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TFE4120 Electromagnetics - Crash course

Exercise 1

In this exercise we will recapitulate some mathematical tools from vector-analysis, which is often found in electromagnetics. Notation: Vectors will be written in bold, e.g $\mathbf{F}$, and unity vectors with circumflex (hat), e.g $\hat{\mathbf{x}}$.

Problem 1

In physics and mathematics there are several types of line integrals. Some of these are

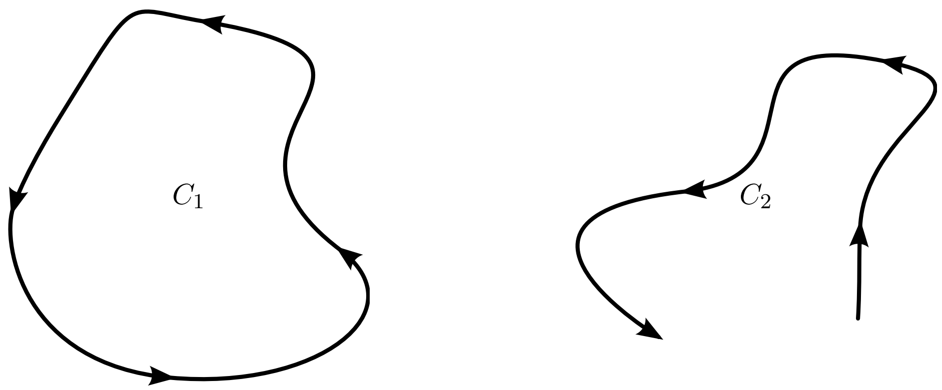
$$1)\ \int_C F\mathrm{d}l, \qquad 2)\ \int_C F\mathrm{d}\mathbf{l}, \qquad 3)\ \int_C \mathbf{F}\mathrm{d}l, \qquad 4)\ \int_C \mathbf{F}\cdot\mathrm{d}\mathbf{l}$$

a) Which of the integrals fit to the following physical situations:

i) When the mass density of a wire is F, and you want to find the total mass of the wire

ii) Given a force $\mathbf{F}$ which acts on a body moving along a curve C, and you want to find the total work done by the force.

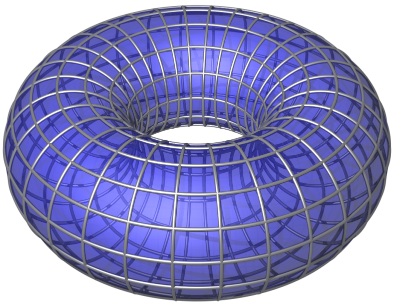
b) Let $\mathbf{F}$ and F be constants, unequal to 0, and use the integration curves C_1 and C_2 from the figure below. In which of the cases 1)-4), above, is the integral along these curves equal to 0? Sketch $\mathrm{d}\mathbf{l}$ for different positions on each curve.



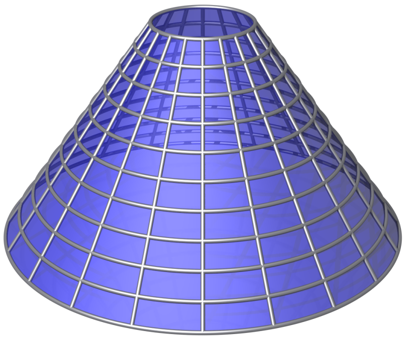
For surface integrals the corresponding integrals are

$$1)\ \int_S F \mathrm{d}S, \qquad 2)\ \int_S F \mathrm{d}\mathbf{S}, \qquad 3)\ \int_S \mathbf{F} \mathrm{d}S, \qquad 4)\ \int_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$$

c) Repeat task **b)** for the surface integrals. Use the surfaces below.



Torus



Open cone (both ends)

d) How is the direction of $\mathrm{d}\mathbf{S}$ for the torus defined? Sketch $\mathrm{d}\mathbf{S}$ at three different points.

e) Consider the surface S_1 defined by the curve C_1. In what direction does $\mathrm{d}\mathbf{S}$ for S_1 point?

Problem 2

a) Define and explain the following terms using your own words.

- i) Flux
- ii) Divergence
- iii) Curl
- iv) Conservative field.

b) Write down and briefly explain the following theorems using your own words.

- i) Divergence theorem
- ii) Stoke's theorem

Optional tasks: The following tasks are optional, but offers good practice. It is recommended to do them.

Problem 3

Calculate the integral

$$I = \int_V (\nabla \cdot \mathbf{F}) \mathrm{d}V \tag{1}$$

where $\mathbf{F} = r\hat{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$, and the volume V is a sphere with radius R placed in the origin.

a) Calculate the integral directly.

b) Calculate the integral using the divergence theorem.

Problem 4

a) Calculate the line integral

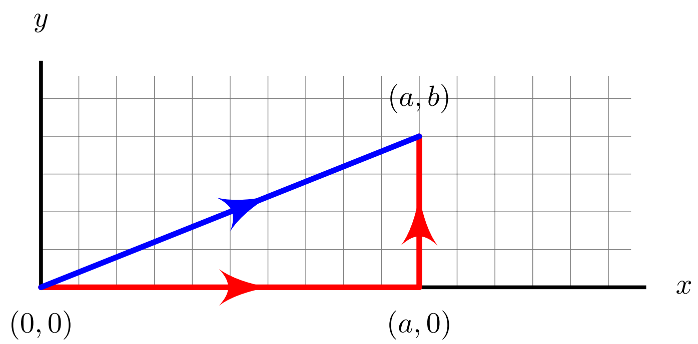
$$I = \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{l}, \tag{2}$$

Where $\mathbf{F} = (xy^2 + 2y)\hat{\mathbf{x}} + (x^2y + 2x)\hat{\mathbf{y}}$,

i) Along the curve C_1 which consists of two straight lines connecting the points $(0, 0)$, $(a, 0)$ and (a, b), see figure below.

ii) Along the curve C_2 which consists of one straight line connecting the points $(0, 0)$ and (a, b), see figure below.

iii) Why do these calculations produce the same answer? Explain using Stoke's theorem.



b) For the following scenarios, sketch the curve C and calculate the integral of the function along that curve (i.e calculate the integral $\int_C f \mathrm{d}l$ for i) and ii), and $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{l}$ for iii))

i) $f = x + y$, C is given by: $x = 1 + e^{2t}$, $y = e^{2t}$, $t \in [0, \ln 2]$.

ii) $f = \frac{2y}{x}\sqrt{1 + x^2}$, C is given by: $y = \frac{1}{2}x^2$, $x \in [0, 2]$.

iii) $\mathbf{F} = \hat{\mathbf{x}} \cos^2 t + \hat{\mathbf{y}} 2 \sin t$, along the line $\mathbf{l}(t) = \hat{\mathbf{x}} \tan t - \hat{\mathbf{y}} \cos t$, $t \in [-\pi/3, \pi/3]$.