NTNU

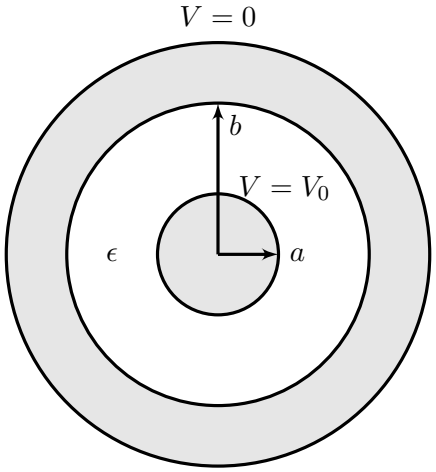
Dpt. of Electronics and Telecommunications,
IME faculty,
NTNU

TFE4120 Electromagnetics - Crash course

Exercise 3

Problem 1

A coaxial cable consists of an inner conductor with radius a and an outer conductor with inner radius b (see figure below). Between the conductors we have a dielectric material with a dielectric constant $\epsilon = \epsilon_r \epsilon_0$. The inner conductor has the potential V_0 while the outer conductor is grounded (potential equal to zero).



Cross section of a coaxial cable

a) Find the **E**-field between the conductors and the per length capacitance C' using

i) Gauss' law in integral form,

ii) Gauss' law in differential form,

iii) Poisson's (Laplace') equation.

Hint: Express ii) $\nabla \cdot \mathbf{D}$ and iii) $\nabla^2 V$ in cylindrical coordinates.

b) Find the numerical answer for C' when $\epsilon_r = 3$ and $\frac{b}{a} = 7$.
(Answer: $C' = 85.8\text{pF/m}$)

c) Find the stored electrostatic energy per unit length of the cable by using

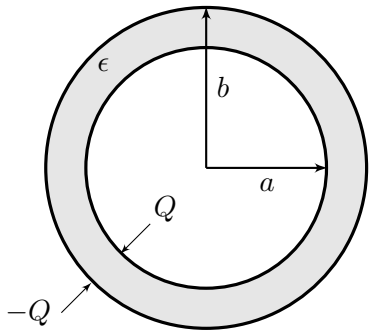
i) the expression for stored energy in a capacitor,

ii) the expression for energy density in an electric field.

d) What is the magnitude of the net force acting on the outer conductor from the inner conductor?

Problem 2

A spherical capacitor consists of two electrically conductive, concentric spherical caps with radius a and b respectively. The volume between the two conductors (gray area in the figure) is filled with a dielectric material with a permittivity of ϵ.



Cross section of a spherical capacitor

a) What is the capacitance of such a capacitor?

Hint: Find the potential difference between the conductors when we have charges $+Q$ on the inner conductor and $-Q$ on the outer conductor.

Check your answer by showing that the capacitance is equal to that of parallel plate capacitor when the thickness of the layer between the conductors become very small. $d = b - a \ll a$.

b) What is the capacitance C for a single conductive sphere, if we consider the spherical capacitor in **a)** when $b \to \infty$ (and $\epsilon \to \epsilon_0$ when the sphere is surrounded by vacuum/air)?