2 V - \epsilon\mu \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$$

$$\nabla^2 \mathbf{A} - \epsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

Impedance

$$Z = \frac{E}{H}$$

E: Amplitude of electric field

H: Amplitude of magnetic field

Skin depth

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

ρ: Resistivity

ω: Angular frequency

Poynting's theorem and Poynting vector

Poynting's theorem states that the rate of energy transfer per unit volume from a region of space equals the rate of work done on the charge distribution in the region, plus the energy flux leaving that region.

$$-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon\mathbf{E}^2+\frac{1}{2}\mu\mathbf{H}^2\right) = \mathbf{J}\cdot\mathbf{E}+\nabla\cdot(\mathbf{E}\times\mathbf{H})$$

$$\boxed{\mathbf{P}=\mathbf{E}\times\mathbf{H}}$$

Poynting's vector: Power density

$$\int_{v}\left(\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}+\boxed{\mathbf{E}\cdot\mathbf{J}}\right)dv=-\oint_{s}(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{s}$$

$$\mathbf{W}=-\oint(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{s}$$

Magnetic energy density

Electric energy density

Ohmic power density

Maxwell's equations

$$\nabla \cdot \mathbf{D} = \rho$$
Gauss's law for electric field

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
Faraday's law

$$\nabla \cdot \mathbf{H} = 0$$
Gauss's law for magnetic field

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$
Ampere's law

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Constitutive relations

$$\mathbf{J} = \sigma \mathbf{E}$$
Ohm's law

Lorentz law

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

Coulomb's law

Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

Right-hand rule

Thank you !