TFE4120 Electromagnetism: crash course

Intensive course: Two-weeks.

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Paticipants: should have Bsc in electronic, electrical/ power engineering.

Aim of the course: Give students a minimum pre-requisity to follow a 2-year master program in electronics or electrical /power engineering.

Webpage: All information is posted there .
https://www.ntnu.no/wiki/display/tfe4120/Crash+course+in+Electromagnetics+2024

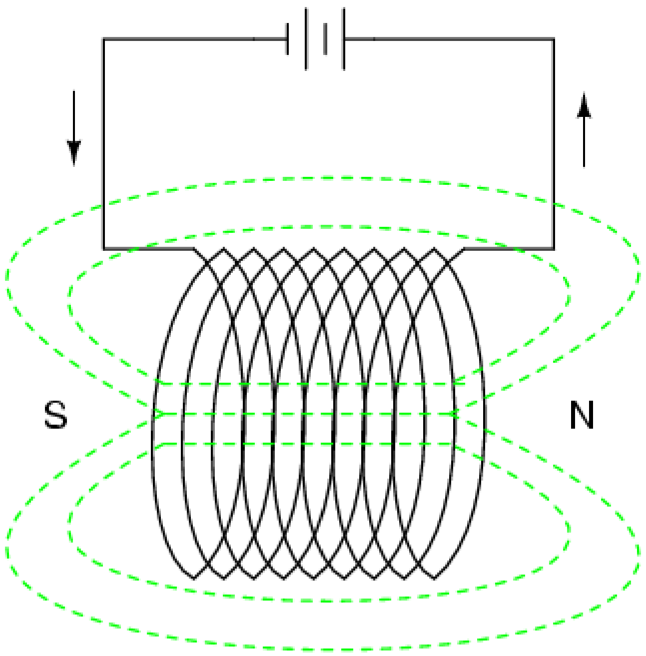
Lecture plan

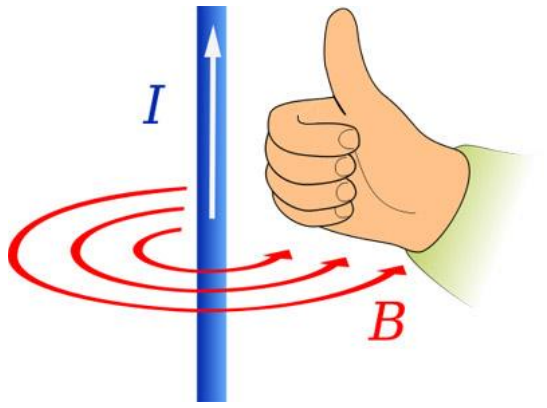
Lecture Plan

Week	Date	Time	Room	Topics
32	Monday 5/8	09.15-12.00	EL6	Maxwell's equations, Vector calculus, Divergence´s Theorem, Stokes´ Theorem
	Tuesday 6/8	09.15-12.00	EL6	Coulomb's law, Gauss' law, Potential, Poisson equation
	Thursday 7/8	09.15-12.00	EL6	Energy in electric field, Electric field in material, Capacitance, Boundary conditions for electric fields, Ideal conductors, Current density
	Thursday 8/8	09.15-12.00	El6	Exercise help session with additional time
33	Tuesday 13/8	09.15-12.00	EL1	Magnetic fields, Biot-Savart's law, Ampere's law, Magnetic field in material, Boundary conditions for magnetic field
	Wednesday 14/8	09.00-12.00	EL1	Faraday´s law, Ampere's law, Induction, Inductance, Lenz's law
	Thursday15/8	09.15-12.00	EL1	Maxwell's equations, Wave equations, Poynting's theorem, Summary
	Friday 16/8	09.15-12.00	EL1	Exercise help session

Lecture1: Electro-magnetism and vector calculus

1) What does electro-magnetism describe?
2) Brief induction about Maxwell equations
3) Vector calculus



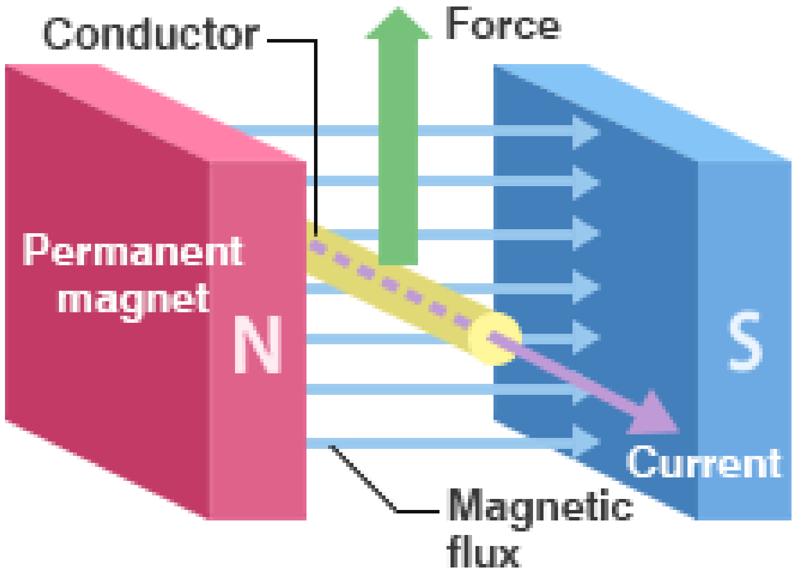


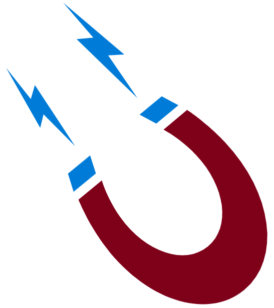
Electro-magnetism

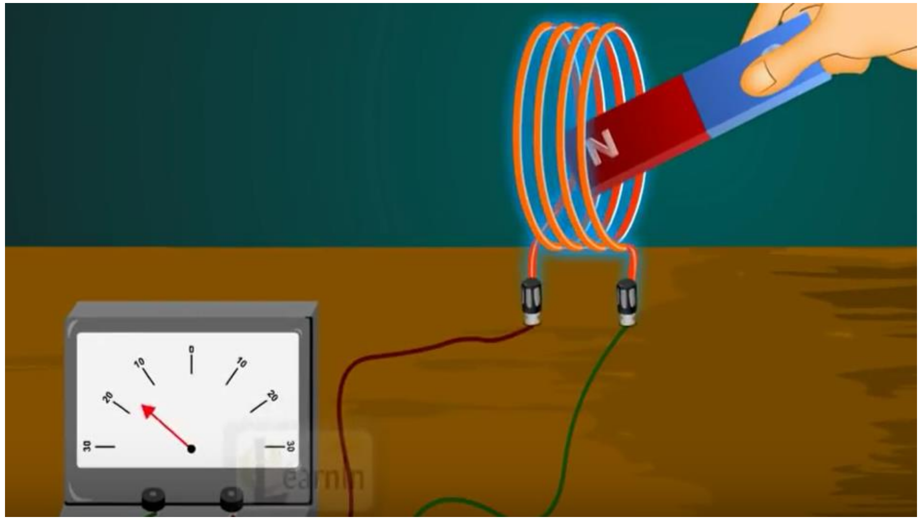
Electricity and magnetism were considered to be two separate phenomena until Maxwell published his masterpiece of electricity and magnetism in 1873.

Electro-magnetism: Physical interaction among electric charges, magnetic moments, and electromagnetic field.

Electro-magnetic force: one of the four fundamental interactions in the nature. (gravitational, electromagnetic, strong and weak forces)







History:

Carl Friedrich Gauss (1777-1855): German mathematician and physicist

The electric flux out of a closed surface = total enclosed charge divided by the permittivity of free space

Electrostatic

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Andre-Marie Ampere (1775-1836): French physicist and mathematician

The magnetic field produced by an electric current is proportional to the magnitude of the current with a proportionality constant equal to the permeability of free space (μ_o)

Magnetostatic

$$\oint \vec{B}.\vec{dl} = \mu_o I$$

Michael Faraday (1791-1867): English Scientist

- In 1831 Faraday observed that a moving magnet could induce a current in a circuit.
- He also observed that a changing current could, through its magnetic effects, induce a current to flow in another circuit.

Magnetodynamic $V = -\frac{d\emptyset}{dt}$

Carl Friedrich Gauss

André-Marie Ampère

Michael Faraday
FRS

Founder of electromagnetism

James Clerk Maxwell: (1839-1879) Scottish Mathematician

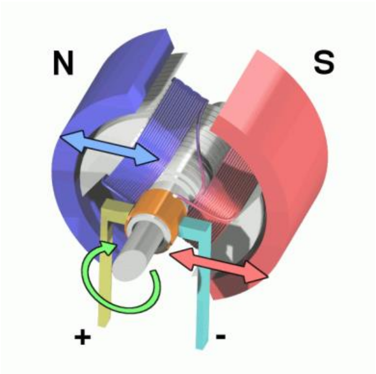
Maxwell published his work“ A treatise on electricity and magnetism” in 1873.

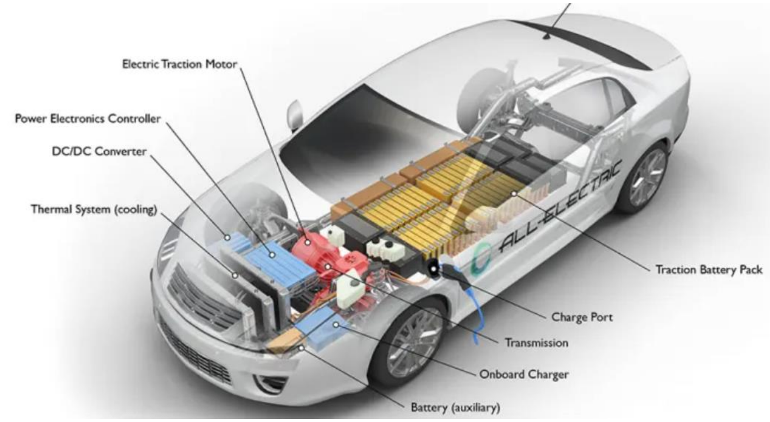
- Maxwell equations
- Developed a scientific theory to explain electromagnetic waves.
- Coupled the electrical fields and magnetic fields together
- Established the foundations of electricity and magnetism as **electromagnetism.**

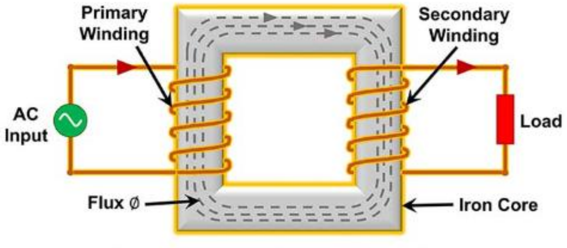
Daily life applications

For example:

- Electric motor/generator:
- Battery charger
- Induction oven
- ……

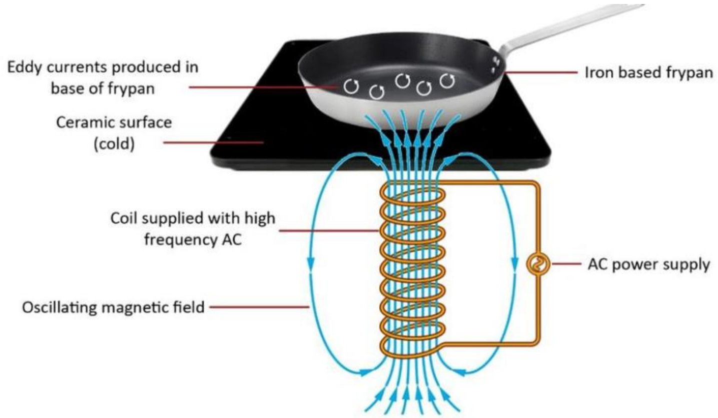






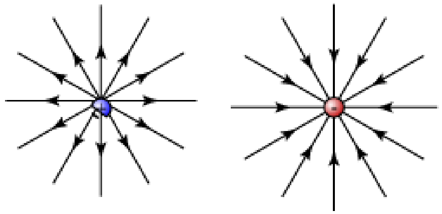
Basic Construction of Transformer

Iphone charger

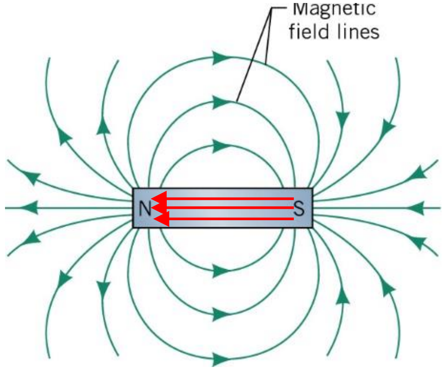


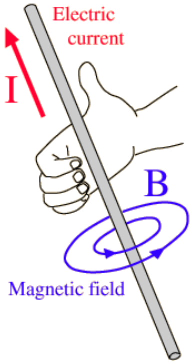
Electromagnetism: Maxwell equations

- A static electric charge produces electric field (Gauss's law in electric form)
- There is no magnetic charge (monopole) (Gauss's law in magnetic form)
- A *changing* magnetic field produces an electric field (Faraday's law)
- Charges in *motion* (an electrical current) produce a magnetic field (Ampere's law)
- A changing electric field produces a magnetic field.



Field pattern of
a pointed electrode





Electric and Magnetic fields can produce forces on charges

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

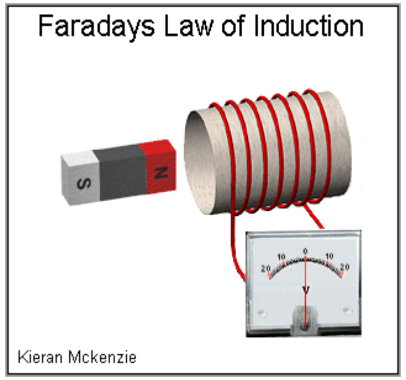
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

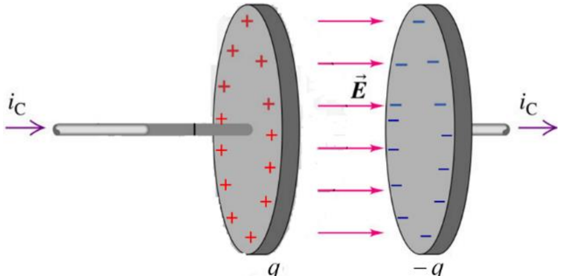
E: Electric field, Vector

B: Magnetic field, Vector

J: Current density, Vector

ρ: Electric charge density, Scalar

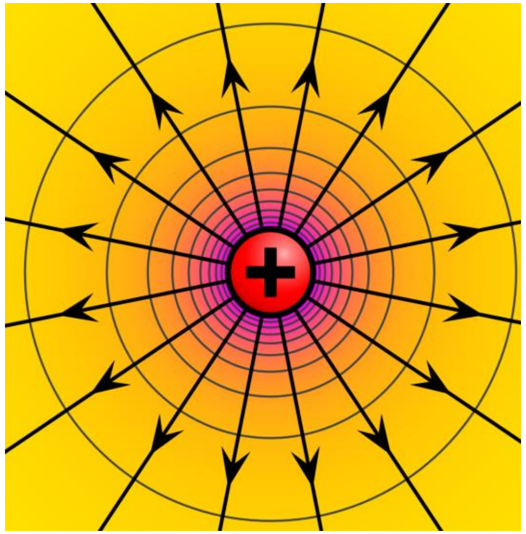


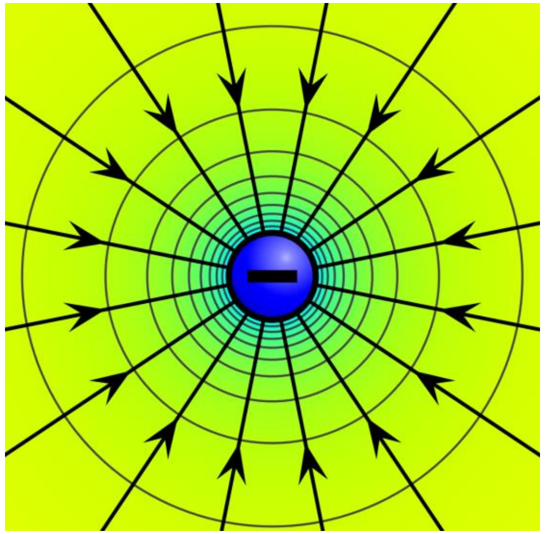


Electricity and magnetism had been unified into electromagnetism!

Gradient, electric potential and field

Gradient: 3-dimension derivative of a scalar function





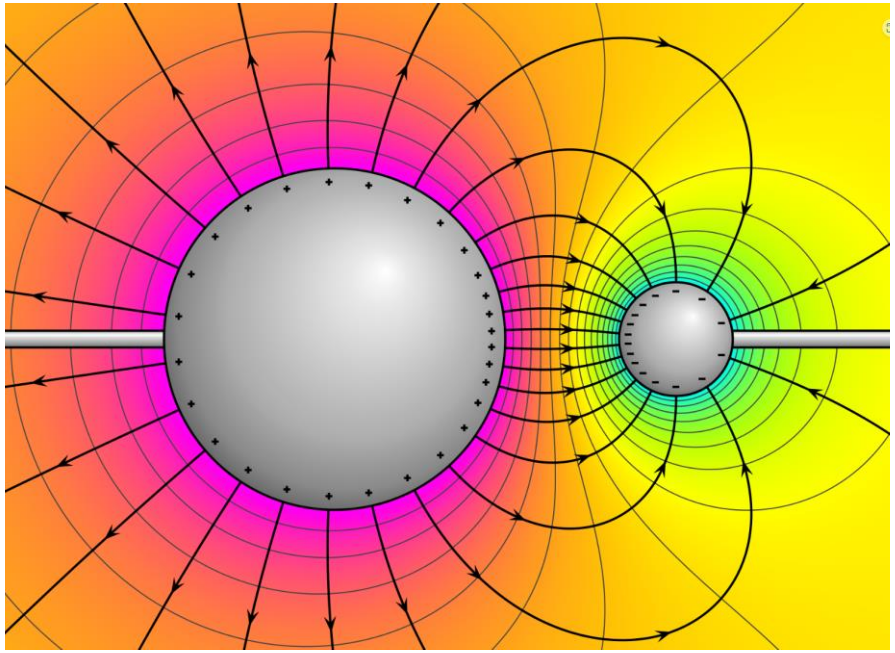
- The gradient is a vector.
- Its **direction is along the fastest increase** of the scalar function at a point space.
- Its magnitude quantifies the change of a scalar field per unit distance.

Indicating how quickly a scalar increase per unit distance in a point space?

$$\nabla f = \text{grad } f = < \frac{\partial f}{\partial x}(x,y,z), \frac{\partial f}{\partial y}(x,y,z), \frac{\partial f}{\partial z}(x,y,z) >$$

$$\nabla \boldsymbol{f} = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

$$\mathbf{E} = -\nabla V_{\mathbf{E}}$$



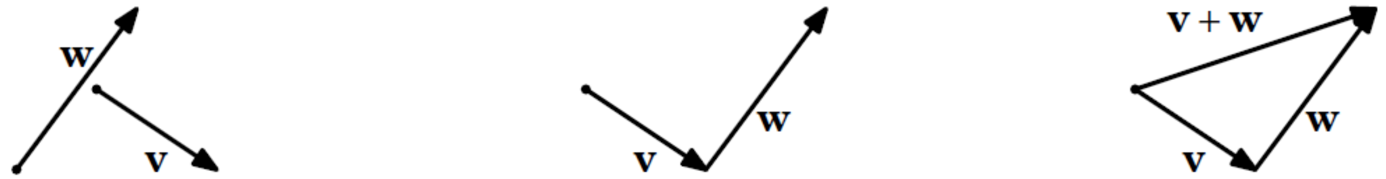
f : Scalar function

$\nabla \boldsymbol{f}$ (Gradient): Vector function

Direction: fastest rate of increase

Basic Vector calculus:

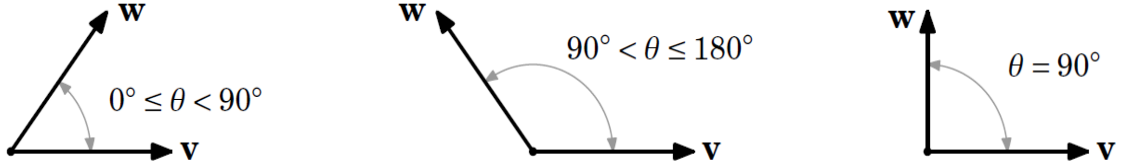
$\mathbf{v} + \mathbf{w}$



(a) Vectors **v** and **w** **(b)** Translate **w** to the end of **v** **(c)** The sum **v** + **w**

Dot product

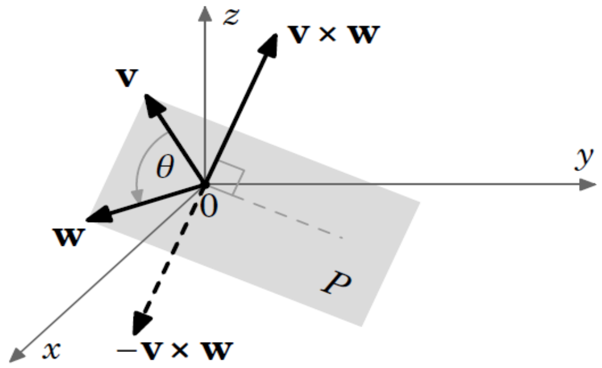
$$\mathbf{v} \cdot \mathbf{w} = \cos\theta \, \|\mathbf{v}\| \, \|\mathbf{w}\|$$



(a) $\mathbf{v} \cdot \mathbf{w} > 0$ **(b)** $\mathbf{v} \cdot \mathbf{w} < 0$ **(c)** $\mathbf{v} \cdot \mathbf{w} = 0$

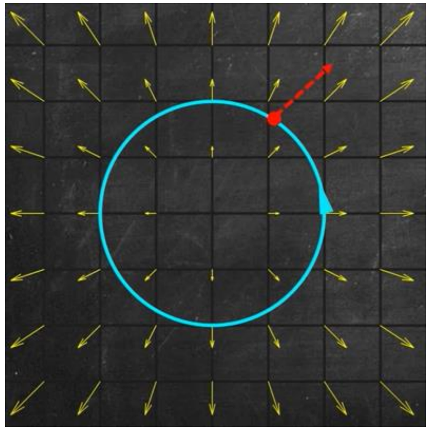
cross product

$$\|\mathbf{v} \times \mathbf{w}\| = \|\mathbf{v}\| \, \|\mathbf{w}\| \sin\theta$$



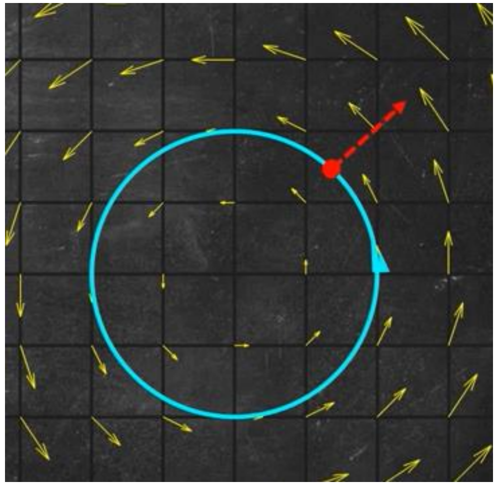
Flux and Flow

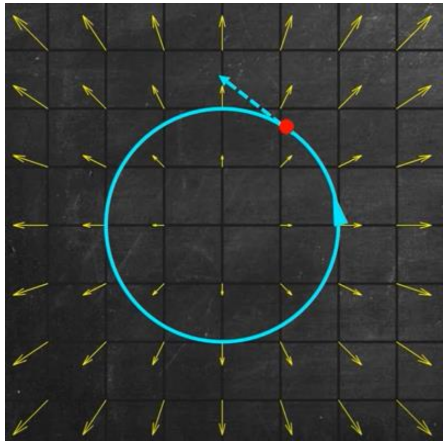
Flux: field perpendicular to the boundary
Flow: field tangential to the boundary



$\vec{n}$ the outward normal

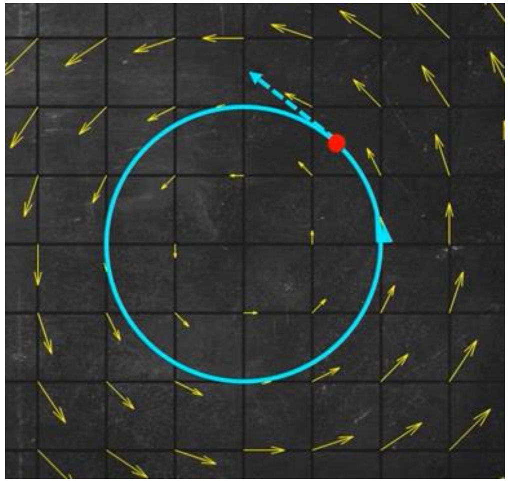
$$Flux = \oint_C \vec{F} \cdot \vec{n} ds$$





$$Flow = \int_C \vec{F} \cdot \vec{T}\, ds$$

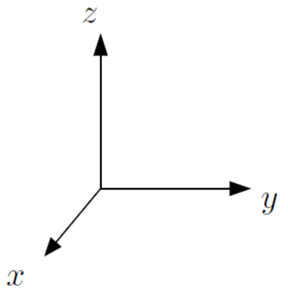
If the curve C is *closed,* called *Circulation*

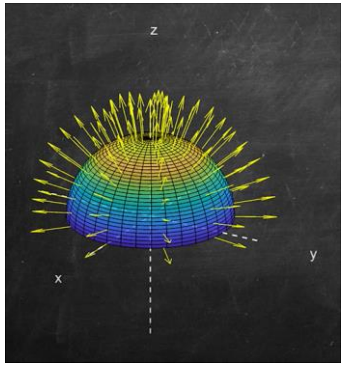


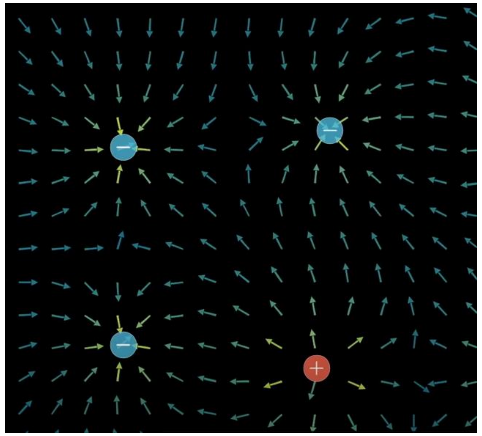
Divergence (Flux density)

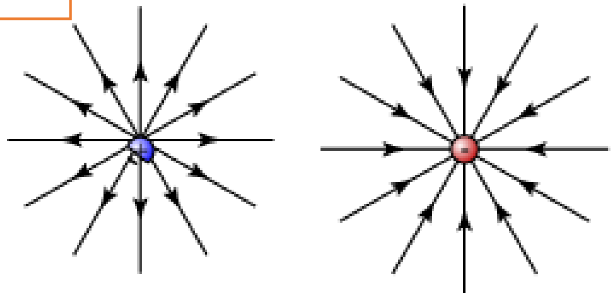
Divergence of a field represents the flux density of the outward flux of a vector field from an infinitesimal volume (boundary) around a given point

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \lim_{V\to 0}\frac{1}{|V|}\oiint_{S(V)} \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$$







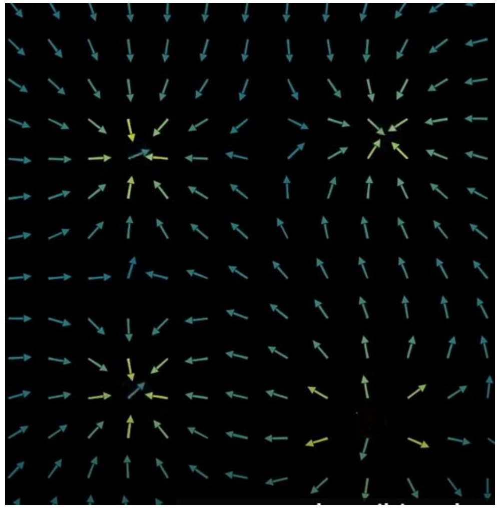


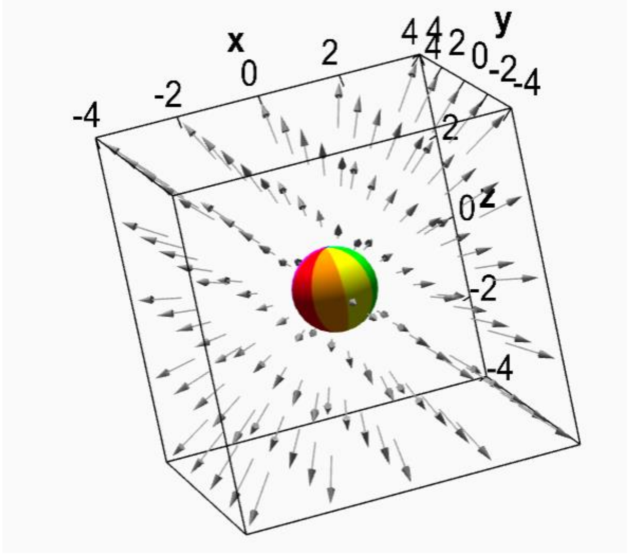
Field pattern of
a pointed electrode

F is a vector

$\nabla\cdot F$ is a scalar.

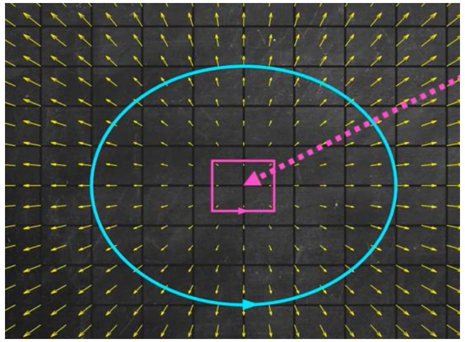
$$\nabla\cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

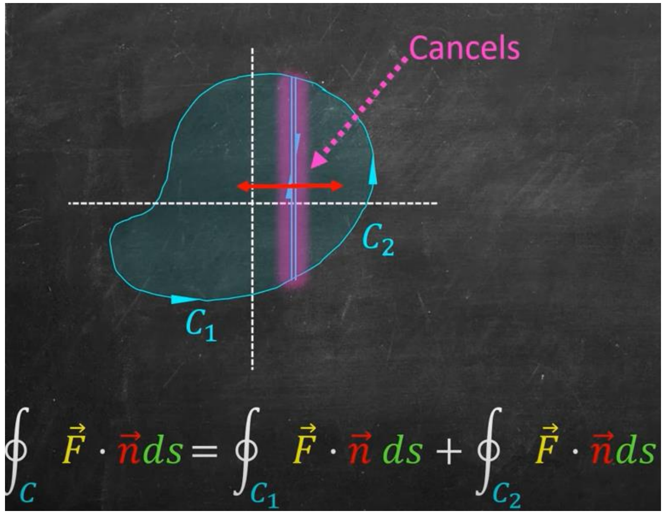




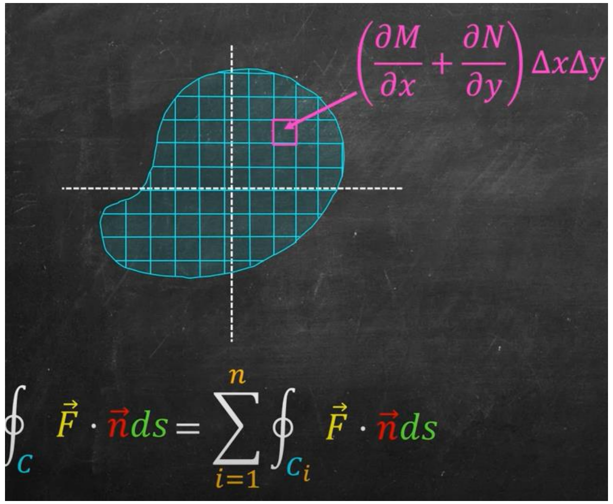
Divergence theorem

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A} dv$$



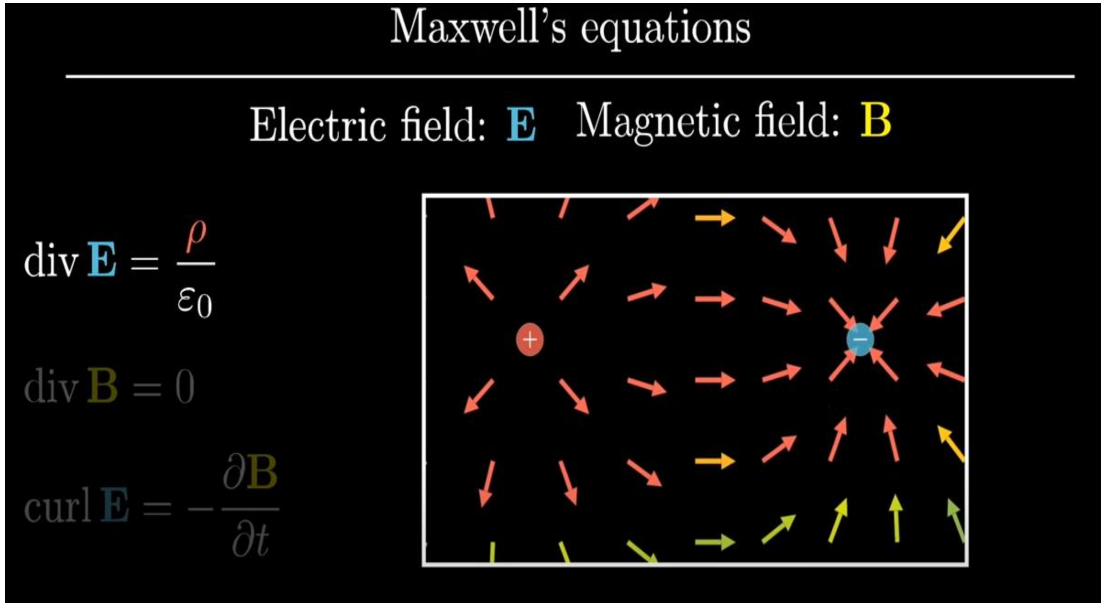


$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \sum_i \oint_{S_i} \mathbf{E} \cdot d\mathbf{S} = \sum_i \left(\frac{1}{\Delta V_i} \oint_{S_i} \mathbf{E} \cdot d\mathbf{S}_i\right) \Delta V_i \rightarrow \int \nabla \cdot \mathbf{E} dV.$$

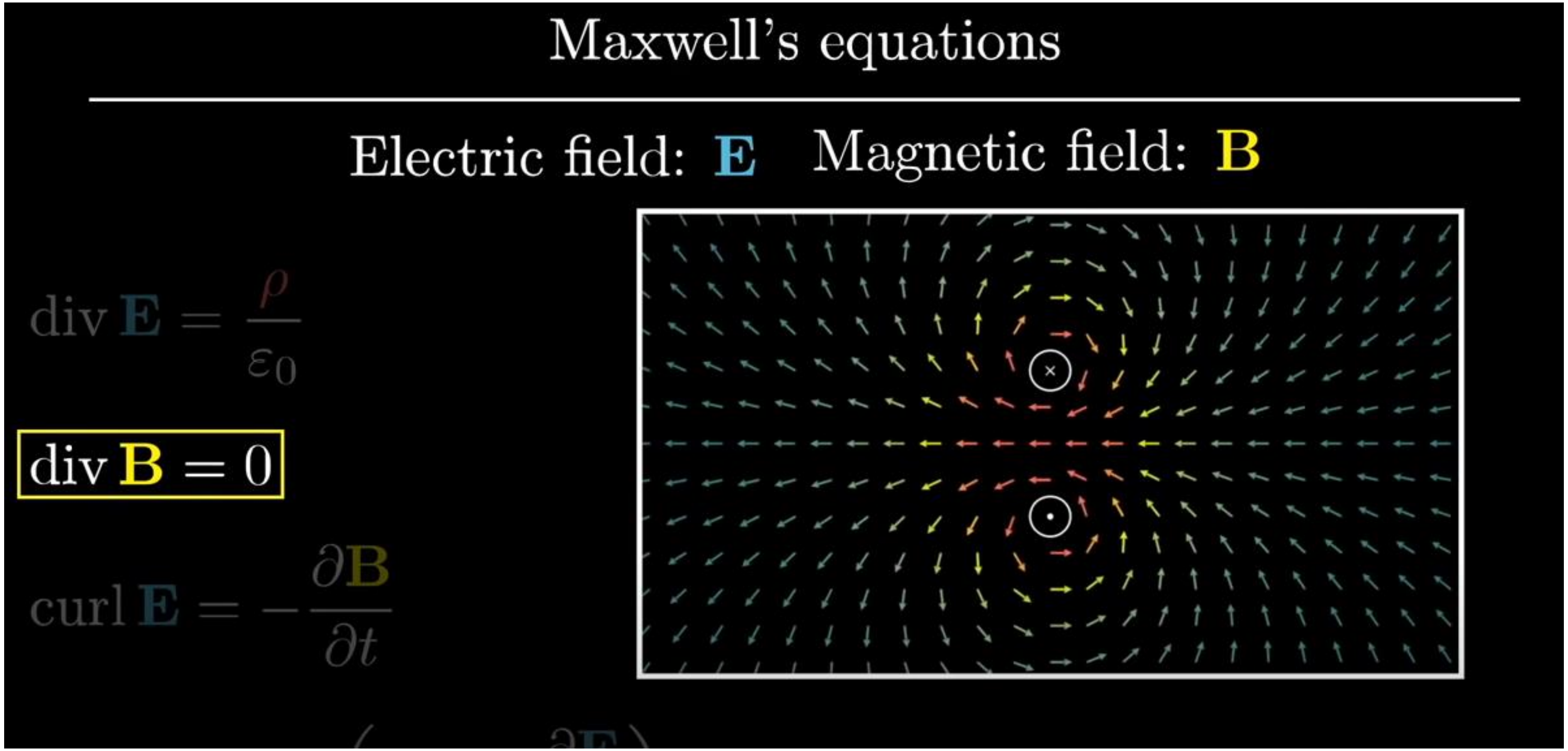


$$\nabla . E = \frac{\rho}{\varepsilon_0}$$

Gauss' Law

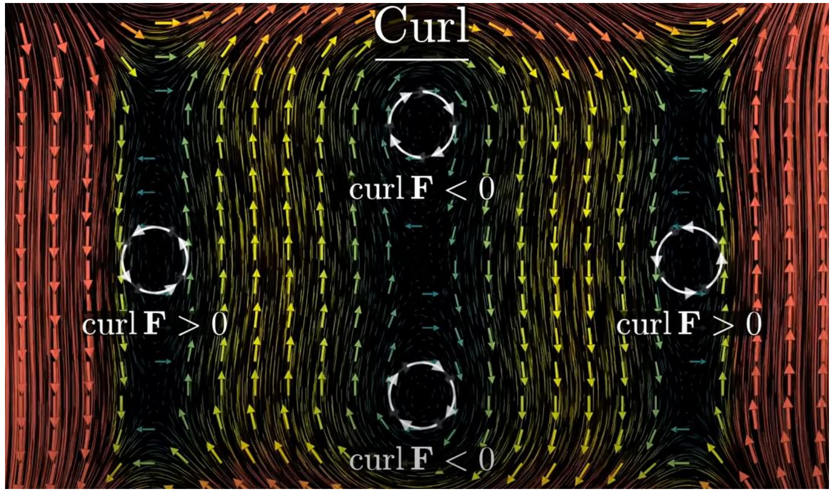


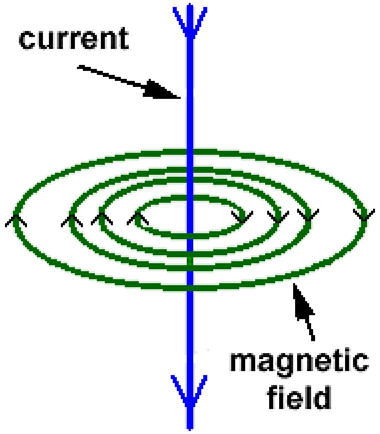
Divergence of magnetic field is zero



Curl: Circulation density

The curl presents the circulation density of vector field at an infinitesimal point



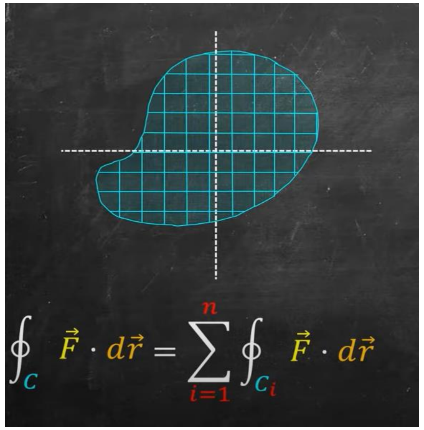


One dimension x

$$(\text{curl}\,\mathbf{A})_x = \lim_{\Delta S \to 0} \frac{\oint_C \mathbf{A} \cdot d\mathbf{l}}{\Delta S}$$

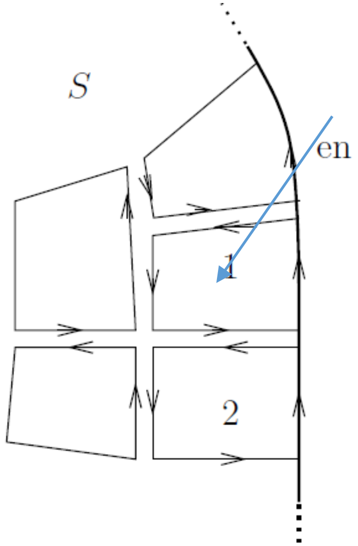
Three dimensions x, y and z

$$\nabla \times A = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

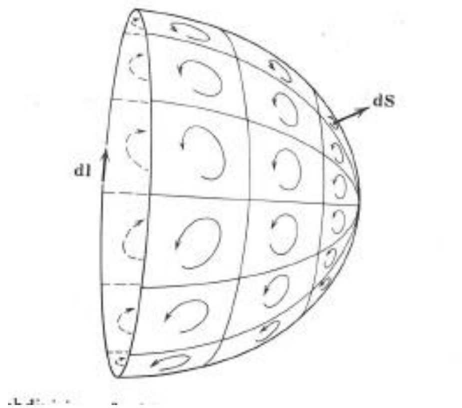


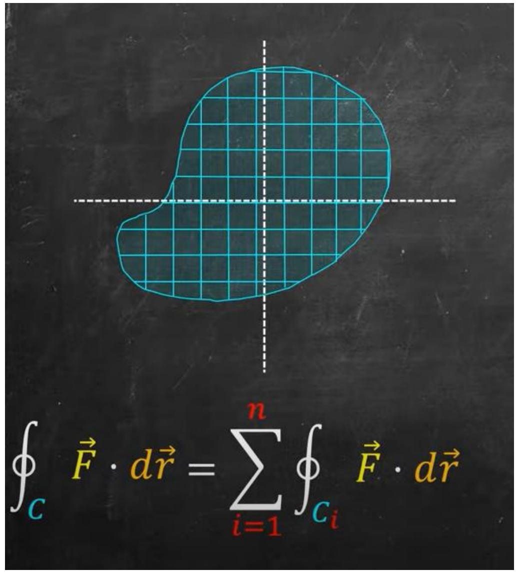
Stokes' Theorem

$$\oint_C \mathbf{A}\cdot d\mathbf{l} = \int_S \nabla\times\mathbf{A}\cdot d\mathbf{S}.$$

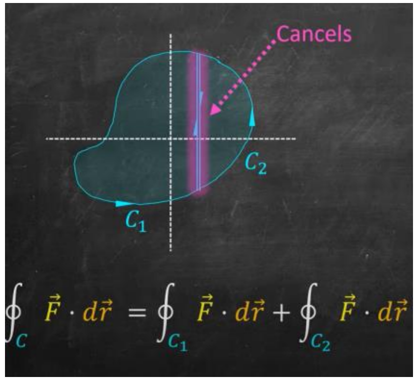


$$\nabla\times\mathbf{A}\cdot d\mathbf{S} = \oint_{C_1} \mathbf{A}\cdot d\mathbf{l}.$$

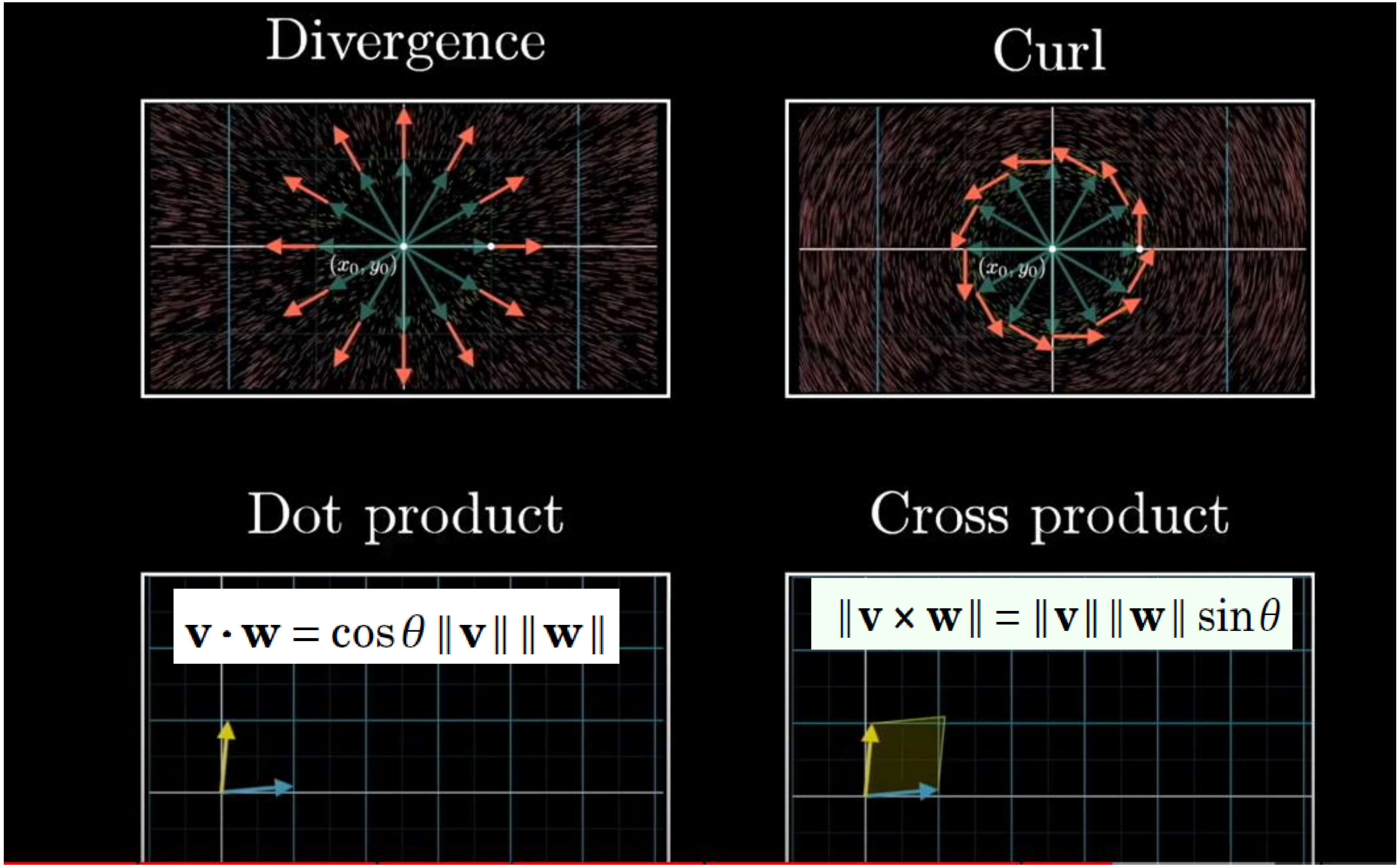




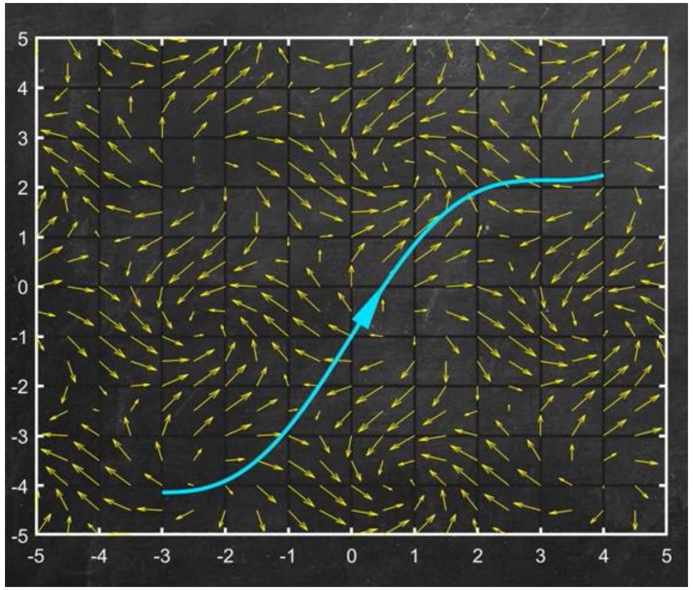
$$\int_S \nabla\times\mathbf{A}\cdot d\mathbf{S} = \sum_i \oint_{C_i} \mathbf{A}\cdot d\mathbf{l}.$$



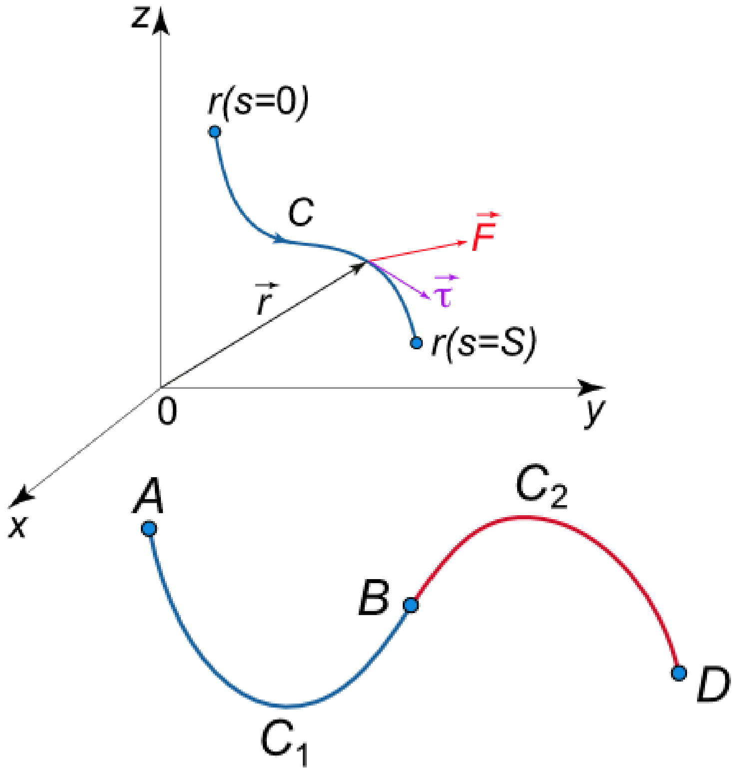
Dot and Cross product



Line integral of vector



Curve direction is important.



$\frac{dr}{dS} = \tau$ ***(Tangent direction at each point of the curve)***

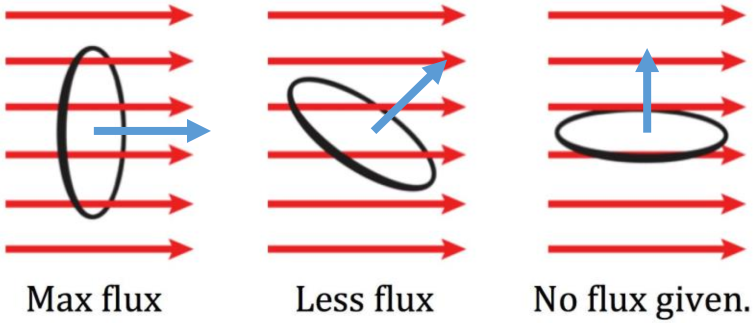
$$\int_C (\mathbf{F} \cdot d\mathbf{r}) = \int_0^S (\mathbf{F}(\mathbf{r}(s)) \cdot \boldsymbol{\tau})\, ds,$$

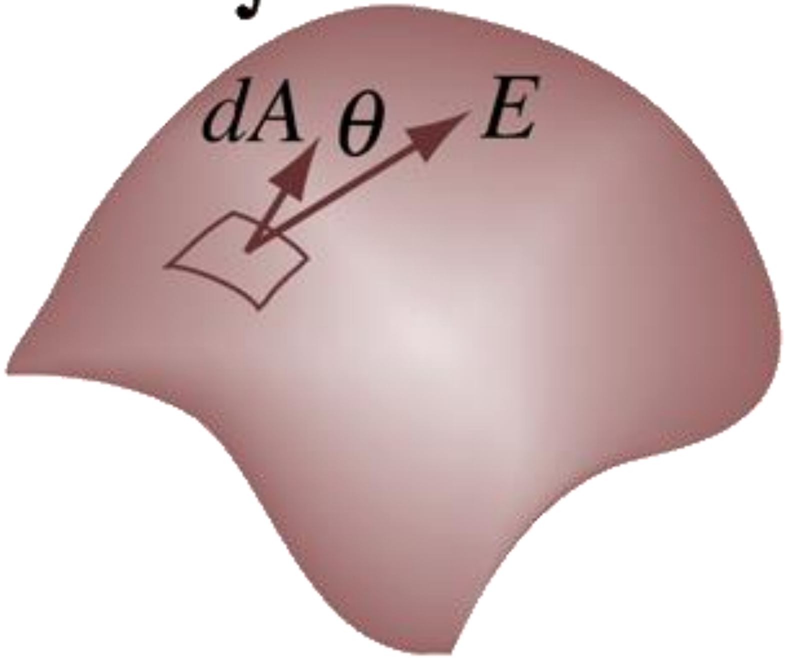
$$\int_C (\mathbf{F} \cdot d\mathbf{r}) = \int_{C_1 \cup C_2} (\mathbf{F} \cdot d\mathbf{r}) = \int_{C_1} (\mathbf{F} \cdot d\mathbf{r}) + \int_{C_2} (\mathbf{F} \cdot d\mathbf{r});$$

Surface integral of vector

dA direction is perpendicular to the tangent plane to that surface at A

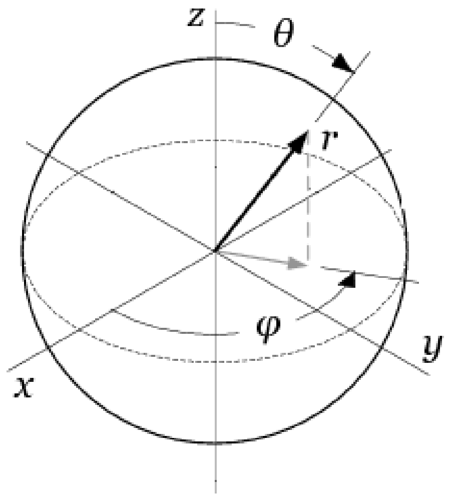
$$\int \vec{E} \cdot d\vec{A} = \int E \cos\theta dA$$



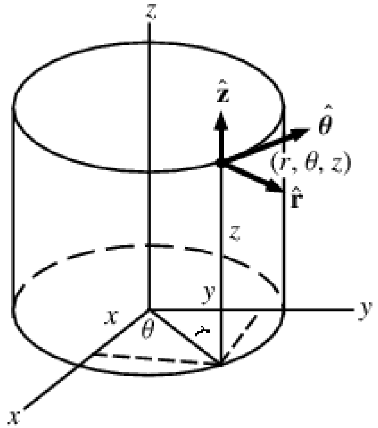


$$\iint_S \vec{F} \bullet d\vec{S} = \iint_S \vec{F} \bullet \vec{n}\, dS$$

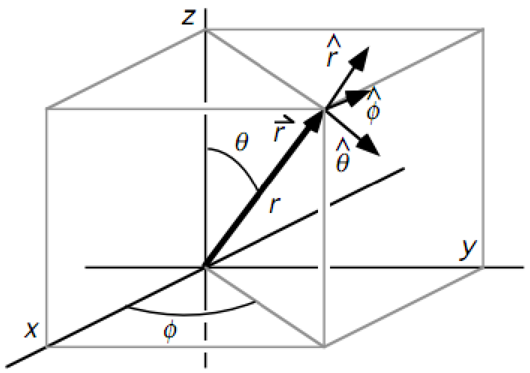
Different coordinates



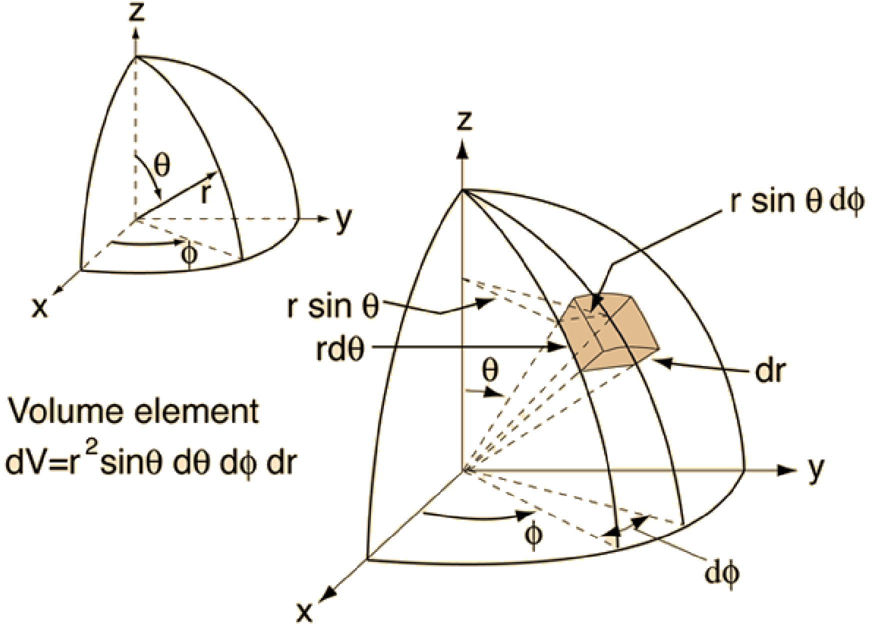
Spherial coordinate

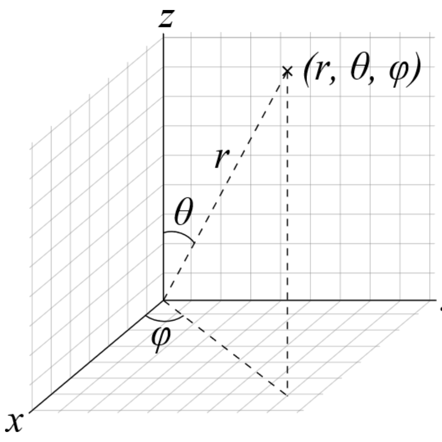


Cylindrical coordinate



Cartesia Coordinate





VECTOR DIFFERENTIAL OPERATIONS

Rectangular Coordinates (x, y, z)

$$\nabla\Phi = \hat{\mathbf{x}}\frac{\partial\Phi}{\partial x} + \hat{\mathbf{y}}\frac{\partial\Phi}{\partial y} + \hat{\mathbf{z}}\frac{\partial\Phi}{\partial z}$$

$$\nabla\cdot\mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla\times\mathbf{H} = \hat{\mathbf{x}}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\mathbf{A} = \hat{\mathbf{x}}\nabla^2 A_x + \hat{\mathbf{y}}\nabla^2 A_y + \hat{\mathbf{z}}\nabla^2 A_z$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla\Phi = \hat{\mathbf{r}}\frac{\partial\Phi}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial\Phi}{\partial\phi} + \hat{\mathbf{z}}\frac{\partial\Phi}{\partial z}$$

$$\nabla\cdot\mathbf{D} = \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla\times\mathbf{H} = \hat{\mathbf{r}}\left[\frac{1}{r}\frac{\partial H_z}{\partial\phi} - \frac{\partial H_\phi}{\partial z}\right] + \hat{\boldsymbol{\phi}}\left[\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right] + \hat{\mathbf{z}}\left[\frac{1}{r}\frac{\partial(rH_\phi)}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial\phi}\right]$$

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\mathbf{A} = \hat{\mathbf{r}}\left(\nabla^2 A_r - \frac{2}{r^2}\frac{\partial A_\phi}{\partial\phi} - \frac{A_r}{r^2}\right) + \hat{\boldsymbol{\phi}}\left(\nabla^2 A_\phi + \frac{2}{r^2}\frac{\partial A_r}{\partial\phi} - \frac{A_\phi}{r^2}\right) + \hat{\mathbf{z}}(\nabla^2 A_z)$$

Spherical Coordinates (r, θ, ϕ)

$$\nabla\Phi = \hat{\mathbf{r}}\frac{\partial\Phi}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \frac{\hat{\boldsymbol{\phi}}}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\nabla\cdot\mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{H} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi\sin\theta) - \frac{\partial H_\theta}{\partial\phi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial r}(rH_\phi)\right] + \frac{\hat{\boldsymbol{\phi}}}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial H_r}{\partial\theta}\right]$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

$$\nabla^2\mathbf{A} = \hat{\mathbf{r}}\left[\nabla^2 A_r - \frac{2}{r^2}\left(A_r + \cot\theta A_\theta + \csc\theta\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_\theta}{\partial\theta}\right)\right] + \hat{\boldsymbol{\theta}}\left[\nabla^2 A_\theta - \frac{1}{r^2}\left(\csc^2\theta A_\theta - 2\frac{\partial A_r}{\partial\theta} + 2\cot\theta\csc\theta\frac{\partial A_\phi}{\partial\phi}\right)\right] + \hat{\boldsymbol{\phi}}\left[\nabla^2 A_\phi - \frac{1}{r^2}\left(\csc^2\theta A_\phi - 2\csc\theta\frac{\partial A_r}{\partial\phi} - 2\cot\theta\csc\theta\frac{\partial A_\theta}{\partial\phi}\right)\right]$$

Examples: Probelm 3

Calculate the integral

$$I = \int_V (\nabla \cdot \mathbf{F}) \mathrm{d}V \tag{1}$$

where $\mathbf{F} = r\hat{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$, and the volume V is a sphere with radius R placed in the origin.

a) Calculate the integral directly.

b) Calculate the integral using the divergence theorem.

$$\mathrm{div}\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \nabla \cdot \mathbf{A}.$$

$$\oint_S \mathbf{A} \cdot \mathrm{d}\mathbf{S} = \int_v \nabla \cdot \mathbf{A} \mathrm{d}v$$

Solutions:

Calculate the integral

$$I = \int_V (\nabla \cdot \mathbf{F}) \mathrm{d}V \tag{1}$$

where $\mathbf{F} = r\hat{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$, and the volume V is a sphere with radius R placed in the origin.

a) Calculate the integral directly.

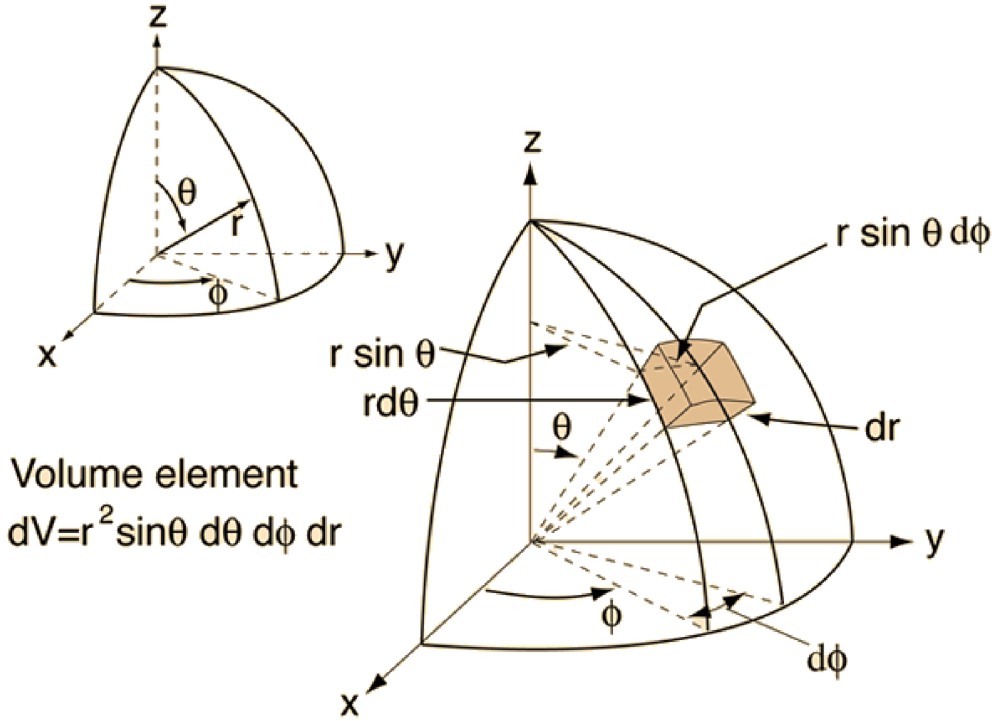
b) Calculate the integral using the divergence theorem.

$$\mathrm{div}\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \nabla \cdot \mathbf{A}.$$

$$\nabla \cdot \mathbf{F} = 3.$$

$$\begin{aligned}\int_v (\nabla \cdot \mathbf{F}) \mathrm{d}V &= \int_0^R \int_0^{2\pi} \int_0^{\pi} 3r^2 \sin\theta \mathrm{d}\varphi \mathrm{d}\theta \mathrm{d}r \\ &= 3 \int_0^R r^2 \mathrm{d}r \int_0^{\pi} \sin\theta \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi \\ &= \underline{\underline{4\pi R^3}}.\end{aligned}$$

$$\underline{\underline{\int_V (\nabla \cdot \mathbf{F}) \mathrm{d}V = 3 \int_V \mathrm{d}V = 4\pi R^3.}}$$



Solution for b

$$
\begin{aligned}
\int_v (\nabla \cdot \mathbf{F}) \mathrm{d}v &= \oint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S} \\
&= \int_0^{2\pi} \int_0^{\pi} (R\hat{\mathbf{r}}) \cdot (R^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi \hat{\mathbf{r}}) \\
&= R^3 \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \sin\theta \mathrm{d}\theta \\
&= \underline{\underline{4\pi R^3}}.
\end{aligned}
$$

$$
\boxed{\oint_S \mathbf{A} \cdot \mathrm{d}\mathbf{S} = \int_v \nabla \cdot \mathbf{A} \mathrm{d}v}
$$

Example:

Calculate the line integral

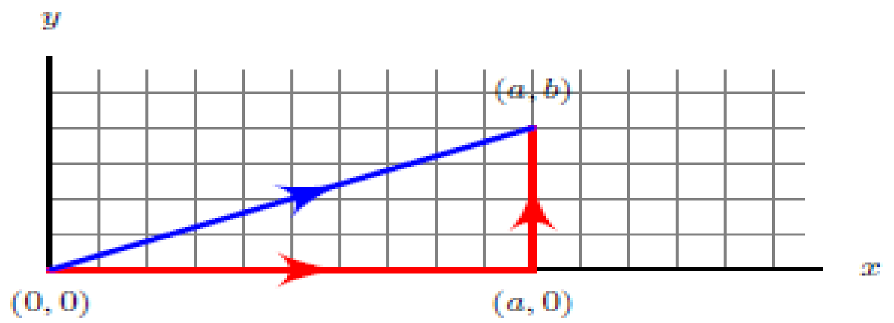
$$I = \int_C \mathbf{F} \cdot d\mathbf{l}, \qquad (2)$$

Where $\mathbf{F} = (xy^2 + 2y)\hat{\mathbf{x}} + (x^2y + 2x)\hat{\mathbf{y}}$,

i) Along the curve C_1 which consists of two straight lines connecting the points $(0,0)$, $(a,0)$ and (a,b), see figure below.

ii) Along the curve C_2 which consists of one straight line connecting the points $(0,0)$ and (a,b), see figure below.

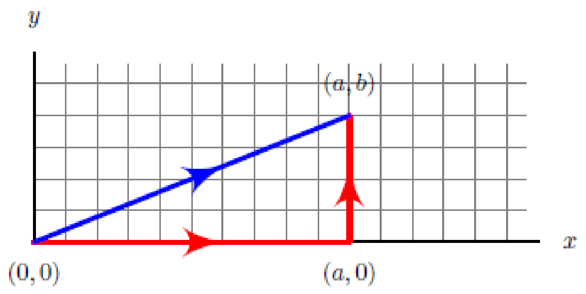
iii) Why do these calculations produce the same answer? Explain using Stoke's theorem.



$$\nabla \times A = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{A} \cdot d\mathbf{S}.$$

Solution for i) and ii)



$$I = \int_V (\nabla \cdot \mathbf{F}) \mathrm{d}V$$

$$\mathbf{F} = (xy^2 + 2y)\hat{\mathbf{x}} + (x^2y + 2x)\hat{\mathbf{y}}$$

i) Along the curve C_1 which consists of two straight lines connecting the points $(0, 0)$, $(a, 0)$ and (a, b), see figure below.

$$\begin{aligned} I &= \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{l} \\ &= \int_0^b (a^2y + 2a)\mathrm{d}y \\ &= \underline{\underline{\frac{1}{2}a^2b^2 + 2ab.}} \end{aligned}$$

ii) Along the curve C_2 which consists of one straight line connecting the points $(0, 0)$ and (a, b), see figure below.

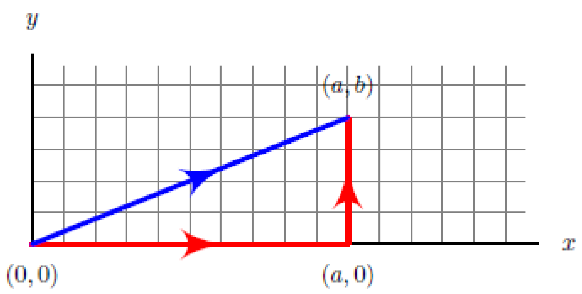
$$\mathrm{d}\mathbf{l} = \mathrm{d}x\hat{\mathbf{x}} + \mathrm{d}y\hat{\mathbf{y}}.$$

$$\begin{aligned} \mathbf{F} \cdot \mathrm{d}\mathbf{l} &= F_x \mathrm{d}x + F_y \mathrm{d}y \\ &= (xy^2 + 2y)\mathrm{d}x + (x^2y + 2x)\mathrm{d}y. \end{aligned}$$

$$y = \frac{bx}{a}$$

$$\begin{aligned} I &= \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{l} \\ &= \int_0^a \left[x \left(\frac{bx}{a}\right)^2 + 2\left(\frac{bx}{a}\right) \right] \mathrm{d}x + \int_0^b \left[\left(\frac{ay}{b}\right)^2 y + 2\left(\frac{ay}{b}\right) \right] \mathrm{d}y \\ &= \underline{\underline{\frac{1}{2}a^2b^2 + 2ab.}} \end{aligned}$$

Conservative vector: solution for iii)



iii) Why do these calculations produce the same answer? Explain using Stoke's theorem.

$$\mathbf{F} = (xy^2 + 2y)\hat{\mathbf{x}} + (x^2y + 2x)\hat{\mathbf{y}}$$

$$\nabla \times A = (\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\hat{x} + (\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{y} + (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\hat{z}$$

These integrals have equal values since **F** is a conservative field:

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{z}} = (2xy - 2xy)\,\hat{\mathbf{z}} = 0.$$

$$\oint_C \mathbf{A} \cdot \mathrm{d}\mathbf{l} = \int_S \nabla \times \mathbf{A} \cdot \mathrm{d}\mathbf{S}.$$