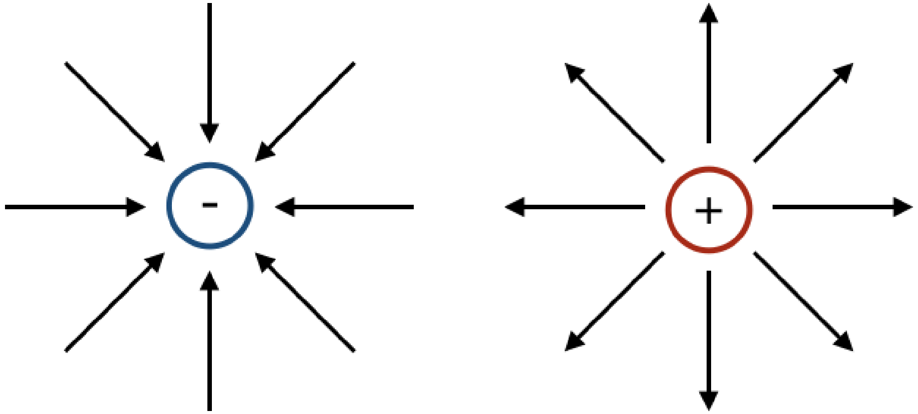
Lecture 3: stationary electric field

1) Energy in electric field
2) Boundary conditon in electro-statics
3) perfect conductor
4) Ohm’s law

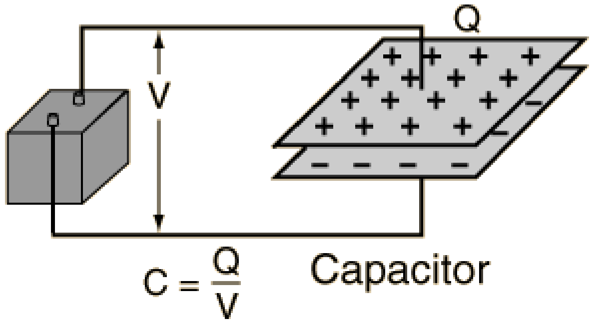
Electric field and energy

Electric potential energy : energy required to move a charge through an electric field.

$$E_p = \frac{qQ}{4\pi\varepsilon_0 r}$$



Capacitors are devices that can store electric potential energy and release it as charge through an electric circuit.



Electric energy in capacitor

The energy stored on a capacitor can be expressed in terms of the work done by the battery.

Voltage represents energy per unit charge

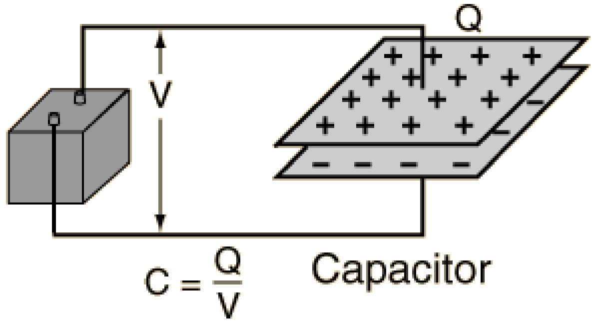
$$W_e = \int_a^b F d\mathbf{l} = \int_a^b qE d\mathbf{l} = qV$$

The work to move a charge element *dq* from the negative plate to the positive plate is equal to V *dq*

$$\mathrm{d}W_e = V dq$$

$$Q = CV$$

$$W_e = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$



$$W_e = \int IU dt = \int_0^V C \frac{dU}{dt} U dt = \int_0^V CU d\mathbf{U} = \frac{1}{2} CV^2$$

$$I = C \frac{dU}{dt}$$

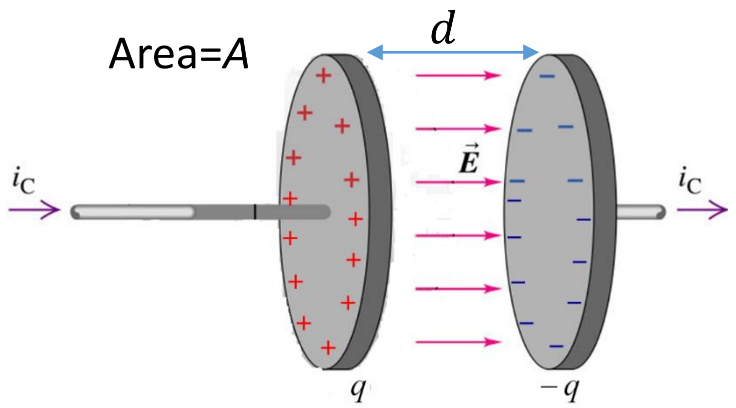
$$\boxed{W_e = \frac{1}{2} CV^2}$$

Energy in electric field

$C = \frac{\varepsilon A}{d}$ Volume=Ad

$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon A}{d}V^2 = \frac{1}{2}\varepsilon Ad\frac{V^2}{d^2} = \frac{1}{2}\varepsilon V_{vol}E^2 = \frac{1}{2}V_{vol}\boldsymbol{DE}$$

$\frac{V^2}{d^2}=E^2$ $\boldsymbol{D}= \varepsilon\boldsymbol{E}$



Electric Energy density $\eta_e=\frac{W_e}{V_{vol}}=\frac{1}{2}\varepsilon E^2=\frac{1}{2}\boldsymbol{DE}$

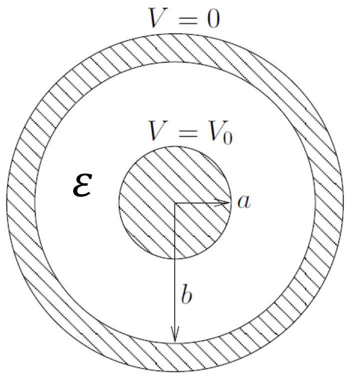
Electric Energy density

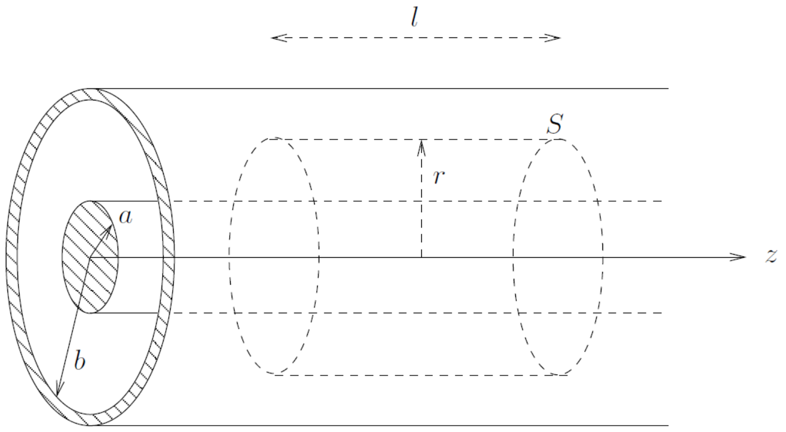
$$\eta_e = \frac{1}{2}ED = \frac{1}{2}\varepsilon E^2$$

Example:

Assumming : infinite long cable and the permittivity of the dielectric in between is ε

1) calculate the capacitance per unit length.
2) calculate the electric energy stored in unit length





$$C' = \frac{Q'}{V_0} \qquad V_0 = \frac{Q'}{2\pi\epsilon} \ln\frac{b}{a} \qquad C' = \frac{Q'}{V_0} = \frac{2\pi\epsilon}{\ln\frac{b}{a}}.$$

$$W_e' = \frac{1}{2} C' V_0^2 = \frac{\pi\epsilon}{\ln\frac{b}{a}} V_0^2.$$

$$W_e = \frac{1}{2}\int_{\text{mellom lederne}} \epsilon E^2 \mathrm{d}v = \frac{\epsilon V_0^2}{2\left(\ln\frac{b}{a}\right)^2}\int_a^b \frac{2\pi r \mathrm{d}r l}{r^2} = \frac{2\pi\epsilon V_0^2}{2\ln\frac{b}{a}} l,$$

$$W_e' = \frac{W_e}{l} = \frac{\pi\epsilon}{\ln\frac{b}{a}} V_0^2.$$

Boundary conditions in electrostatics

Electric field involves more than one materials

Normal component

$$D_{n1}\Delta S - D_{n2}\Delta S = \rho_s \, \Delta S$$

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_S$$

$$D_{n1} - D_{n2} = \rho_s$$

Conservative field: electrostatic electric field

Tangential component

$$E_{t1}\Delta l - E_{t2}\Delta l = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

$$E_{t1} = E_{t2}$$

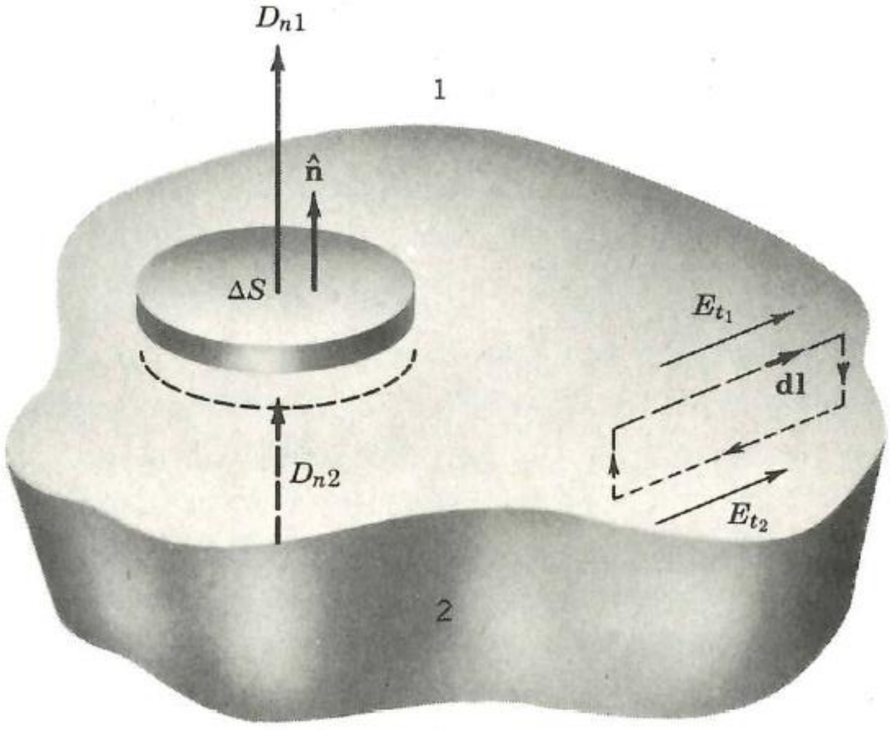
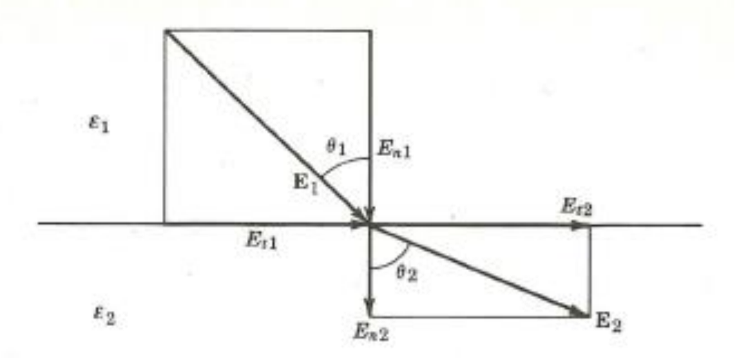


Fig. 1.14*a* Boundary between two different media.

Electric field changes direction accross two different dielectric materials

Considering, no charge on the surface between two dielectric materials



$$D_{n1} - D_{n2} = \rho_s = 0;$$

$$\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$$

$$E_{n2} = \frac{\varepsilon_1 E_{n1}}{\varepsilon_2}$$

$$\theta_2 = \tan^{-1}\frac{E_{t2}}{E_{n2}} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1\right)$$

$$E_{t1} = E_{t2}$$

$$\Rightarrow E_{t2} = E_{t1} = E_{n1}\tan\theta_1$$

$$\tan\theta_1 = \frac{E_{t1}}{E_{n1}}$$

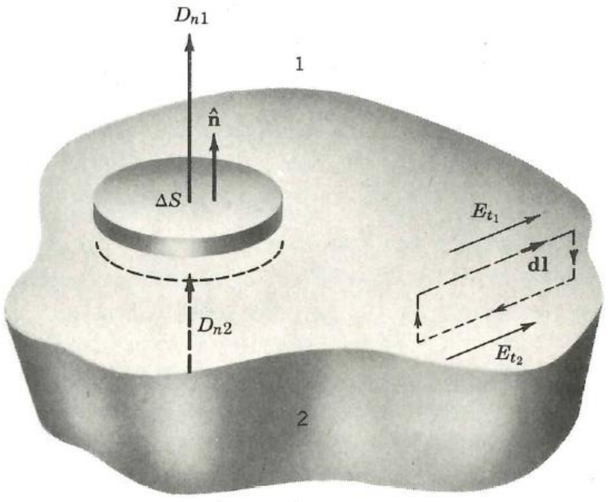


Fig. 1.14*a* Boundary between two different media.

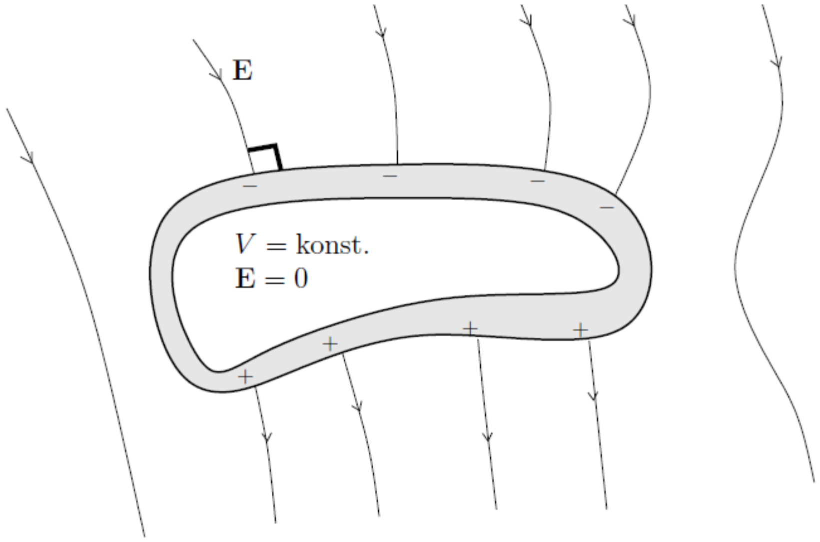
Perfect conductor in electric field

1) E=0 inside the conductor. Both $E_t = E_n$=0.
2) $\rho_{\text{in}} = 0,$ no charge inside the conductor.
3) $\rho_s \neq 0,$ there is surface charge .
4) The electric field outside the boundary of the perfect conduct is

$$E_n = \frac{\rho_s}{\varepsilon},\ E_t = 0,$$

5) The conductor is an equipotential surface.

$$V_{AB} = -\int_A^B E dl = 0.$$



$$D_{n1}\,\Delta S - D_{n2}\,\Delta S = \rho_s\,\Delta S$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = E_{t1}\,\Delta l - E_{t2}\,\Delta l = 0$$

$$D_{n1} - D_{n2} = \rho_s$$

$$E_{t1} = E_{t2}$$

Faraday cage:

https://www.youtube.com/watch?v=t23iXhEiQUc

Material conductivity and ohm's law

Good conductive material:

Silver: $6.2 \times 10^7 S/m$

Copper $5.8 \times 10^7 S/m$

Gold $4.1 \times 10^7 S/m$

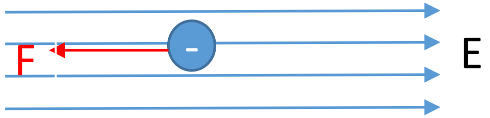
Aluminium $3.5 \times 10^7 S/m$

Non conductive material

Glass $\times 10^{-12} S/m$

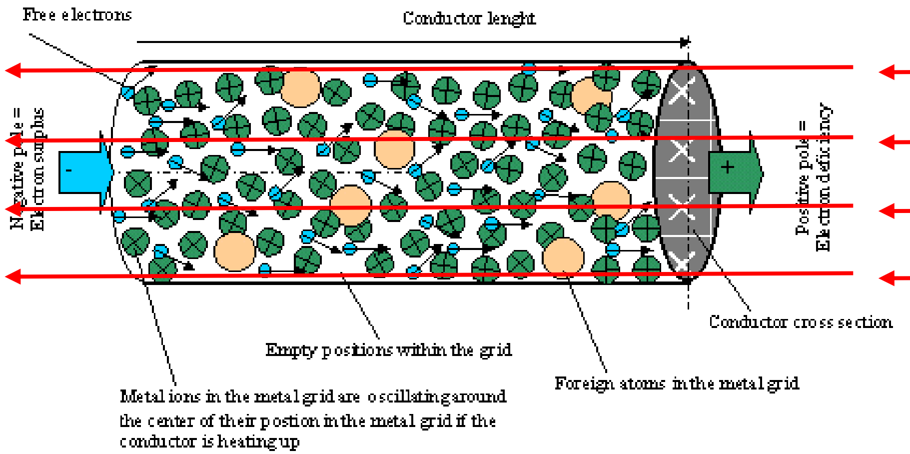
Air $\times 10^{-14} S/m$

Once there are free charges in an electric field, the charges can move along the electric field direction.

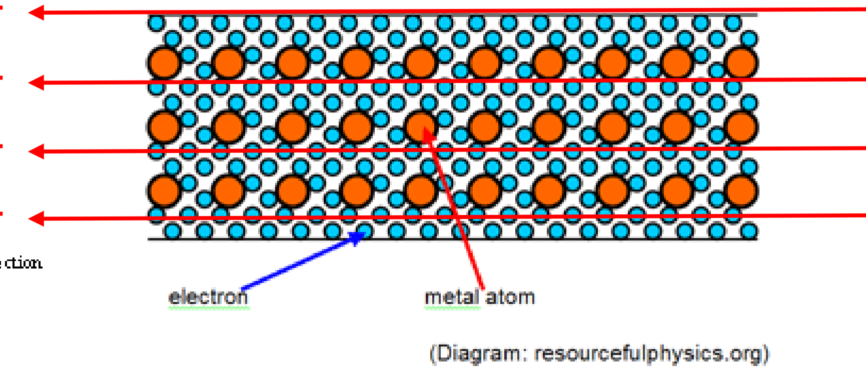


That how many charges can flow through a material is dependent of the material property and the applied electric field.
When electrons move in a material, they collide atoms and lost energy.
The capability is represented by σ, conductivity.
J is called current density. The charges go through the material per unit area per second.

$$\mathbf{J} = \sigma \mathbf{E}$$



Low conductive material



High conductive material

Ohm’s law (II)

An electric current is a flow of electric charge.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Current density *J* is the current per unit area

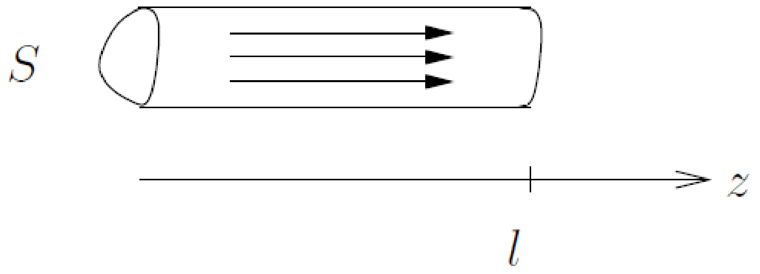
$$I = \int_S \mathbf{J} \cdot \mathrm{d}\mathbf{S}.$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$R = \frac{V}{I}$$

Example:

A conductor with constant conductivity σ, and cross − sectional area S , the length is l and the constant current is I



1) Calculating the resistance of the conductor and deriving $R = \frac{V}{I}$.
2) Calculating the power done by the current.

Area S, and I is also constant. $J = I/S$

Constant σ , the electric field is constant: $E = J/\sigma$

$$V = \int_0^l E\mathrm{d}z = \int_0^l (J/\sigma)\mathrm{d}z = Jl/\sigma$$

$$R = \frac{l}{S\sigma} = \frac{Jl}{\sigma}\frac{1}{JS} = \frac{V}{I}$$

Vector: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

Vector: $\vec{F} = \frac{qQ}{4\pi\varepsilon_0 r^2}\hat{r}$: ***Coulomb's law***

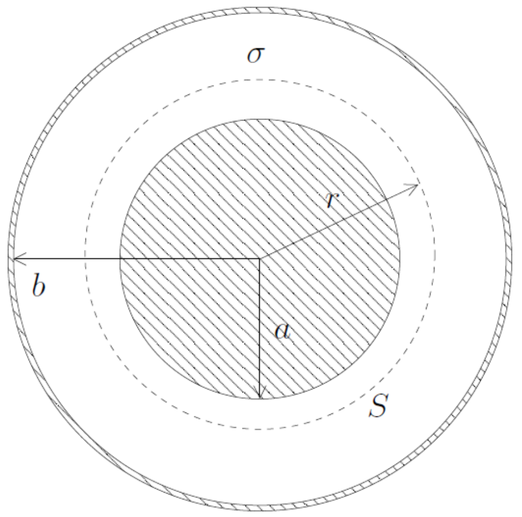
$\vec{E} = \vec{F}/q$ $\quad V = -\int E.dl$

$$W_e = \int_0^l Fdl = \int_0^l qEdl = qV \Rightarrow W_e = \int VIdt$$

q= $\int Idt$

$$P = VI = I^2R$$

Example:



A solid conductive ball with a radius *a* is put into a hollow conductive ball with inner radius *b*, between is a type material with conductivity *σ* .

a) What is the resistance between the two balls?
b) If the solid ball is buried deeply in to earth, what is the earth resistance?

$$a = 0.5 \text{ m} \qquad \sigma = 10^{-2} \text{ m}^{-1}\Omega^{-1}$$

$$\mathbf{J} = \frac{I}{4\pi r^2}\hat{\mathbf{r}}, \quad \text{for } a < r < b.$$

$$\mathbf{E} = \mathbf{J}/\sigma = I\hat{\mathbf{r}}/(4\pi\sigma r^2)$$

$$V = -\int_a^b E\mathrm{d}r = -\int_a^b \frac{I}{4\pi\sigma r^2}\mathrm{d}r = \frac{I}{4\pi\sigma}\left(-\frac{1}{b} + \frac{1}{a}\right)$$

$$R = \frac{V}{I} = \frac{1}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{b}\right)$$

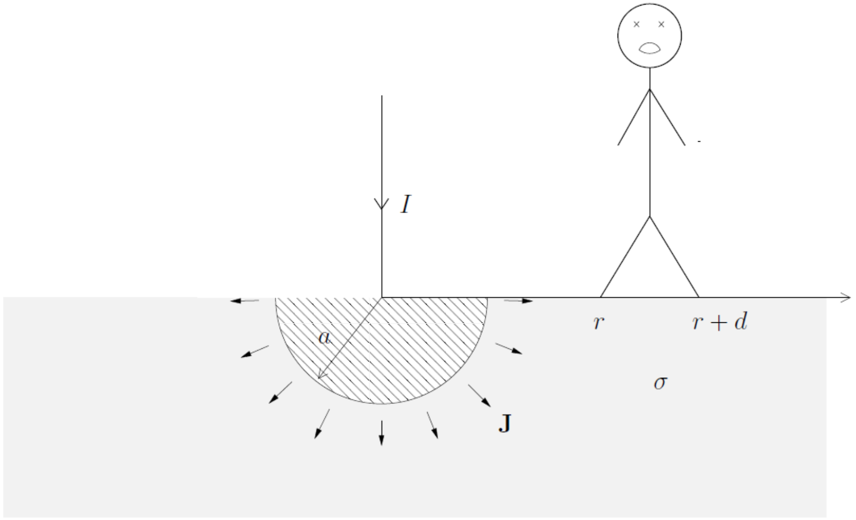
example

Now if a half solid ball is buried in earth as shown in picture, recalculate the resistance?
If the current is 1000A $\sigma = 10^{-2}\,\mathrm{m}^{-1}\Omega^{-1}$, $r = 1\,\mathrm{m}$ og $d = 0.75\,\mathrm{m}$
What is the voltage between the two legs of the people

$$\mathbf{J} = \frac{I}{2\pi r^2}\hat{\mathbf{r}}, \quad \mathbf{E} = \frac{I}{2\pi\sigma r^2}\hat{\mathbf{r}}, \quad \text{for } r > a.$$

$$R = \frac{1}{2\pi\sigma a}$$

$$V = -\int_r^{r+d} E(r)\mathrm{d}r = -\int_r^{r+d} \frac{I}{2\pi\sigma r^2}\mathrm{d}r = \frac{I}{2\pi\sigma}\left(\frac{1}{r} - \frac{1}{r+d}\right)$$



Kirchoff’s law

Current conservation:

Kirchoff’s law: at any point at electric circuit, the sum of the current is zero.

$$\oint_S \mathbf{J} \cdot \mathrm{d}\mathbf{S} = 0.$$