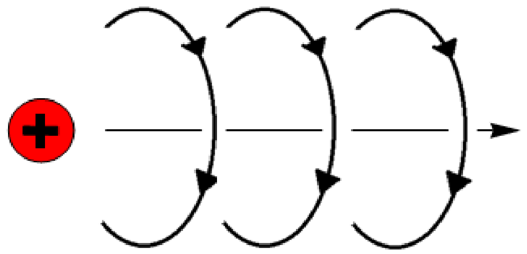
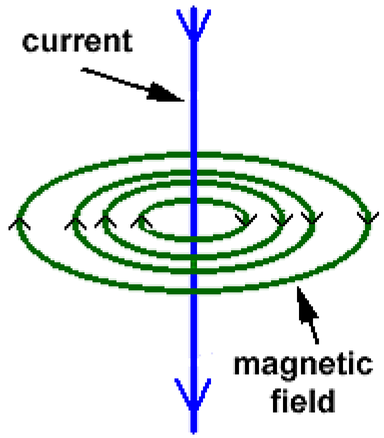
Lecture 4: Stationary magnetic field

- Charge in motion,
- Magnetic field
- Gauss’s law in magnetic field
- Ampere’s Law

Lecture 4: Charge in motor

Electric Charges in *motion* (current) produce a magnetic field



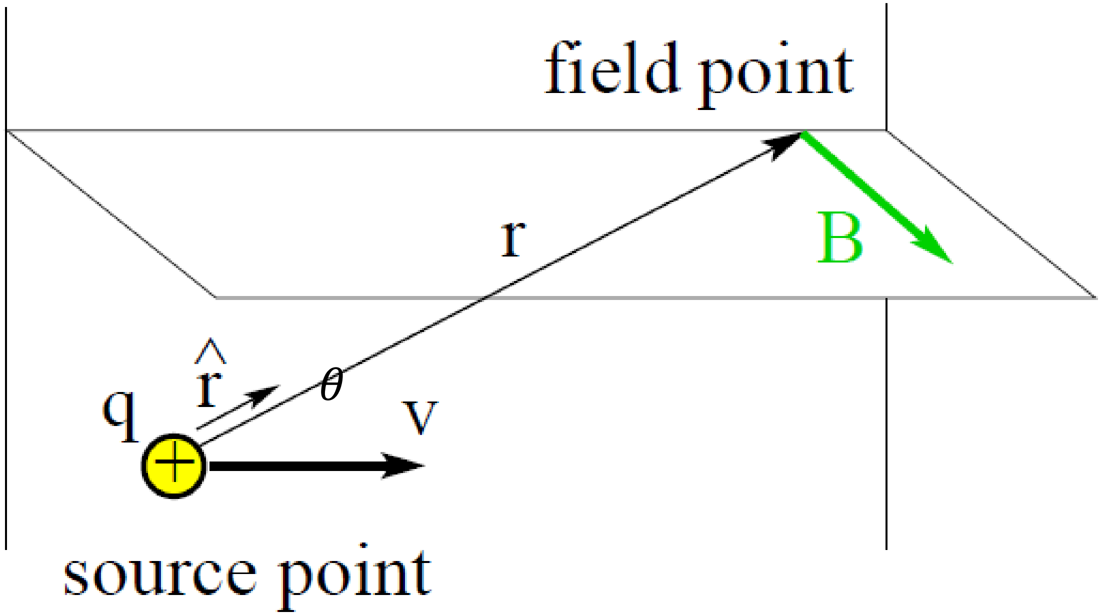


Magnetic flux density B in vacuum generated by moving charge q: $B = \frac{\mu_0}{4\pi}\frac{q\boldsymbol{v}\times\hat{\boldsymbol{r}}}{r^2}$

$$\boldsymbol{v}\times\hat{\boldsymbol{r}} = |v|\sin(\theta)\,\boldsymbol{n}$$

n is a unit vector perpendicular to the plane containing *v* and *r* in the direction given by the right-hand rule

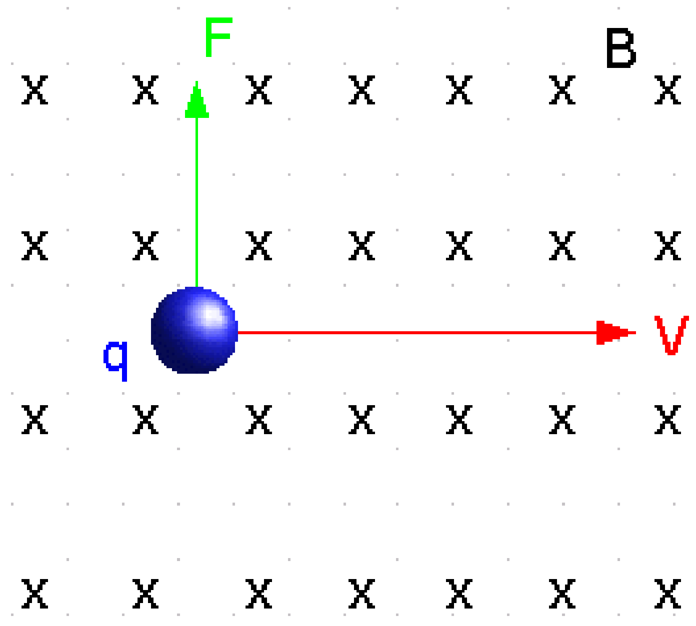
$\mu_0 = 4\pi\times10^{-7}H/m$, permeability of free space



Electro-magnetic force

Force exerted by magnetic field B on a moving point charge Q is:

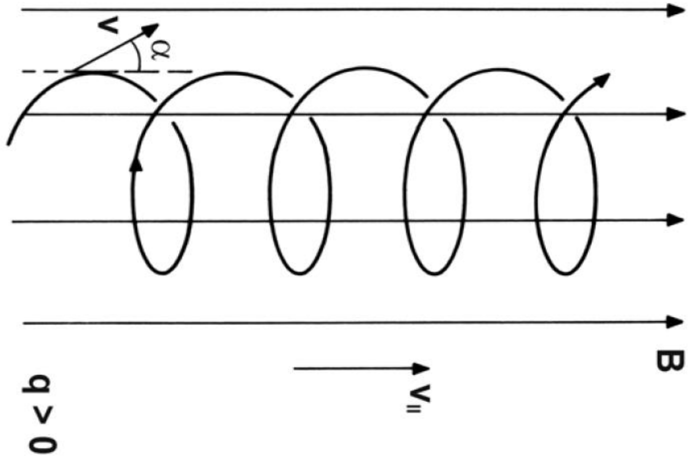
$$\boldsymbol{F} = Q\,\boldsymbol{v} \times \boldsymbol{B}$$



Magnetic force acting on a moving charge is always pependicular to it’s moving directon, so magnetic force does no work on the charges, but changing the charge’s moving direction

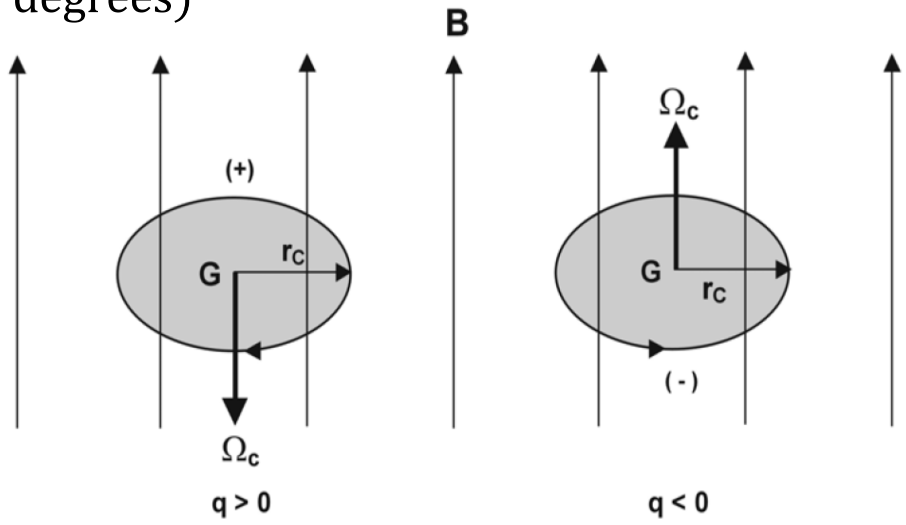
$$W = F \cdot L = \int \boldsymbol{F} \cdot d\boldsymbol{l} = \int \boldsymbol{F} \cdot \boldsymbol{v} dt$$

Example: Point charge's movement in constant uniform magnetic filed



A constant uniform magnetic field B, a charge –q with mass m is shot pependicularly into the magnetic field with speed **V**(0), what is the radius of charge?

Centrifugal force $F = \frac{mV^2}{r} = qvBsin\theta$ (θ is the angle between v and B, here 90 degrees)



$$r = \frac{mV^2}{qvB}$$

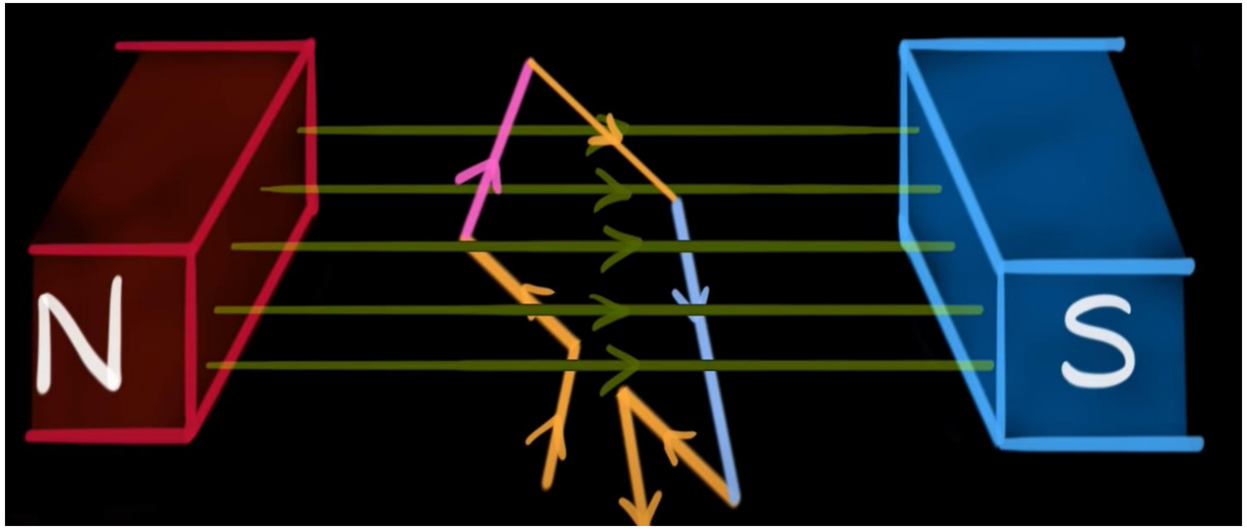
https://www.youtube.com/watch?v=orsMYomjwIw

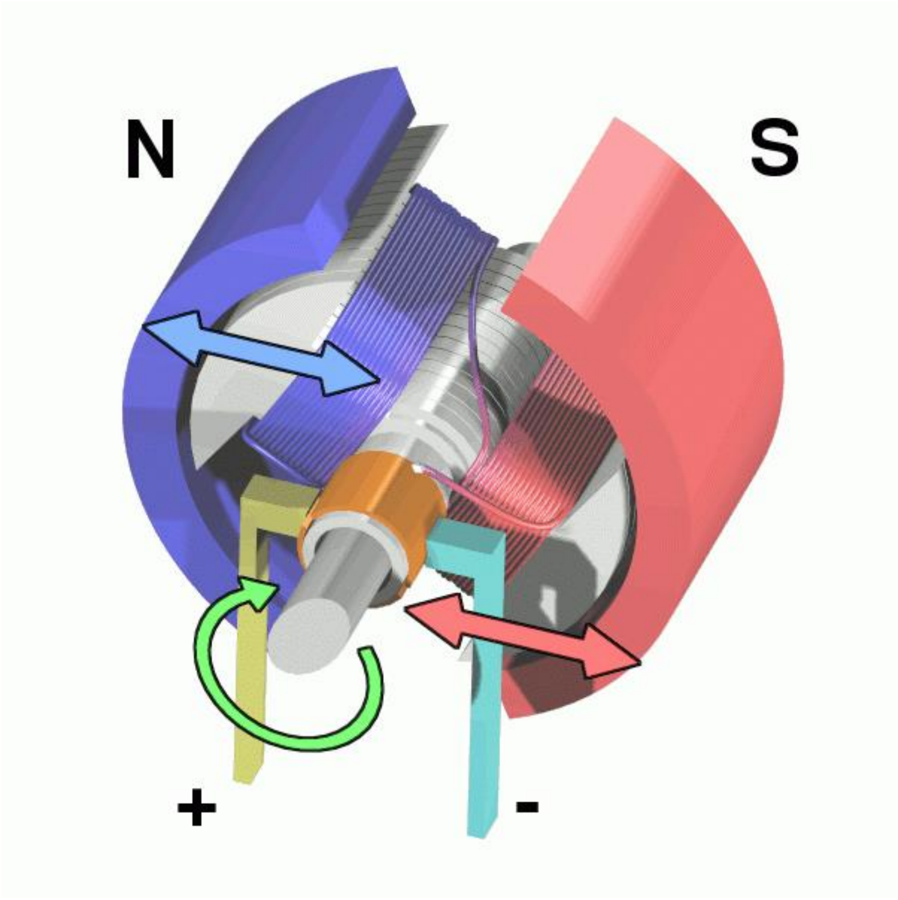
Electric motor

An electrical motor consist of: stator and rotor

One produces magnetic field by either **current** or **magnets**: (Rotor)

Stator winding: Electric current. (moving charge)





Magnetic field density and strength

Magnetic field strength H= $\frac{B}{\mu}$,

μ is called permeability and material dependent, the value of μ for free space is $\mu_0 = 4\pi \times 10^{-7} H/m$.

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} \frac{q\boldsymbol{v} \times \hat{\boldsymbol{r}}}{r^2}$$

$$\boldsymbol{H} = \frac{1}{4\pi} \frac{q\boldsymbol{v} \times \hat{\boldsymbol{r}}}{r^2}$$

The H field is independent of material

Biot-savart's law : Magnetic field generated by current

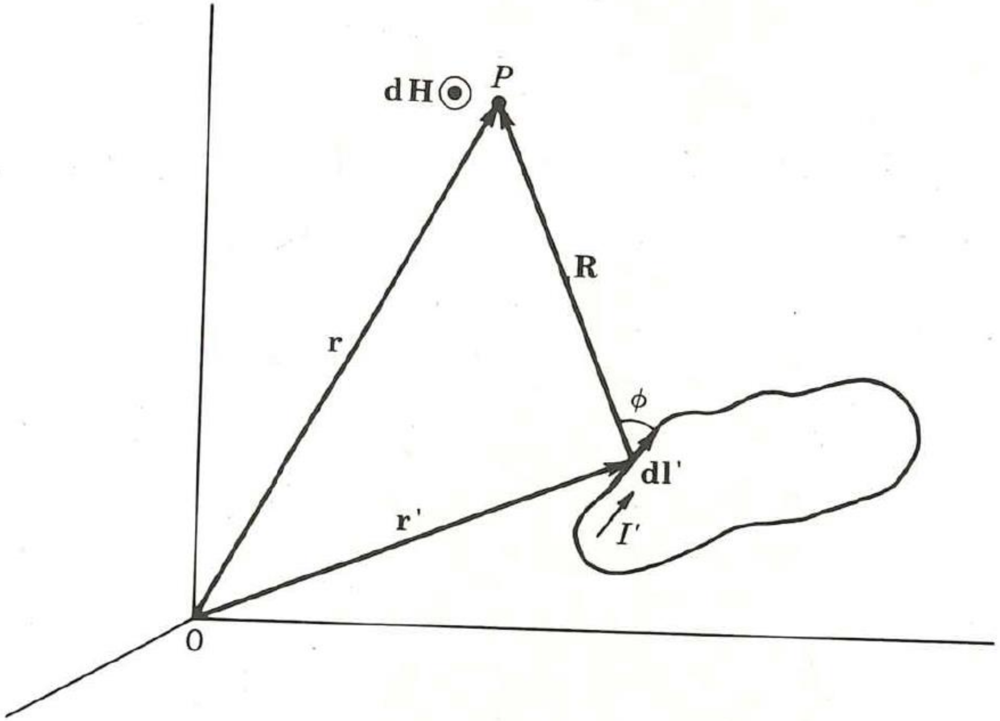
The Biot–Savart law is used for computing the resultant magnetic field B at position r in 3D-space generated by a steady current I .

Vector expression: $\boldsymbol{H}(\boldsymbol{r}) = \int_{\boldsymbol{c}} \frac{I'(r)d\boldsymbol{l}' \times \widehat{\boldsymbol{R}}}{4\pi R^2}$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_{\boldsymbol{c}} \frac{I'(r)d\boldsymbol{l}' \times \widehat{\boldsymbol{R}}}{R^2}$$

$$Id\boldsymbol{l} \times \boldsymbol{R} = |I||R|dl' \sin(\phi)\, \boldsymbol{n}$$

n is a unit vector perpendicular to **the plane containing I and R in t**he direction given by the right-hand rule



Scalar calculation

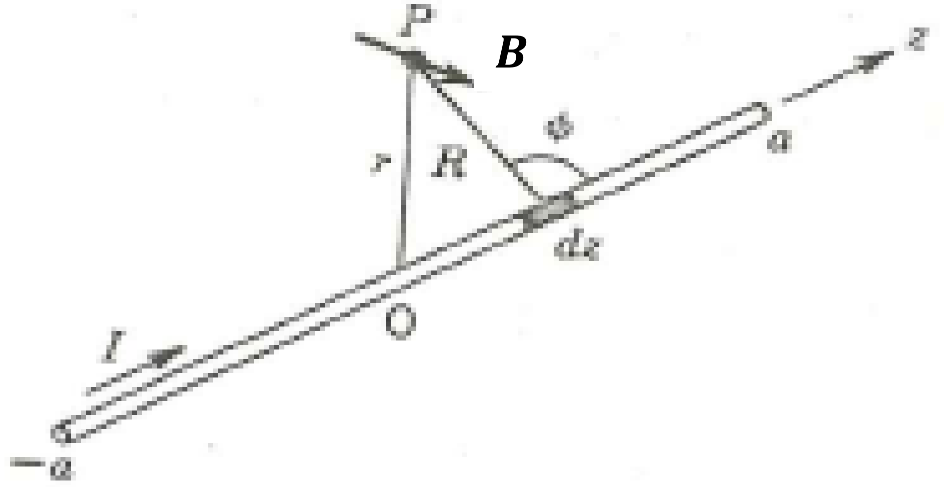
$$H(r) = \int_{c} \frac{I'(r)dl' sin\phi}{4\pi R^2}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_{\boldsymbol{c}} \frac{I'(r)dl' sin\phi}{R^2}$$

Example: Magnetic flux density

Find the magnetic field B at a point **P** a perpendicular distance r from the center of a finite length of current I, the total current length is 2a.

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I d\boldsymbol{l} \times \hat{\boldsymbol{R}}}{R^2}$$



$$\boldsymbol{B} = \int_{-a}^{a} \frac{\mu_0}{4\pi}\frac{I dz}{R^2}\hat{R} = \int_{-a}^{a} \frac{\mu_0 \boldsymbol{I} dz}{4\pi R^2}\hat{R} = \int_{-a}^{a} \frac{\mu_0 I \sin\phi dz}{4\pi R^2}$$

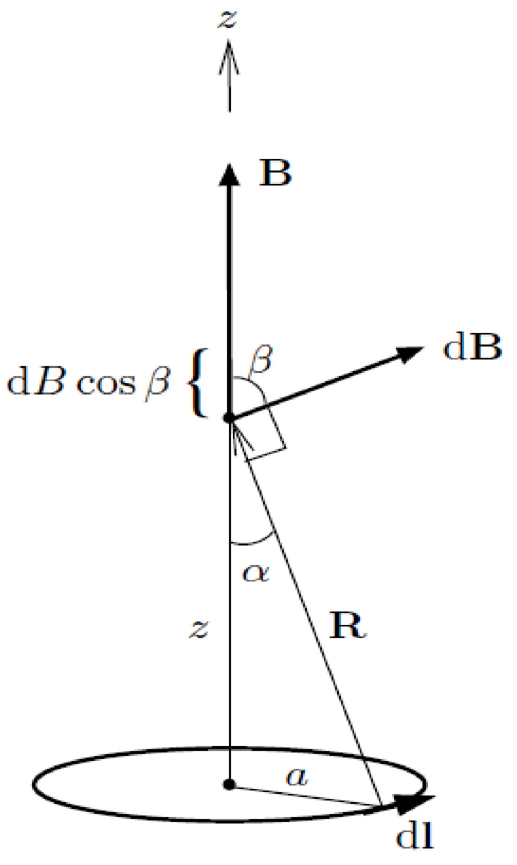
$$\sin(\phi) = \frac{r}{\sqrt{r^2+z^2}} \qquad R^2 = r^2 + z^2$$

$$B = \int_{-a}^{a} \frac{\mu_0 I \sin\phi dz}{4\pi R^2} = \frac{\mu_0 I r}{4\pi}\int_{-a}^{a} \frac{dz}{(r^2+z^2)^{3/2}} = \frac{\mu_0 I}{2\pi r}\frac{1}{[(r/a)^2+1]^{1/2}}$$

Asumming a>>r, what is B?

Field on Axis of circular loop

A ring with radius *a* and current I, calculate B at point on the z axis.



$$d\mathbf{B} \cdot \hat{\mathbf{z}} = dB\cos\beta = dB\sin\alpha,$$

B and d**B** directions are different

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad dB = \frac{\mu_0}{4\pi}\frac{I dl}{R^2}.$$

$$B = \oint_{\text{ring}} dB\sin\alpha = \frac{\mu_0 I \sin\alpha}{4\pi R^2}\oint dl = \frac{\mu_0 I \sin\alpha(2\pi a)}{4\pi R^2} = \frac{\mu_0 I a^2}{2R^3}$$

$$R = \sqrt{Z^2 + a^2}$$

$$\sin\alpha = \frac{a}{R}$$

$$\mathbf{B} = \frac{\mu_0 I a^2}{2R^3}\hat{\mathbf{z}} = \frac{\mu_0 I a^2}{2(z^2 + a^2)^{3/2}}\hat{\mathbf{z}}$$

Magnetic flux and flux contiunity

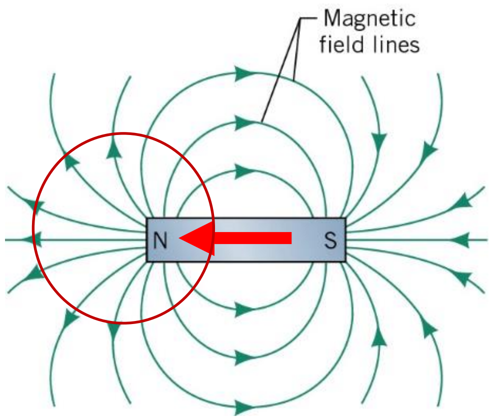
Magnetic flux ϕ is the integral of the flux density accross surface

$$\phi = \int_s \boldsymbol{B}.d\boldsymbol{S},$$

For en enclosed surface, the flux is zero

$$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0.$$

$$\nabla \cdot B = 0$$



Theres is no magnetic monopole, continuous magnetic field.

$$\int_V \nabla \cdot \mathbf{B} dV = \oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0,$$

Ampere’s law

Ampere's Law states that for any closed loop path, the line integral of the magnetic field around closed curve C is equal to the electric current enclosed in the loop.

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int_s \boldsymbol{J}.d\boldsymbol{s}$$

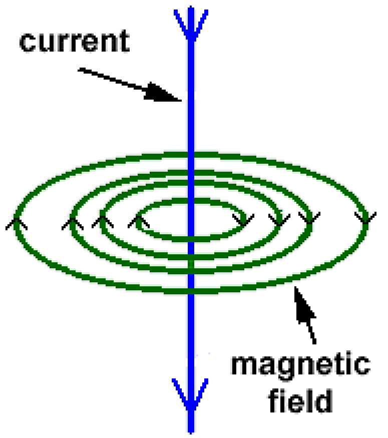
$$\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \int_S \nabla \times \mathbf{B} \cdot \mathrm{d}\mathbf{S} = \mu_0 \int_S \mathbf{J} \cdot \mathrm{d}\mathbf{S}.$$

Stoke’s theorem

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

$$\frac{1}{\mu_0} \nabla \times B = \nabla \times H = J$$

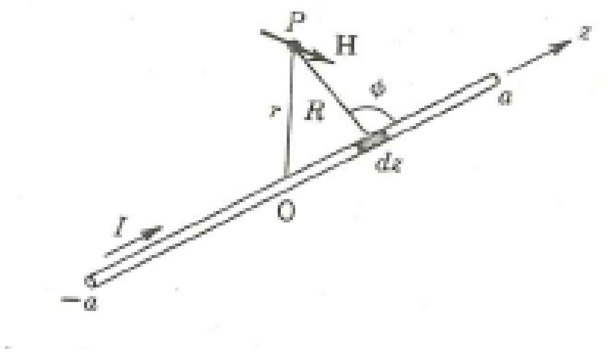
$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$



In magnetostatic, for example: constant DC current. $\frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}} = \mathbf{0}$

Ampere's law: calculating the magentic field around a conductor

$$B = \int_{-a}^{a} \frac{\mu_0 I \sin \phi\, dz}{4\pi R^2} = \frac{\mu_0 I r}{4\pi} \int_{-a}^{a} \frac{dz}{(a^2+z^2)^{3/2}} = \frac{\mu_0 I}{2\pi r} \frac{1}{[(r/a)^2+1]^{1/2}}$$



Stoke's theorem:

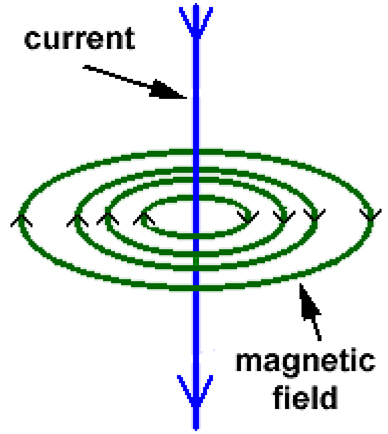
$$\int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \int_C \mathbf{B} \cdot d\mathbf{l}$$

$$\left. \int_s \nabla \times B \cdot dS = \oint B dl = 2\pi r B(r) \right)$$

$$\int_s \nabla \times B \cdot dS = \int_s \mu_0 J \cdot dS = \mu_0 I$$

$$\nabla \times B = \mu_0 J$$

$$2\pi r B(r) = \mu_0 I \Rightarrow B(r) = \frac{\mu_0 I}{2\pi r}$$

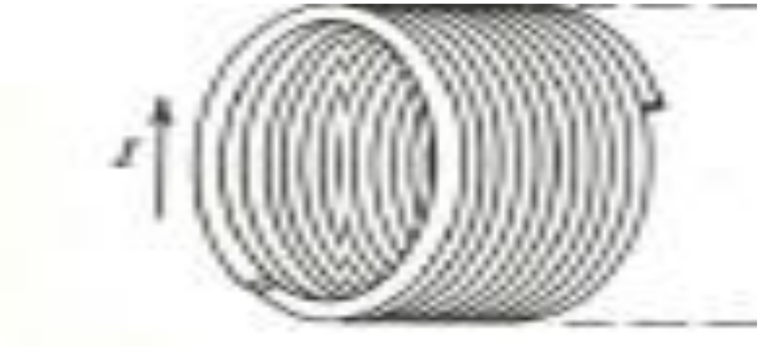


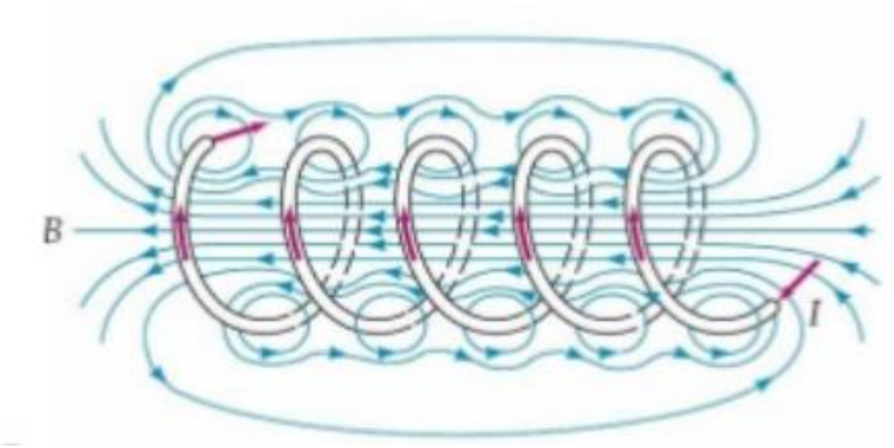
Example: Solenoid

$$\int_s \nabla \times H \cdot dS = \oint H dl = Hl$$

$$\int_s \nabla \times H \cdot dS = \int_s J \cdot dS = NI$$

$$\Rightarrow H = \frac{NI}{l}$$





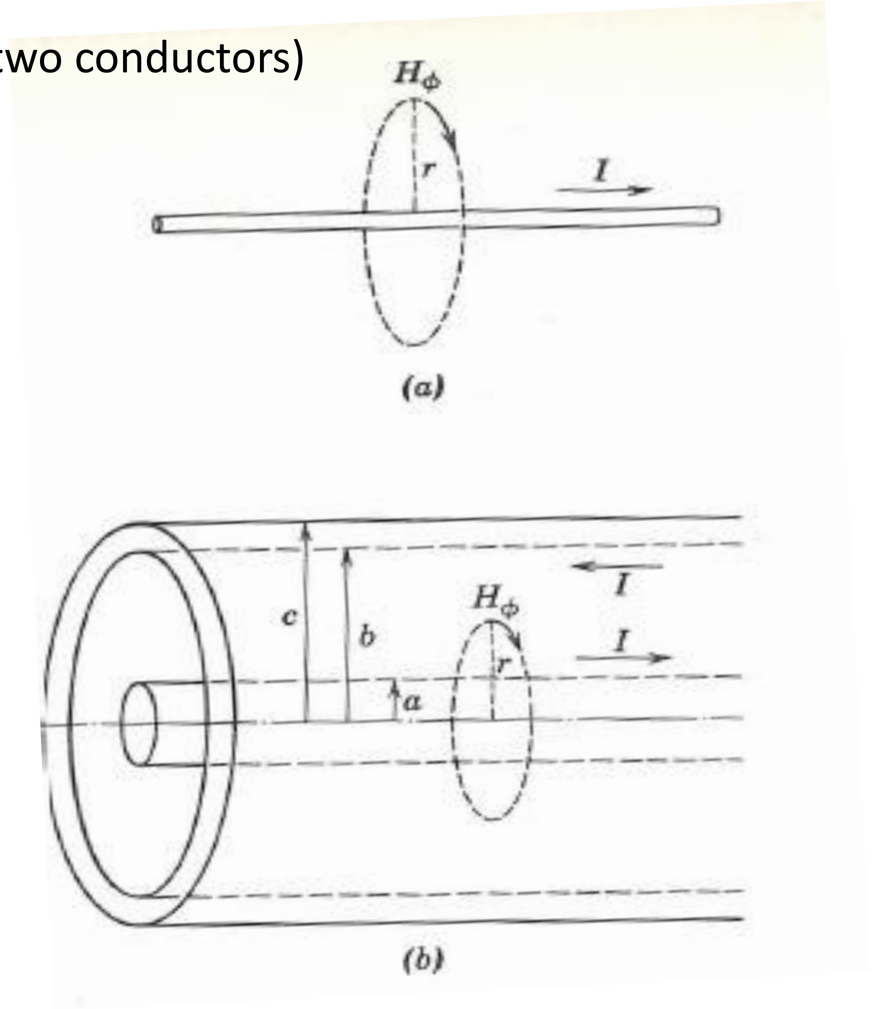
$$\boldsymbol{B} = \mu \boldsymbol{H}$$

If the core is iron instead of air, the flux density **B** is much stronger.

Example: Magnetic field

A coaxial line carrying curent I on the inner conductor and –I on the outer .
Calculate the magnetic field H at r distance, (Current evenly distributed in the two conductors)

1) $0<r<a$,
2) $a<r<b$,
3) $b<r<c$
4) $r>c$



$$I(r) = \left(\frac{r}{a}\right)^2 I$$

$$H_\phi(r) = \frac{I(r)}{2\pi r} = \frac{Ir}{2\pi a^2}$$

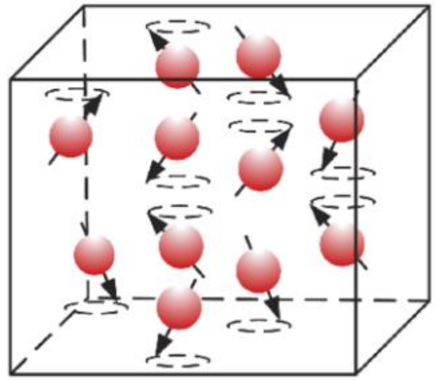
$$H_\phi = \frac{I}{2\pi r} \qquad a < r < b$$

Magnetic field in ferromagnetic materials

$\boldsymbol{B} = \mu_0 \boldsymbol{H}$ in vacuum

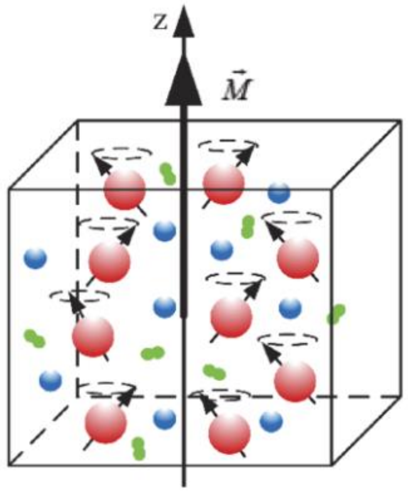
Once there is magentic field applied to medium, the electronic spin motions in the atoms can be thougth of as circulating current that produce a field M, magnetization, which adds to magnetic field H in ferromagetic materials.

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$$



Magnetic susceptibillity χ_m is used to quantify the addtional field M.

$$\boldsymbol{B} = \mu_0(1 + \chi_m)\boldsymbol{H} = \mu_0\mu_r\boldsymbol{H} = \mu\boldsymbol{H}$$



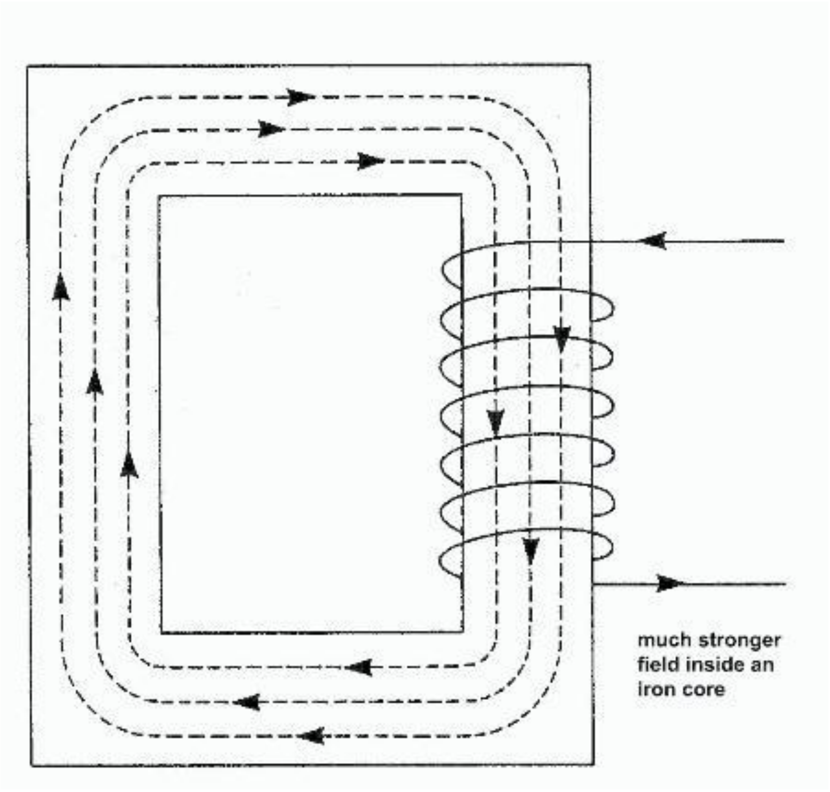
μ_r *is around* 5000 *for iron.*

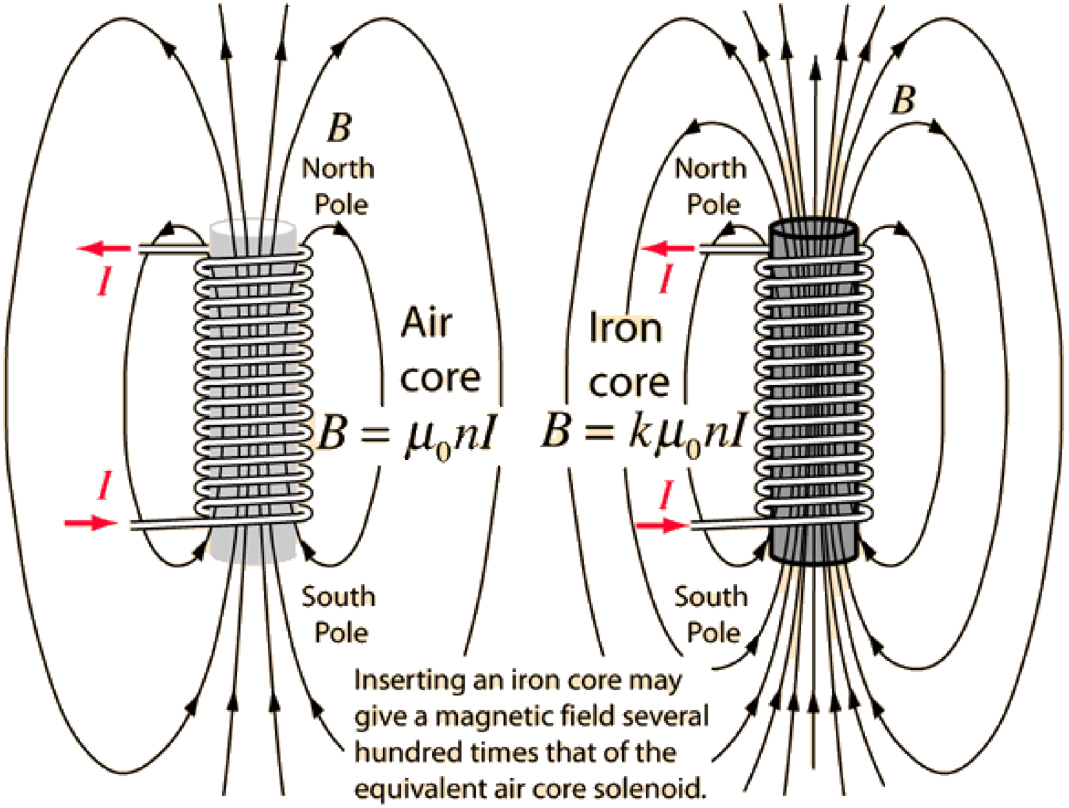
Bad magnetic material relative permeability
Silver:1
Copper : 1
Gold: 1
Aluminium: 1

Good magnetic materail relative permeability
Iron alloy : 100 - 7000

Field in magnetic material

Magnetic material can be used to guide magnetic field path.





Permanent magnet

Some materials after magnetization, the electron movement can remain when external magnetic field dispears. The electron movement can be distroted at high temperatures, strong opposite external field or strong shock.

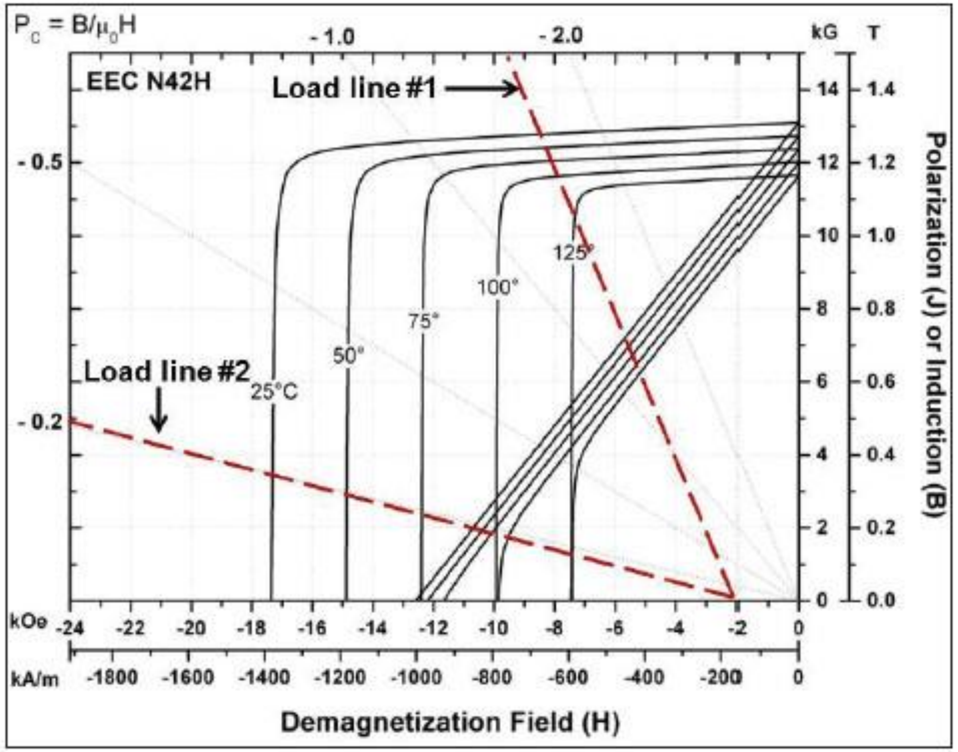
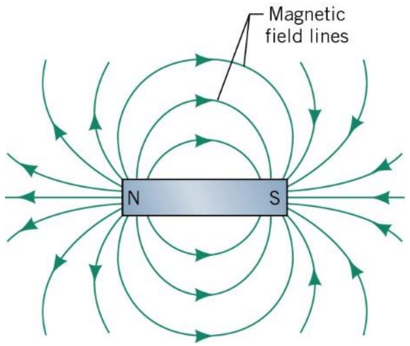
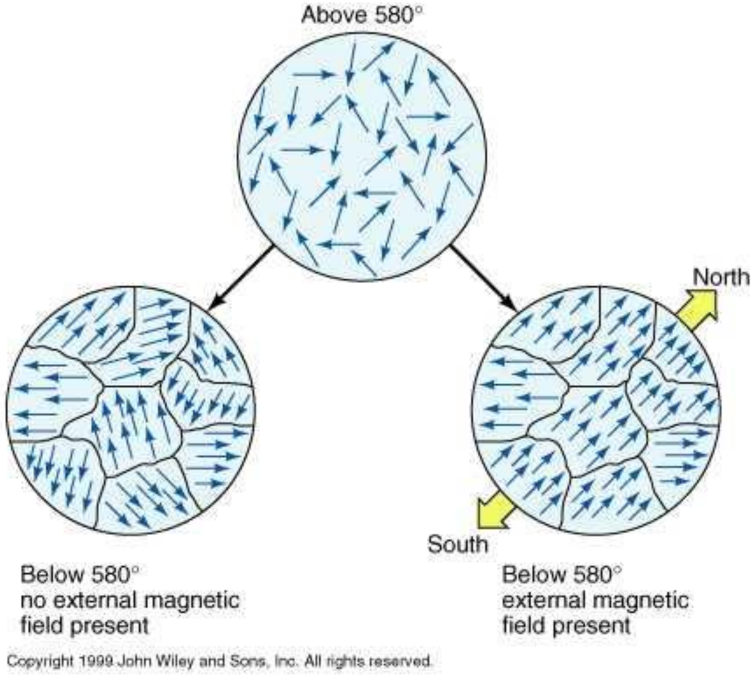
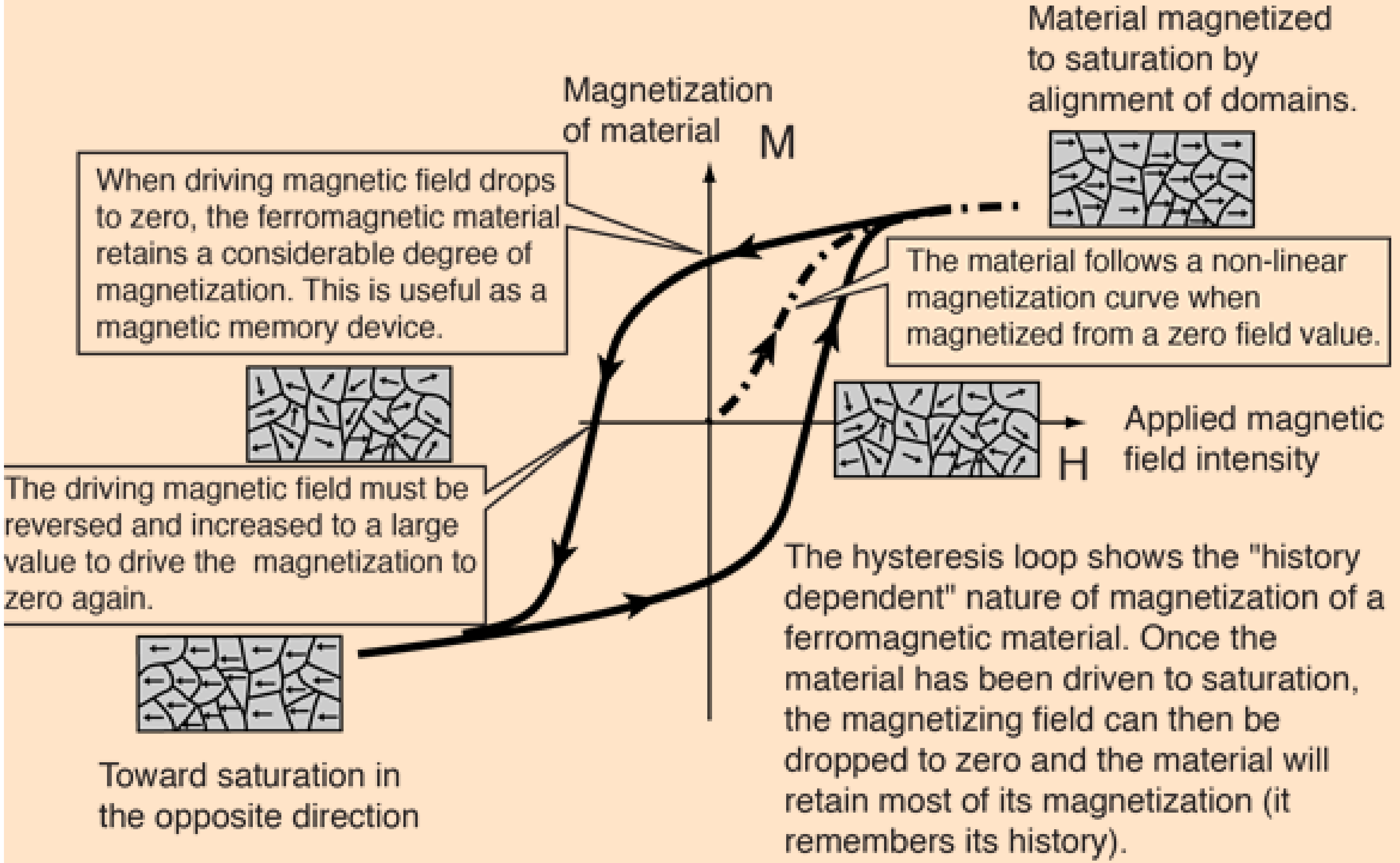


Figure 5. Typical Demagnetization Curves of NdFeB N42H Magnets

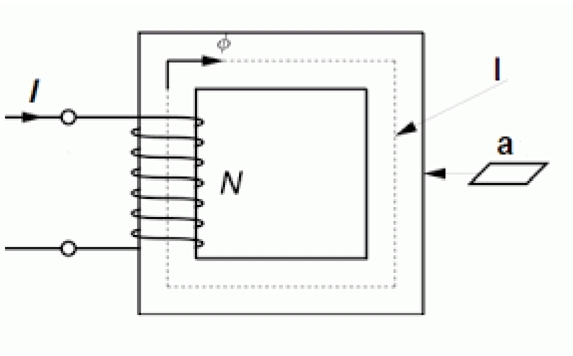




Hysteresis Loop



Magnetic circuit:



MMF: magnetomotive force: NI, HL

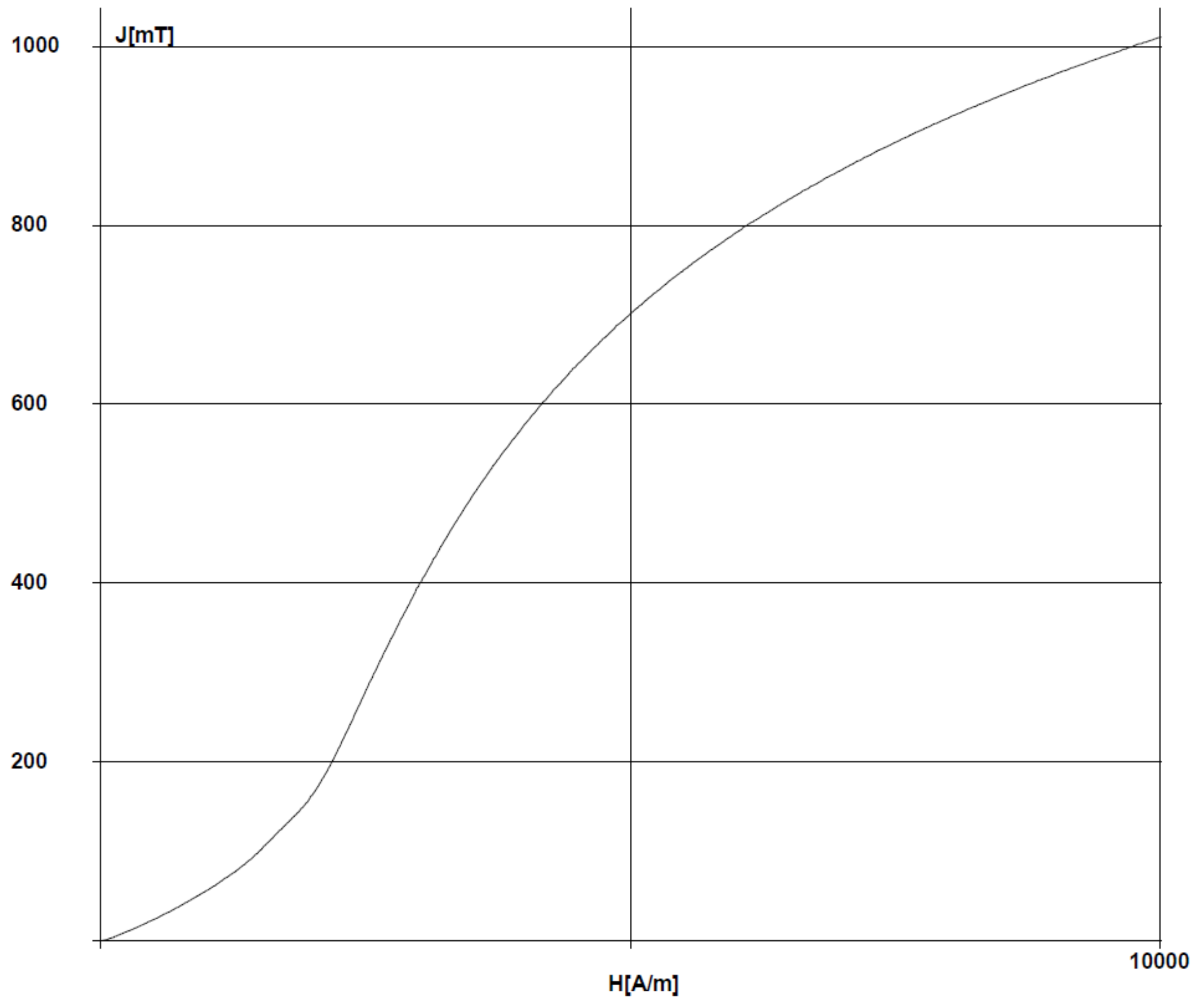
$$R_m = \frac{l}{\mu S}$$

- The magnet (H*l*) or current (NI) possesses a magneto-motive force (MMF).
- The m.m.f generates a magnetic flux.
- The flux exists within the magnet and the air gap between the poles. The enclosed flux path is called a magnetic circuit.
- A stronger m.m.f. will produce more flux.
- The lower the reluctance of the magnetic circuit, the more flux will be produced

Magnetic circuit and electric circuit

Magnetic Circuit	***Electrical Circuit***
1. The closed path for magnetic flux is called a magnetic circuit.	1. The closed path for electric circuit is called an electric circuit.
2. Flux , ø = $\frac{mmf}{Reluctance}$	2. Current, $I = \frac{emf}{resistance}$
3. mmf (Ampere – turns)	3. emf (Volts)
4. Reluctance, $S = \frac{l}{a\mu_0\mu_r}$	4. Resistance, $R = \rho \frac{l}{a}$
5. Flux density, $B = \frac{\emptyset}{a}$ Wb/m^2	5. Current density $\delta = \frac{I}{a}$ A/m^2
6. mmf drop = $\emptyset\ S$	6. Voltage drop = $I\ R$
7. Magnetic Intensity, $H = \frac{N\,I}{l}$	7. Electric intensity, $E = \frac{V}{d}$

Magnetic material: B-H curve



Measuring results

Jmax	=	1010,4 mT
Hmax	=	10008,46 A/m
mumax	=	114,8188
mui	=	4,691205
mur	=	80,33712
Ph(28,0A/m)	=	0

Sample data

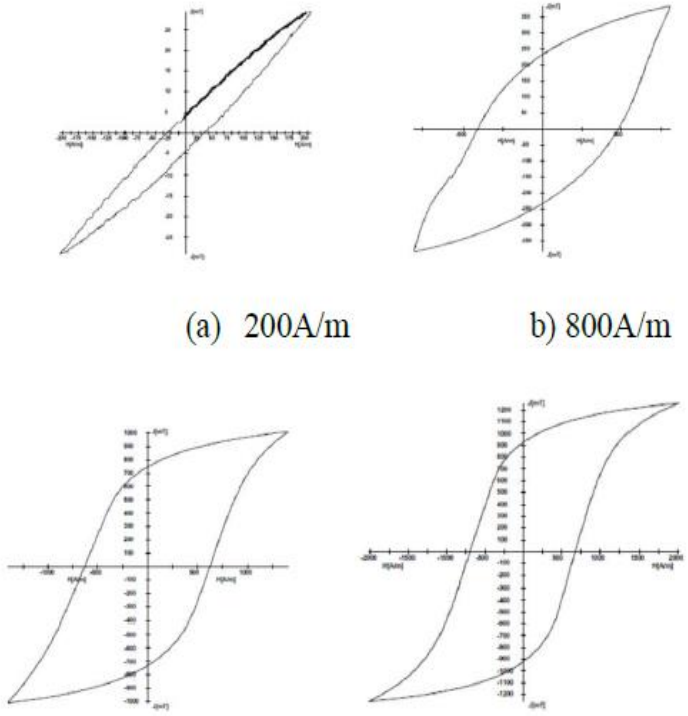
Name	:	blasting_anyuar	
Mass	:	242,6	g
Density	:	7,85	g/cm^3
Length	:	400	mm
Diameter	:	10	mm
Grade	:	DT 63	

Coil : SST 10 x 150

Identification

Operator : Anyuan
Date : 30.03.2017

BROCKHAUS MESSTECHNIK

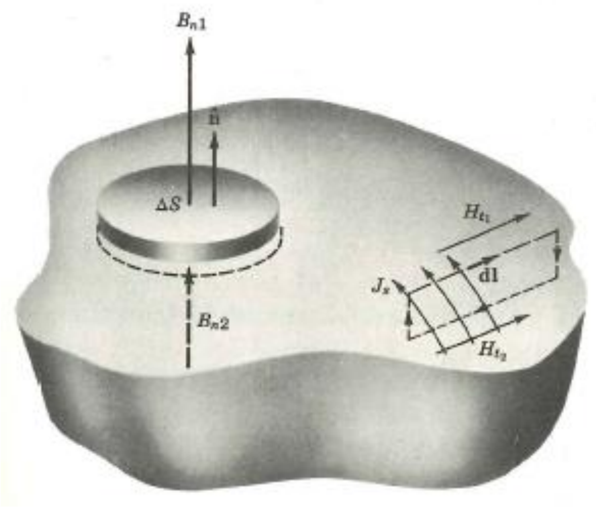


(a) 200A/m
b) 800A/m
c) 1400A/m
d) 2000A/m

Boundary condition for static magentic field

$$\int_V \nabla \cdot \mathbf{B} dV = \oint_S \mathbf{B} \cdot d\mathbf{S} = 0,$$

$$B_{n1}\,\Delta S = B_{n2}\,\Delta S$$



$$B_{n1} = B_{n2}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = H_{t1}\,\Delta l - H_{t2}\,\Delta l = J_s\,\Delta l$$

$$H_{t1} - H_{t2} = J_s$$

$$\frac{1}{\mu_0} \nabla \times B = \nabla \times H = J$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{S}.$$