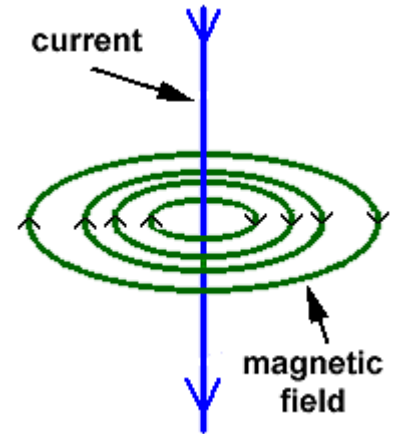
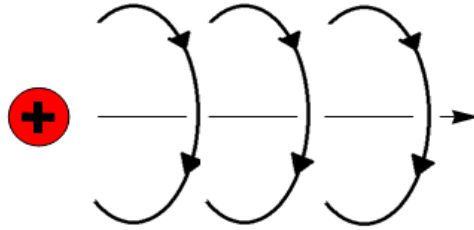


# Lecture 4: Stationary magnetic field

- Charge in motion,
- Magnetic field
- Gauss's law in magnetic field
- Ampere's Law

# Lecture 4: Charge in motor

Charges in *motion* (electrical current) produce a magnetic field

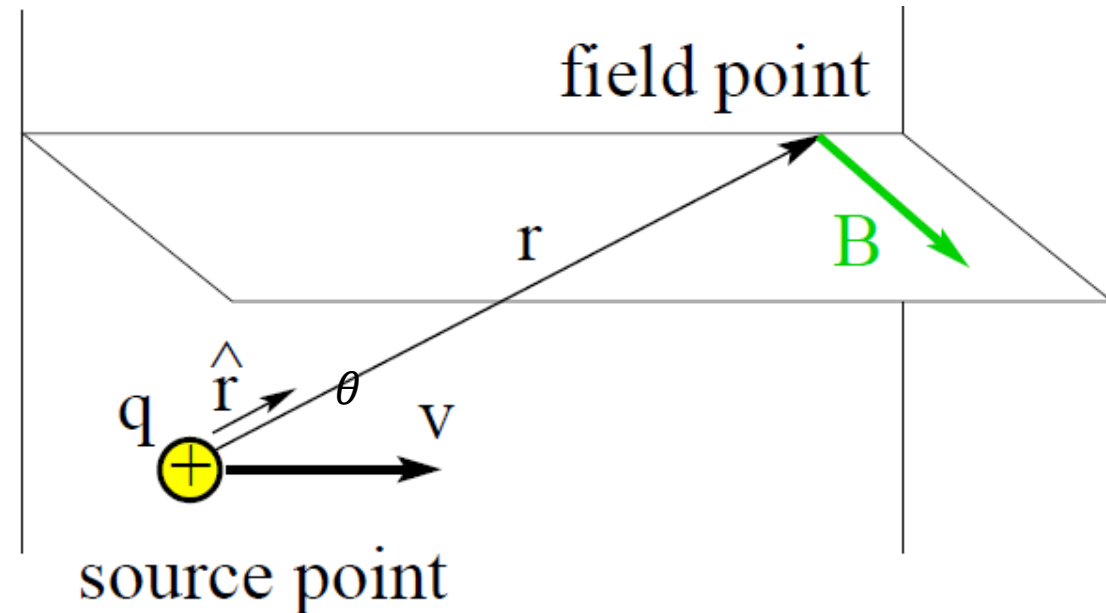


Magnetic flux density  $B$  in vacuum generated by moving charge  $q$ : 
$$B = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{v} \times \hat{\mathbf{r}} = |\mathbf{v}| \sin(\theta) \mathbf{n}$$

$\mathbf{n}$  is a unit vector perpendicular to the plane containing  $\mathbf{v}$  and  $r$  in the direction given by the right-hand rule

$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ , permeability of free space



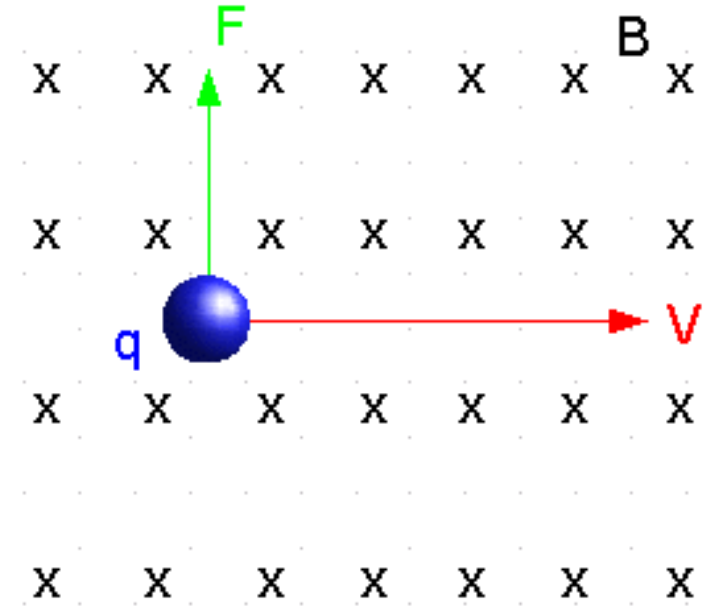
# Electro-magnetic force

Force exerted by magnetic field  $B$  on a moving point charge  $Q$  is:

$$\mathbf{F} = Q \mathbf{v} \times \mathbf{B}$$

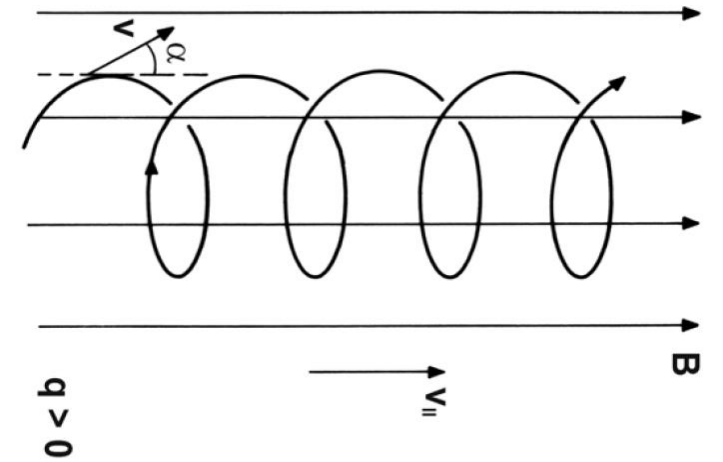
Magnetic force acting on a moving charge is always perpendicular to its moving direction, so magnetic force does no work on the charges, but changing the charge's moving direction

$$W = \mathbf{F} \cdot \mathbf{S} = \int \mathbf{F} \cdot d\mathbf{l} = \int \mathbf{F} \cdot \mathbf{v} dt$$



# Example: Point charge's movement in constant uniform magnetic field

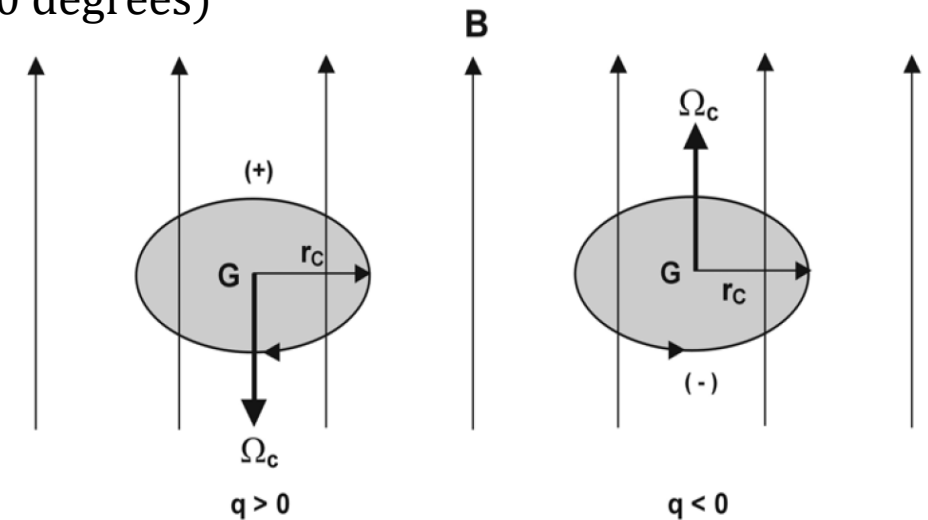
A constant uniform magnetic field  $B$ , a charge  $-q$  with mass  $m$  is shot perpendicularly into the magnetic field with speed  $V(0)$ , what is the radius of charge?



Centrifugal force  $F = \frac{mV^2}{r} = qvB \sin\theta$  ( $\theta$  is the angle between  $v$  and  $B$ , here 90 degrees)

$$r = \frac{mV^2}{qvB}$$

<https://www.youtube.com/watch?v=orsMYomjwlw>

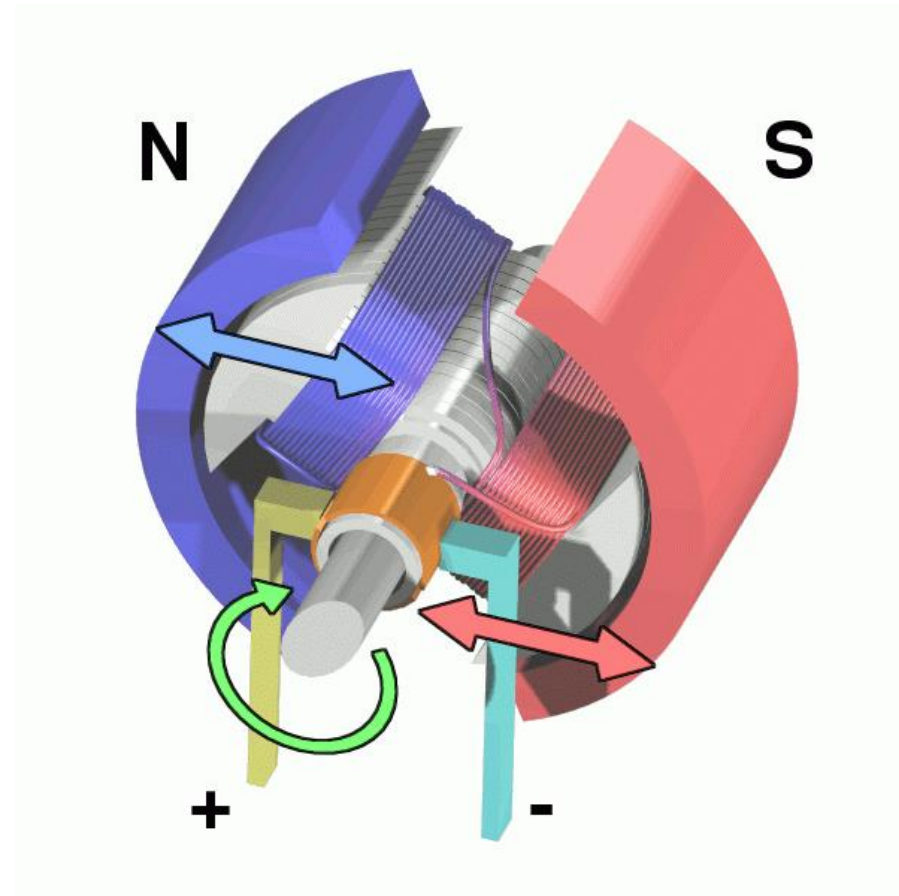
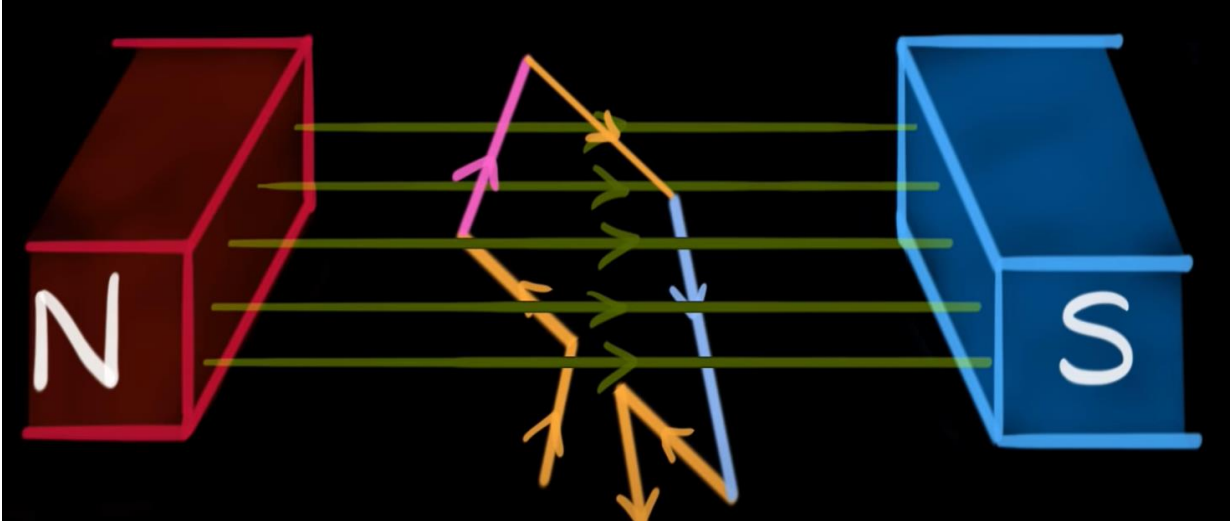


# Electric motor

An electrical motor consist of: stator and rotor

One produces magnetic field by either **current** or **magnets**: ( Rotor)

Stator winding: Electric current. ( moving charge)



# Magnetic field

Magnetic field  $H = \frac{B}{\mu}$ ,  $\mu$  is called permeability and material dependent, the value of  $\mu$  for free space is  $\mu_0 = 4\pi \times 10^{-7} H/m$ .

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{H} = \frac{1}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

The magnetic field intensity is independent of material property.

# Biot-savart's law : Magnetic field generated by current

The Biot–Savart law is used for computing the resultant [magnetic field](#)  $\mathbf{B}$  at position  $\mathbf{r}$  in 3D-space generated by a *steady* [current](#)  $I$ .

Vector expression:  $\mathbf{H}(\mathbf{r}) = \int_c \frac{I'(r)d\mathbf{l}' \times \hat{\mathbf{R}}}{4\pi R^2}$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_c \frac{I'(r)d\mathbf{l}' \times \hat{\mathbf{R}}}{R^2}$$

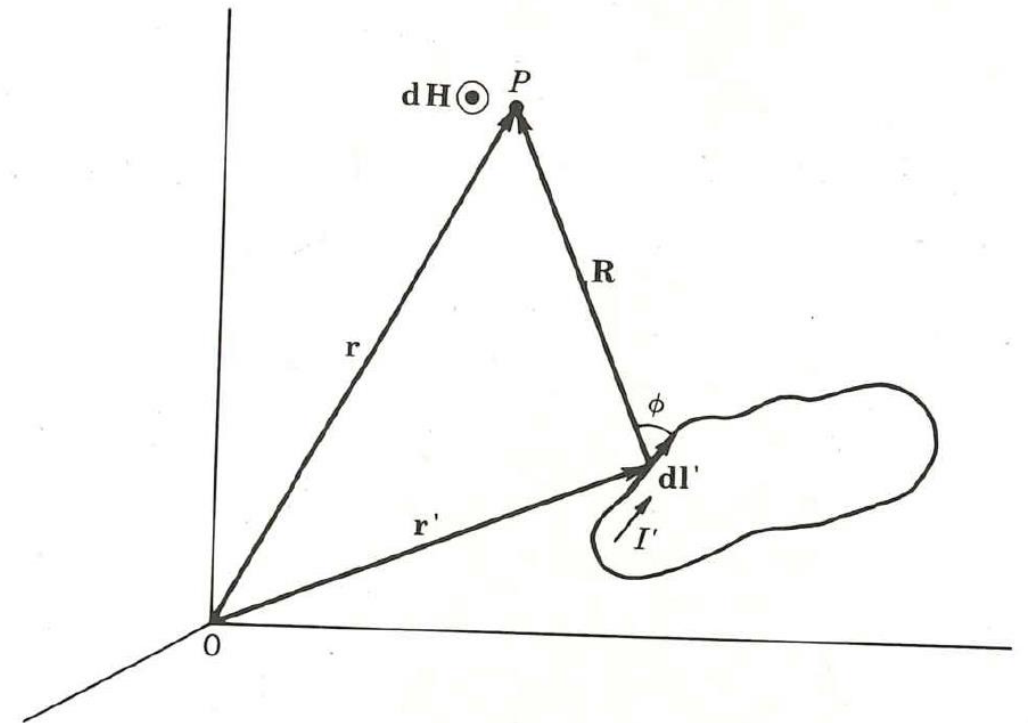
$$I d\mathbf{l} \times \mathbf{R} = |I||R|dl' \sin(\theta) \mathbf{n}$$

$\mathbf{n}$  is a [unit vector perpendicular](#) to the plane containing  $I$  and  $R$  in the direction given by the right-hand rule

Scalar calculation

$$H(r) = \int_c \frac{I'(r)dl' \sin\phi}{4\pi R^2}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_c \frac{I'(r)dl' \sin\phi}{R^2}$$

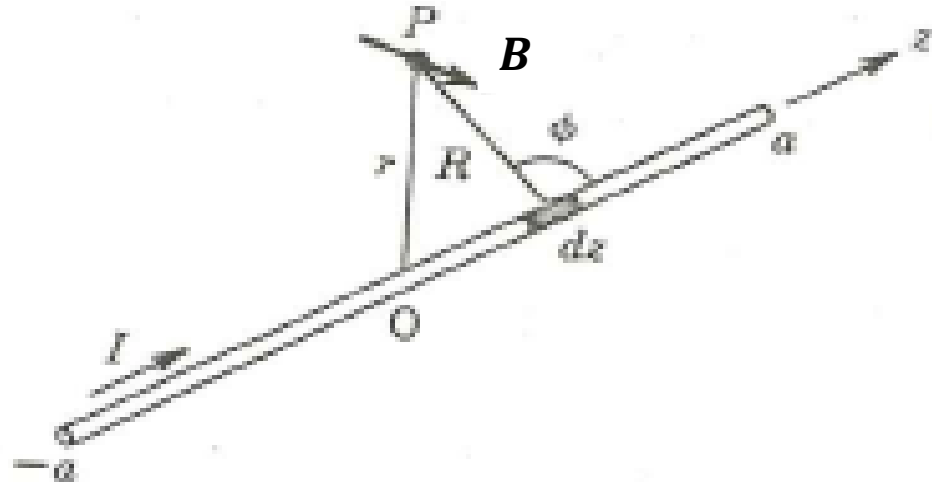


# Example: Magnetic flux density

Find the magnetic field  $B$  at a point  $P$  a perpendicular distance  $r$  from the center of a finite length of current  $I$ , the total current length is  $2a$ .

$$d\mathbf{B} = \frac{\mu_0 I d\mathbf{l} \times \hat{\mathbf{R}}}{4\pi R^2}$$

$$\mathbf{B} = \int_{-a}^a \frac{\mu_0 I dz}{4\pi R^2} \hat{\mathbf{R}} = \int_{-a}^a \frac{\mu_0 I \sin \phi dz}{4\pi R^2}$$



$$\sin(\phi) = \frac{r}{\sqrt{r^2 + z^2}} \quad R^2 = r^2 + z^2$$

$$B = \int_{-a}^a \frac{\mu_0 I \sin \phi dz}{4\pi R^2} = \frac{\mu_0 I r}{4\pi} \int_{-a}^a \frac{dz}{(r^2 + z^2)^{3/2}} = \frac{\mu_0 I}{2\pi r} \frac{1}{[(r/a)^2 + 1]^{1/2}}$$

Assuming  $a \gg r$ , what is  $B$ ?



# Field on Axis of circular loop

A ring with radius  $a$  and current  $I$ , calculate  $B$  at point on the  $z$  axis.

$$d\mathbf{B} \cdot \hat{\mathbf{z}} = dB \cos \beta = dB \sin \alpha,$$

**B** and **dB** directions are different

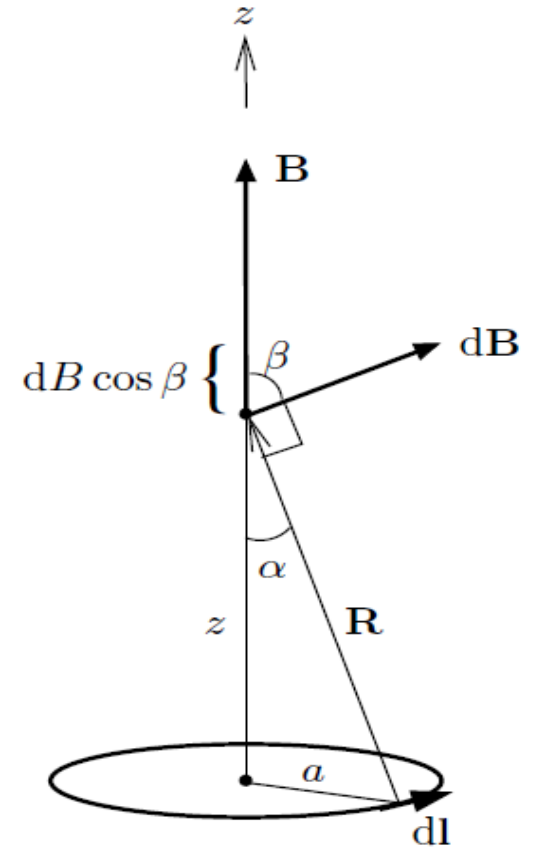
$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad dB = \frac{\mu_0}{4\pi} \frac{I dl}{R^2}.$$

$$B = \oint_{\text{ring}} dB \sin \alpha = \frac{\mu_0 I \sin \alpha}{4\pi R^2} \oint dl = \frac{\mu_0 I \sin \alpha (2\pi a)}{4\pi R^2} = \frac{\mu_0 I a^2}{2R^3}$$

$$R = \sqrt{z^2 + a^2}$$

$$\sin \alpha = \frac{a}{R}$$

$$\mathbf{B} = \frac{\mu_0 I a^2}{2R^3} \hat{\mathbf{z}} = \frac{\mu_0 I a^2}{2(z^2 + a^2)^{3/2}} \hat{\mathbf{z}}$$



# Magnetic flux and flux continuity

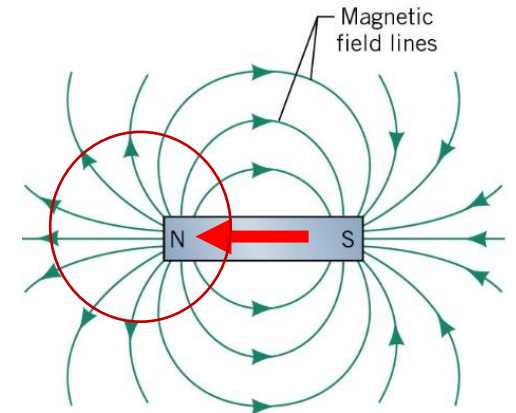
Magnetic flux  $\phi$  is the integral of the flux density across surface

$$\phi = \int_S \mathbf{B} \cdot d\mathbf{S},$$

For an enclosed surface, the flux is zero

$$\boxed{\int_S \mathbf{B} \cdot d\mathbf{S} = 0.}$$

$$\nabla \cdot \mathbf{B} = 0$$



There is no magnetic monopole, continuous magnetic field.

$$\int_V \nabla \cdot \mathbf{B} dV = \oint_S \mathbf{B} \cdot d\mathbf{S} = 0,$$

# Ampere's law

Ampere's Law states that for any closed loop path, the line integral of the magnetic field around closed curve C is equal to the electric current enclosed in the loop.

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{S}.$$

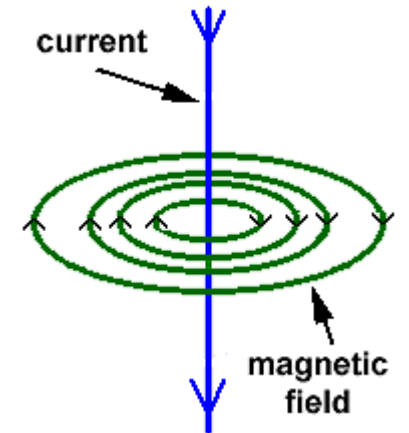
Stoke's theorem

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

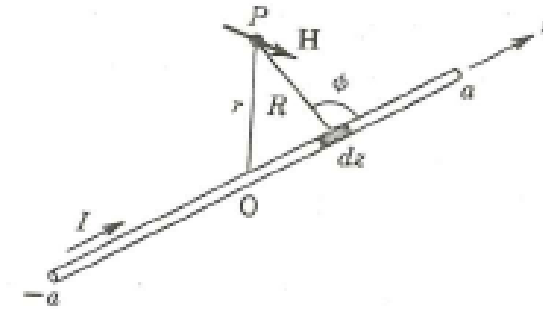
$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

In magnetostatic, for example: constant DC current.  $\frac{\partial \mathbf{D}}{\partial t} = \mathbf{0}$



# Ampere's law: calculating the magnetic field around a conductor



$$B = \int_{-a}^a \frac{\mu_0 I \sin \phi dz}{4\pi R^2} = \frac{\mu_0 I r}{4\pi} \int_{-a}^a \frac{dz}{(a^2+z^2)^{3/2}} = \frac{\mu_0 I}{2\pi r} \frac{1}{[(r/a)^2+1]^{1/2}}$$

Stoke's theorem:

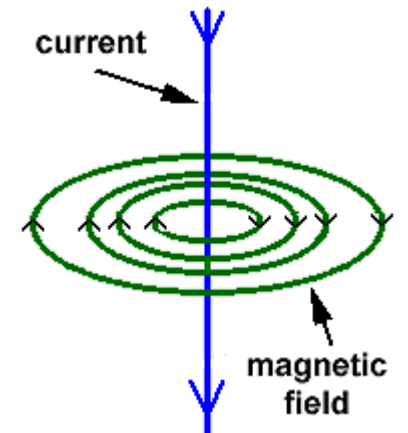
$$\int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \int_C \mathbf{B} \cdot d\mathbf{l}$$

$$\int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \oint B dl = 2\pi r B(r)$$

$$\int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \int_S \mu_0 \mathbf{J} \cdot d\mathbf{S} = \mu_0 I$$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

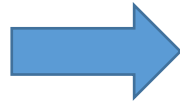
$\Rightarrow 2\pi r B(r) = \mu_0 I \Rightarrow B(r) = \frac{\mu_0 I}{2\pi r}$



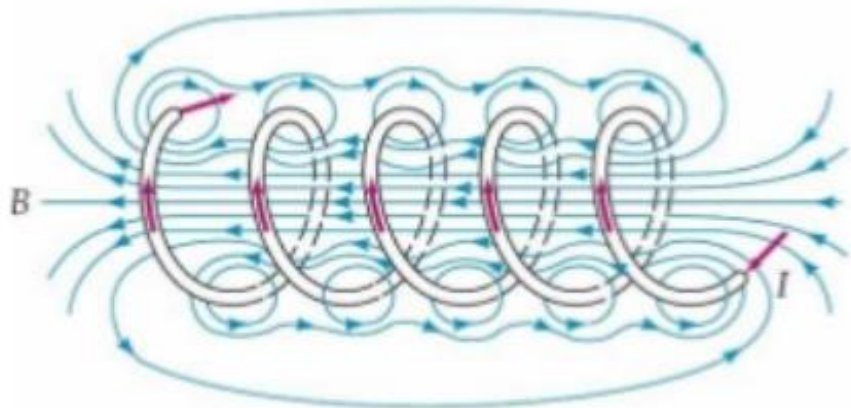
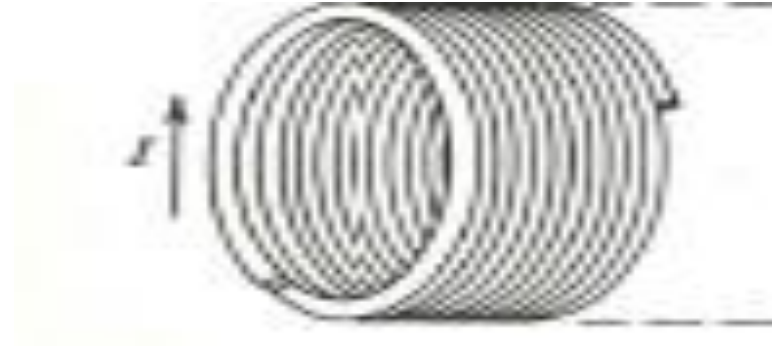
# Example: Solenoid

$$\int_s \nabla \times \mathbf{H} \cdot d\mathbf{S} = \oint H dl = Hl$$

$$\int_s \nabla \times \mathbf{H} \cdot d\mathbf{S} = \int_s \mathbf{J} \cdot d\mathbf{S} = NI$$



$$H = \frac{NI}{l}$$



$$\mathbf{B} = \mu\mathbf{H}$$

If the core is iron instead of air, the flux density  $\mathbf{B}$  is much stronger.

# Example: Magnetic field

A coaxial line carrying current  $I$  on the inner conductor and  $-I$  on the outer .

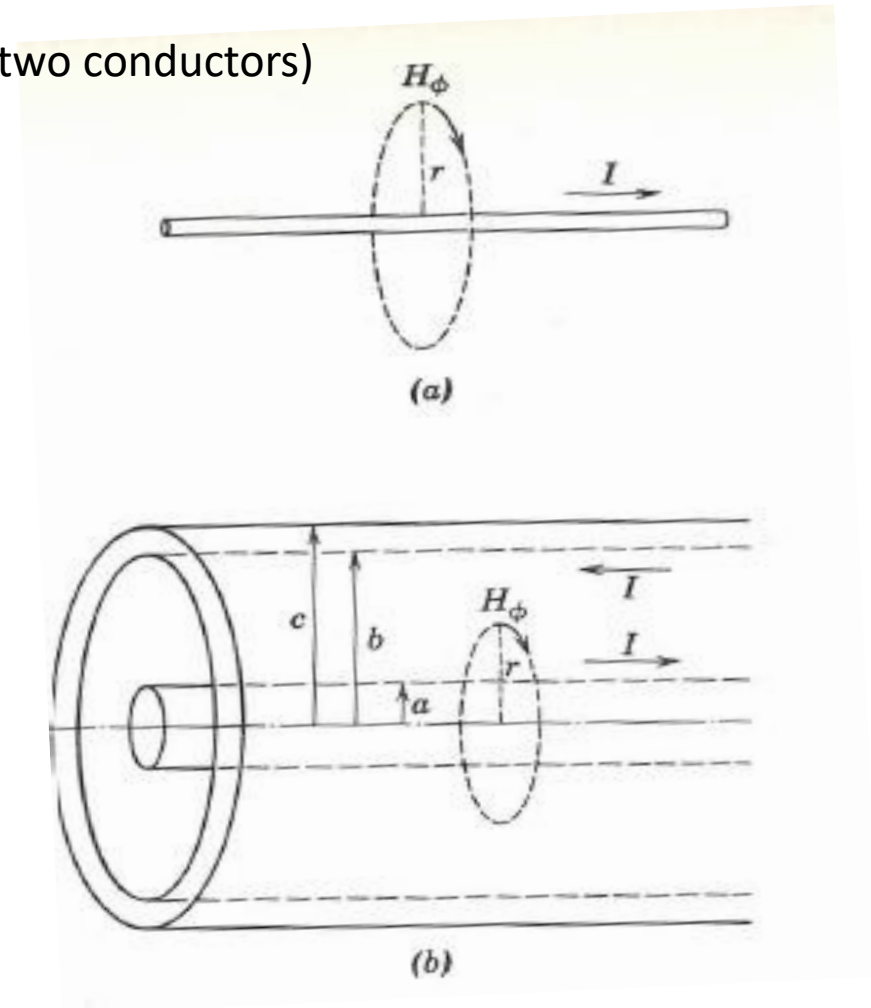
Calculate the magnetic field  $H$  at  $r$  distance, ( Current evenly distributed in the two conductors)

- 1)  $0 < r < a$ ,
- 2)  $a < r < b$ ,
- 3)  $b < r < c$
- 4)  $r > c$

$$I(r) = \left(\frac{r}{a}\right)^2 I$$

$$H_\phi(r) = \frac{I(r)}{2\pi r} = \frac{Ir}{2\pi a^2}$$

$$H_\phi = \frac{I}{2\pi r} \quad a < r < b$$



# Magnetic field in material

$$\mathbf{B} = \mu_0 \mathbf{H} \text{ in vacuum}$$

Once there is magnetic field applied to medium, the electronic spin motions in the atoms can be thought of as circulating current that produce a field  $\mathbf{M}$ , magnetization, which adds to magnetic field  $\mathbf{H}$ .

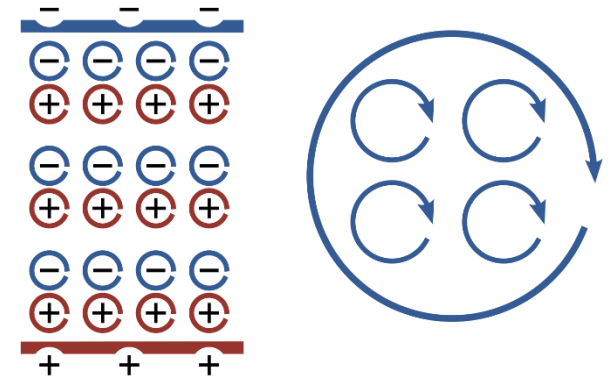
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

Magnetic susceptibility  $\chi_m$  is used to quantify the additional field  $\mathbf{M}$ .

$$\mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 \mu_r \mathbf{H} = \mu \mathbf{H}$$

$\mu_r$  is around 5000 for iron.

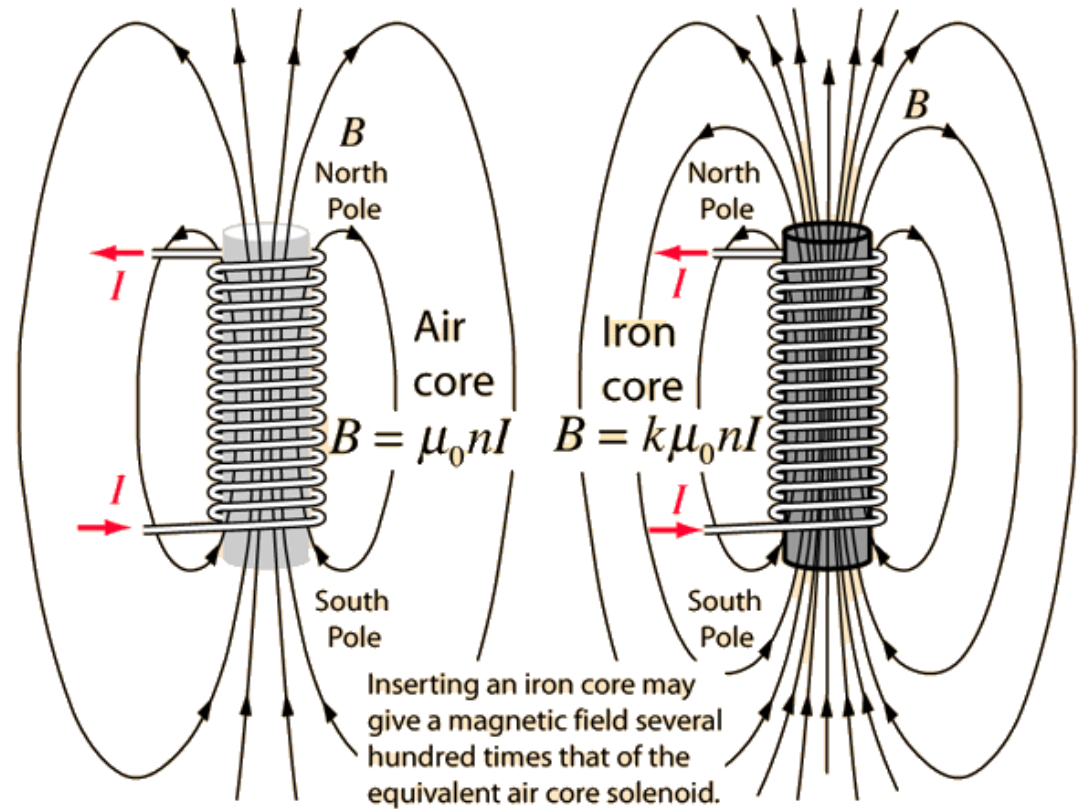
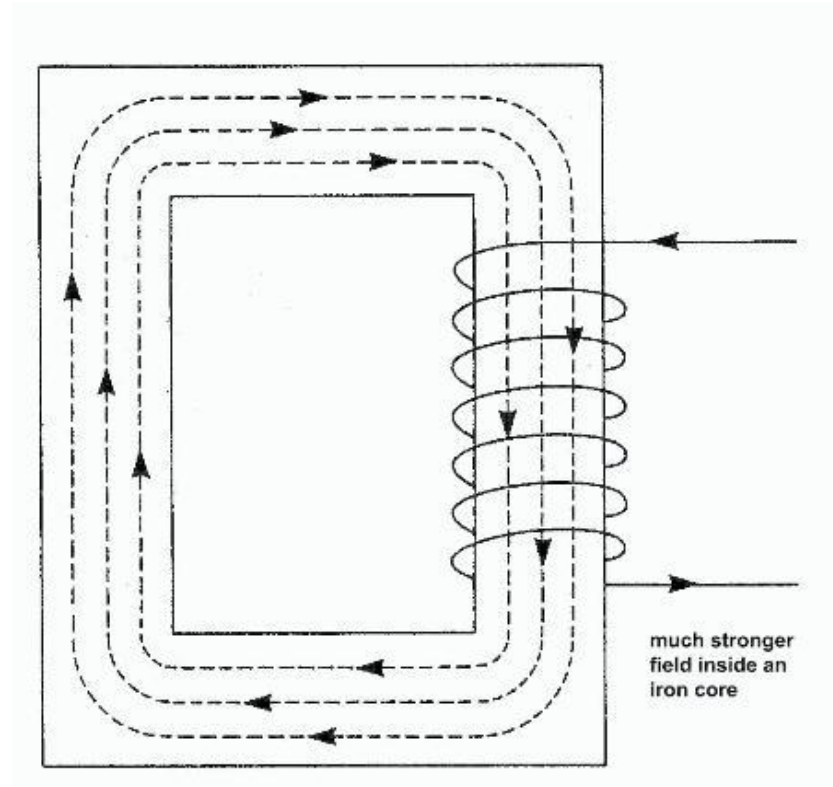
Bad magnetic material  
relative permeability  
Silver: 1  
Copper : 1  
Gold: 1  
Aluminium: 1



Good magnetic material  
relative permeability  
Iron alloy : 100 - 7000

# Field in magnetic material

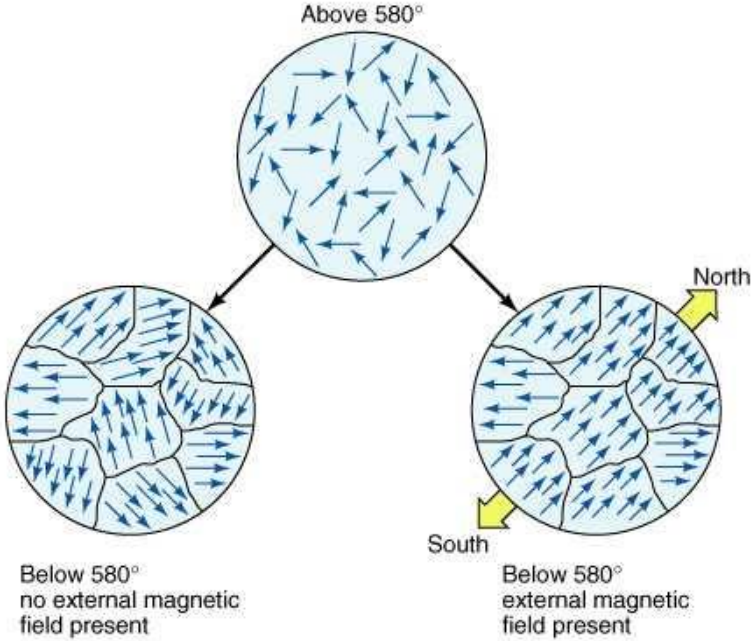
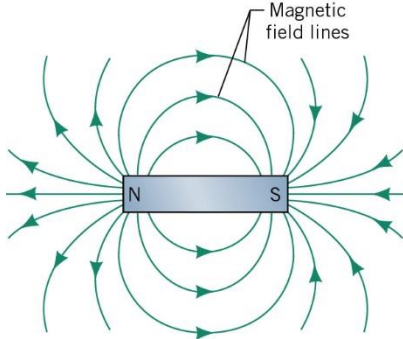
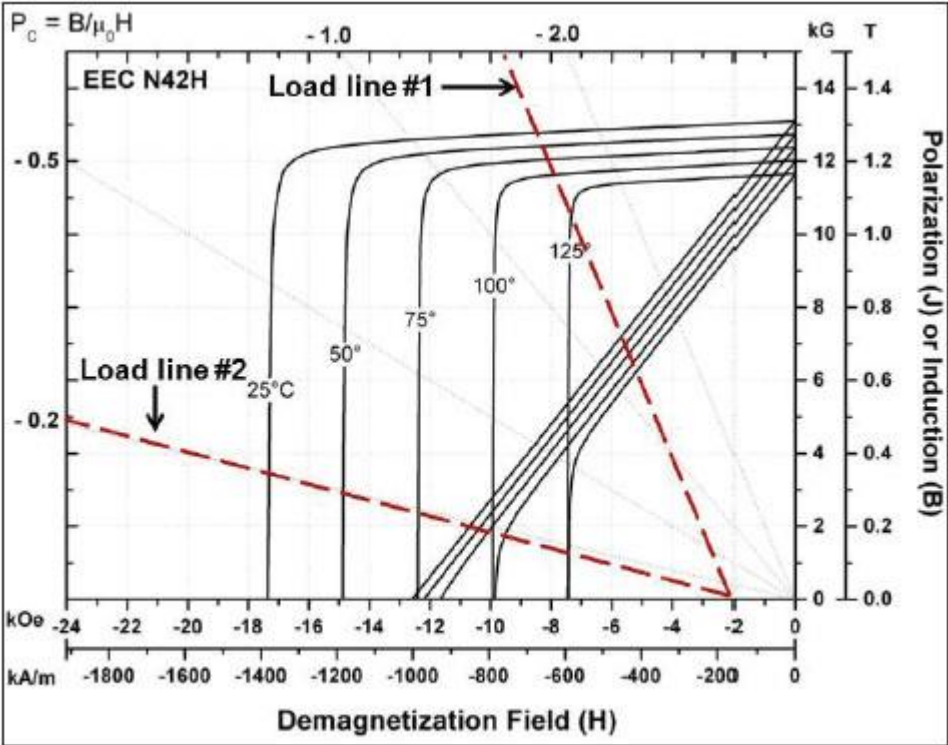
Magnetic material can be used to guide magnetic field path.





# Permanent magnet

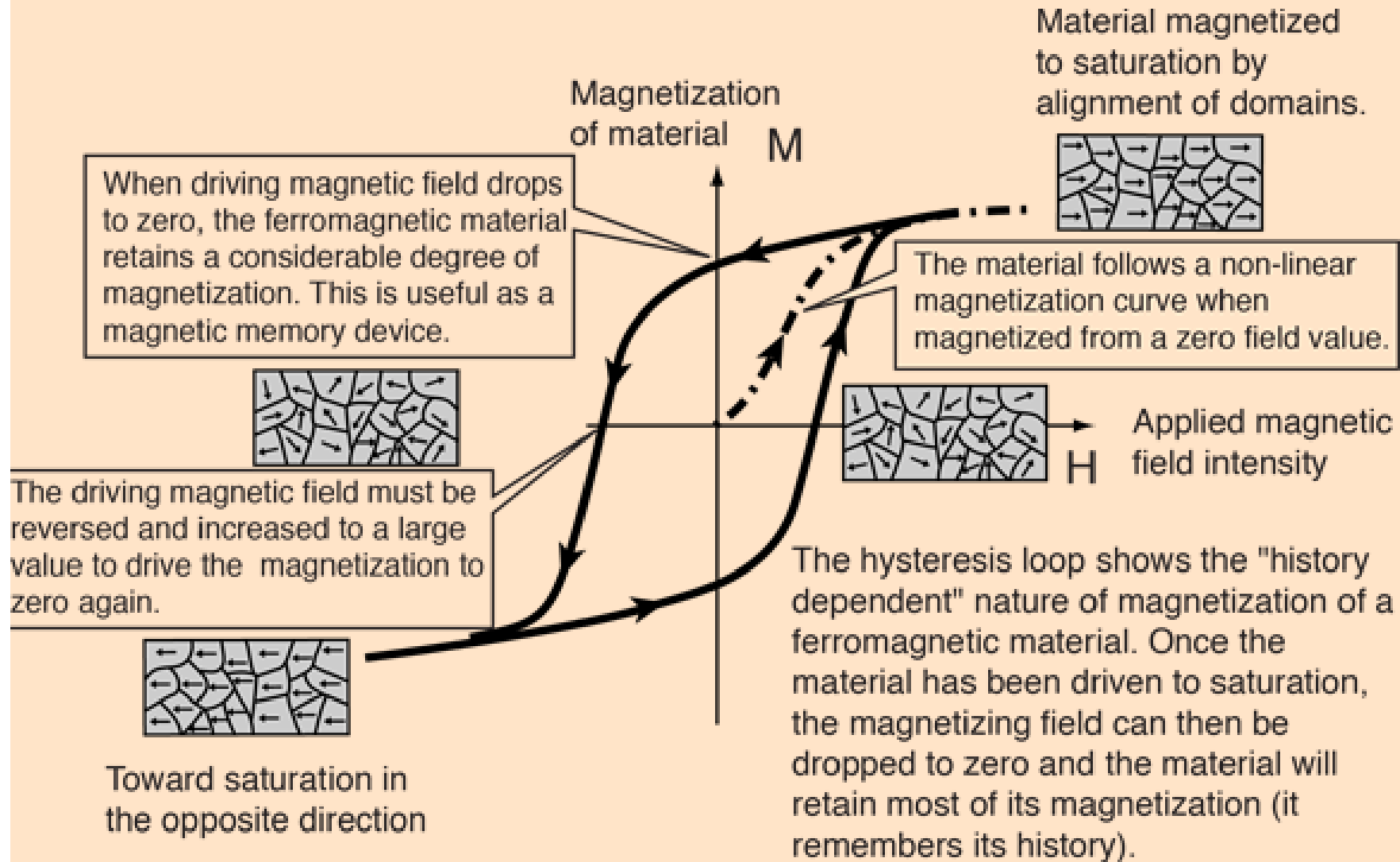
Some materials after magnetization, the electron movement can remain when external magnetic field disappears. The electron movement can be distorted at high temperatures, strong opposite external field or strong shock.



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Figure 5. Typical Demagnetization Curves of NdFeB N42H Magnets

# Hysteresis Loop

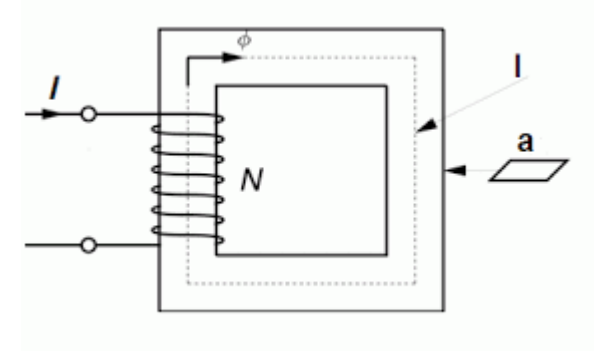


# Magnetic circuit:

MMF: magnetomotive force:  $NI$ ,  $HL$

$$R_m = \frac{l}{\mu S}$$

- The magnet ( $Hl$ ) or current ( $NI$ ) possesses a magneto-motive force (MMF).
- The m.m.f generates a magnetic flux.
- The flux exists within the magnet and the air gap between the poles. The enclosed flux path is called a magnetic circuit.
- A stronger m.m.f. will produce more flux.
- The lower the reluctance of the magnetic circuit, the more flux will be produced



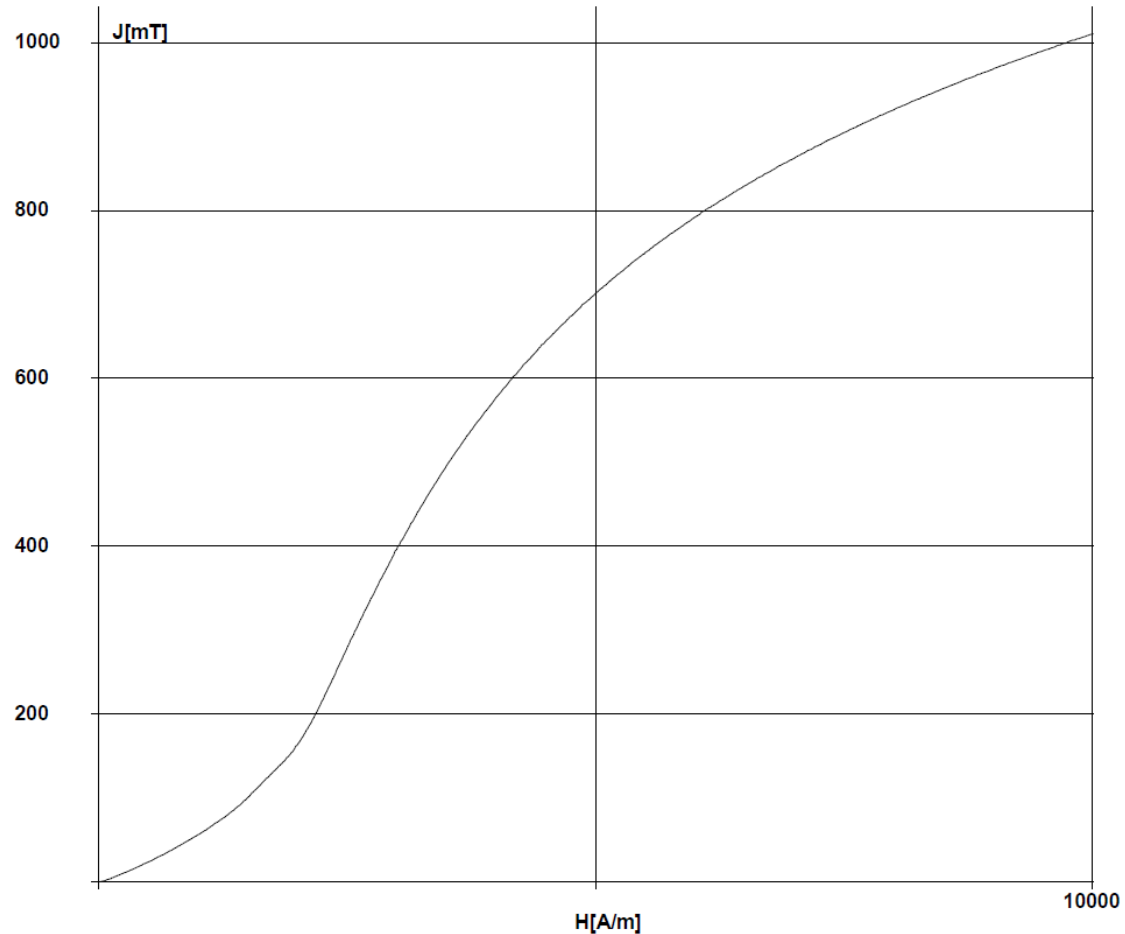
# Magnetic circuit and electric circuit

## *Magnetic Circuit*

## *Electrical Circuit*

1. The closed path for magnetic flux is called a magnetic circuit.	1. The closed path for electric circuit is called an electric circuit.
2. Flux, $\phi = \frac{mmf}{Reluctance}$	2. Current, $I = \frac{emf}{resistance}$
3. mmf (Ampere – turns)	3. emf (Volts)
4. Reluctance, $S = \frac{l}{a\mu_0\mu_r}$	4. Resistance, $R = \rho \frac{l}{a}$
5. Flux density, $B = \frac{\phi}{a}$ Wb/m <sup>2</sup>	5. Current density $\delta = \frac{I}{a}$ A/m <sup>2</sup>
6. mmf drop = $\phi S$	6. Voltage drop = $I R$
7. Magnetic Intensity, $H = \frac{NI}{l}$	7. Electric intensity, $E = \frac{V}{d}$

# Magnetic material: B-H curve



## Measuring results

Jmax = 1010,4 mT  
Hmax = 10008,46 A/m  
mumax = 114,8188  
mui = 4,691205  
mur = 80,33712  
Ph(28,0A/m) = 0

## Sample data

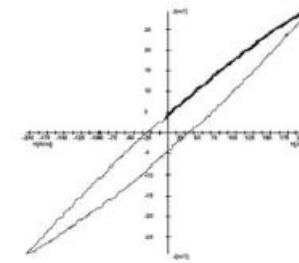
Name : blasting\_anyuar  
Mass : 242,6 g  
Density : 7,85 g/cm<sup>3</sup>  
Length : 400 mm  
Diameter : 10 mm  
Grade : DT 63

Coil : SST 10 x 150

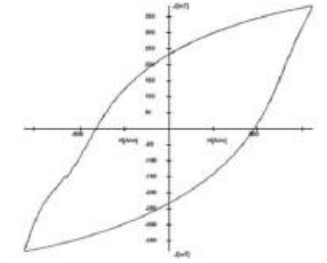
## Identification

Operator : Anyuan  
Date : 30.03.2017

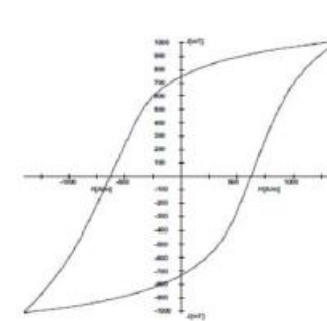
BROCKHAUS MESSTECHNIK



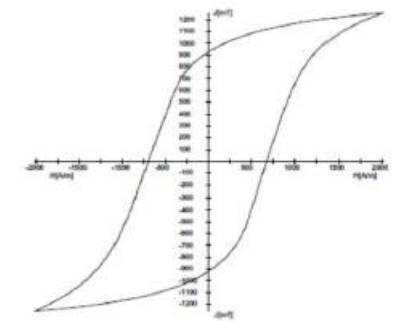
(a) 200A/m



(b) 800A/m



(c) 1400A/m



(d) 2000A/m

# Boundary condition for static magnetic field

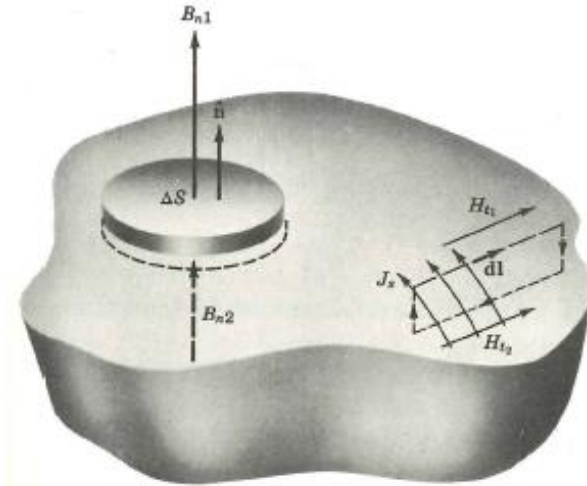
$$\int_V \nabla \cdot \mathbf{B} dV = \oint_S \mathbf{B} \cdot d\mathbf{S} = 0,$$

$$B_{n1} \Delta S = B_{n2} \Delta S$$

$$B_{n1} = B_{n2}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = H_{t1} \Delta l - H_{t2} \Delta l = J_s \Delta l$$

$$H_{t1} - H_{t2} = J_s$$



$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \nabla \times \mathbf{H} = \mathbf{J}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{S}.$$