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# Open-Quantum-Systems: Precision Thermometry at the Extremes

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**Some selected references**:

A.R. "Heavy Quarkonium in Extreme Conditions" Phys.Rept. **858** (2020)
T. Miura, Y. Akamatsu, M. Asakawa, A.R. PRD**101** (2020) 034011
M. Mehboudi et.al. PRL**122** (2019) 030403
Olf et.al. Nature Physics **11** (2015) 720

NPACT
Norwegian Particle, Astroparticle & Cosmology Theory network

PHYSICS-SEMINAR – NTNU – OCTOBER 28TH 2021 – TRONDHEIM, NORWAY

# Temperatures at the extremes

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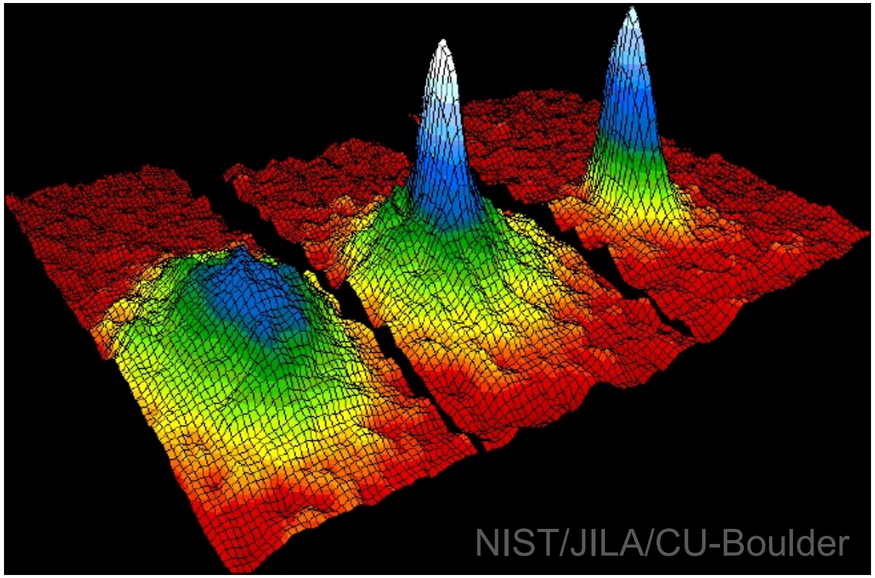
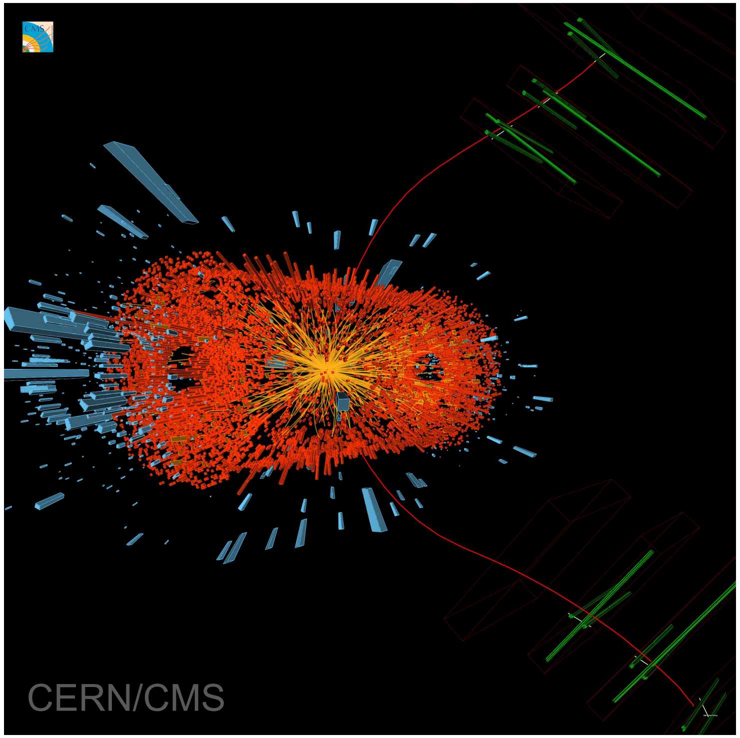



Bose Einstein Condensate (BEC) from trapped ultracold atoms




Quark-Gluon Plasma (QGP) from relativistic heavy-ion collisions

Outer Space — Sun's Corona

$10^{-9}$K — 2.7K — $5\text{x}10^{6}$K — $10^{12}$K

# What is Temperature



For a system with large enough number of degrees of freedom

$$\rho(T) = \sum_k \frac{1}{Z} e^{-E_k/k_B T} |E_k\rangle\langle E_k|$$

density matrix

probability for a state with $E_k$

stationary state of energy $E_k$

T is a **parameter** governing the **statistical distribution** of **energy**

# Thermometry – Measuring Temperature

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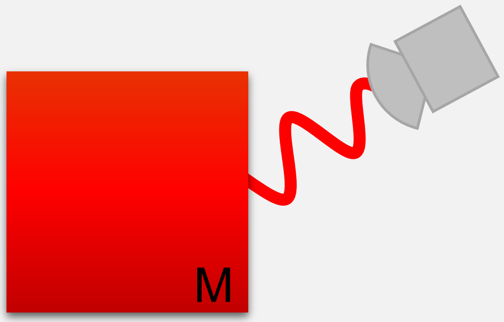
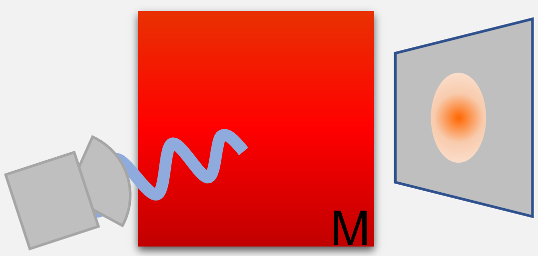
## Probeless

**measure statistics** of E or other T dep. observable

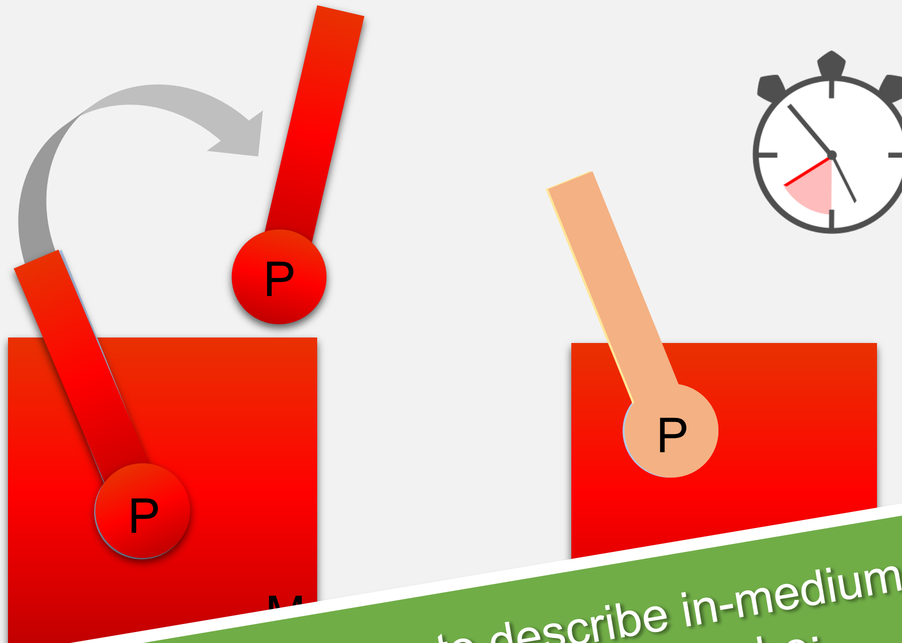



passive imaging




active imaging

## Probe based




pr

uilibrium

Dynamic

Efficient formalism to describe in-medium quantum properties of the probe: **Open-Quantum Systems**

**measure** the **(non-)equilibrium properties** of the small probe system

# Open quantum systems

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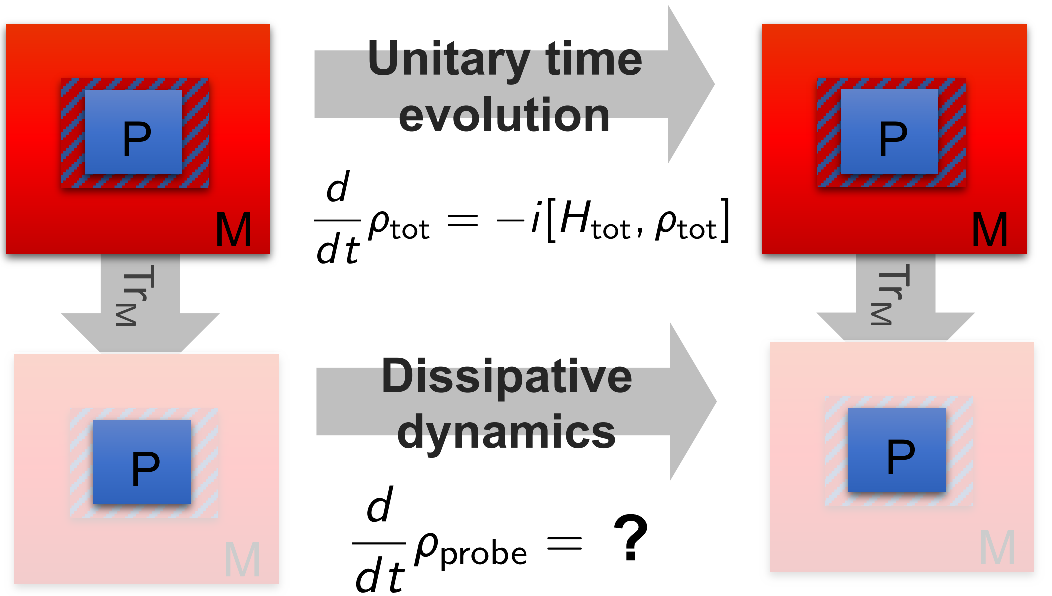
Need general approach to describe (small) probe coupled to a thermal medium

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^{\dagger} \qquad \rho_{\rm tot} = \sum_k \frac{1}{Z_{\rm tot}} e^{-E_k/k_B T} |E_k\rangle\langle E_k|$$




$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

**reduced density matrix**

see e.g. Breuer, Pettrucione
The theory of Open Quantum Systems

In general, e.o.m. is a dissipative **master equation** with memory of past:

**BEYOND THE STANDARD SCHRÖDINGER EQUATION**

**Separation of time-scales** determines the nature of the e.o.m. :

Medium relaxation scale $\tau_{\rm M}$ : intrinsic probe scale $\tau_{\rm P}$ : probe relaxation scale $\tau_{\rm rel}$ :

$$\langle \hat{O}_{\rm M}(t)\hat{O}_{\rm M}(0)\rangle \sim e^{-t/\tau_{\rm M}} \qquad \tau_{\rm P} \sim 1/|\omega_i - \omega_j| \qquad \langle p(t)\rangle \propto e^{-t/\tau_{\rm rel}}$$

# The coldest matter in the universe

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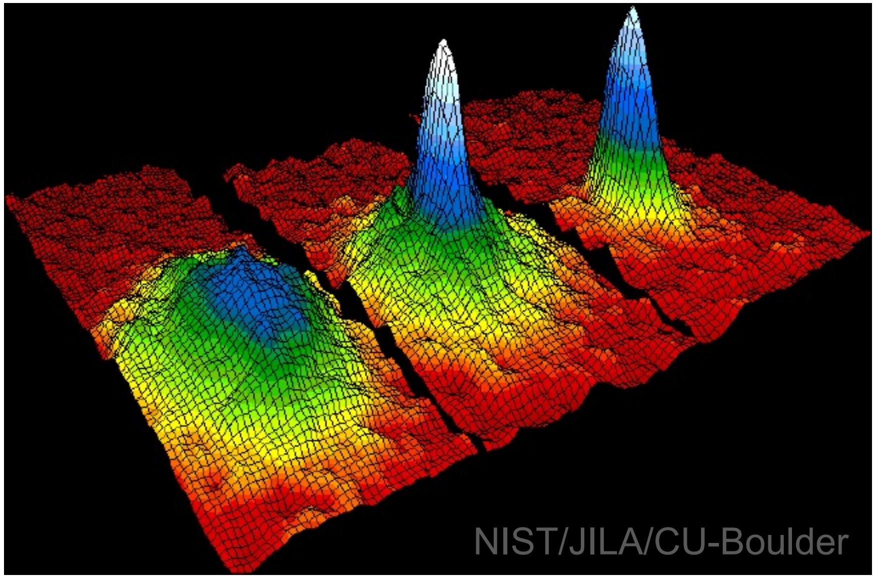


Bose Einstein Condensate (BEC)
from trapped ultracold atoms

# Bose-Einstein Condensation

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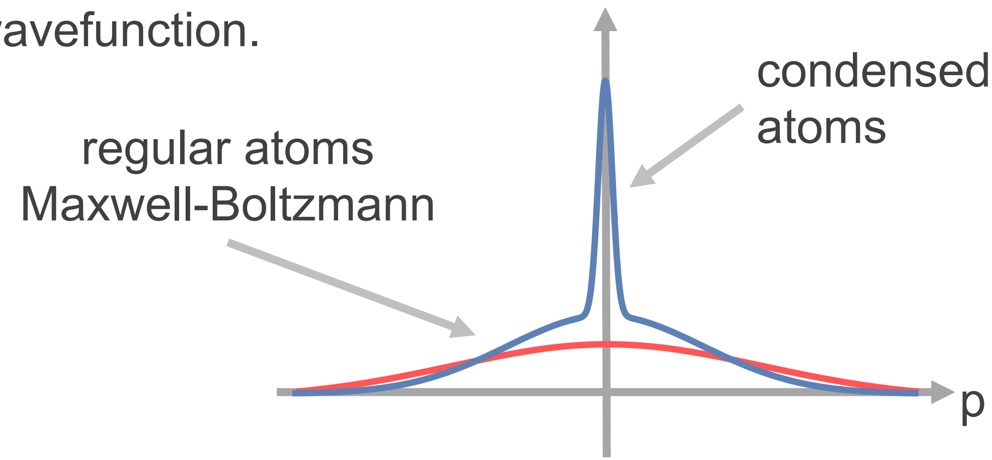
In 1924 Einstein & Bose predicted a **new phase** of bosonic matter at low T: **BEC**

A. Einstein König. Preuß. Akad. Wiss. (1924) 261
S. N. Bose *Zeitschrift für Physik*. 26 (1924) 178

Below a **critical temperature**, significant fraction of bosons **condense** into ground state with **macroscopic** wavefunction.

**boson gas momentum profile**




**Laser light** shone on monoatomic gas induces viscous force, which reduces gas kinetic energy: **T ~ 100nK**

D. J. Wineland et.al. PRL40 (1978) 1639

By changing the trapping potential the Maxwell tail of the gas can be **evaporated** off: **T < nK**

see e.g. Anderson et.al. Science 269 (1995) 198

# A common way to measure T

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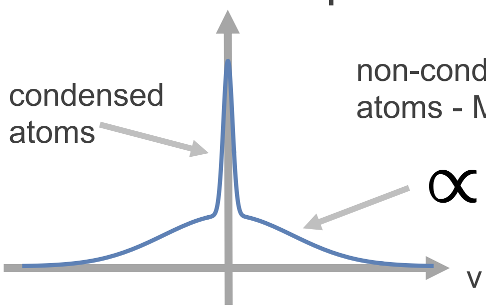
Goal: measure statistical distribution of v at low T via active imaging: **Time-Of-Flight method**

see e.g. P. Lett, et.al. PRL61 (1988) 169

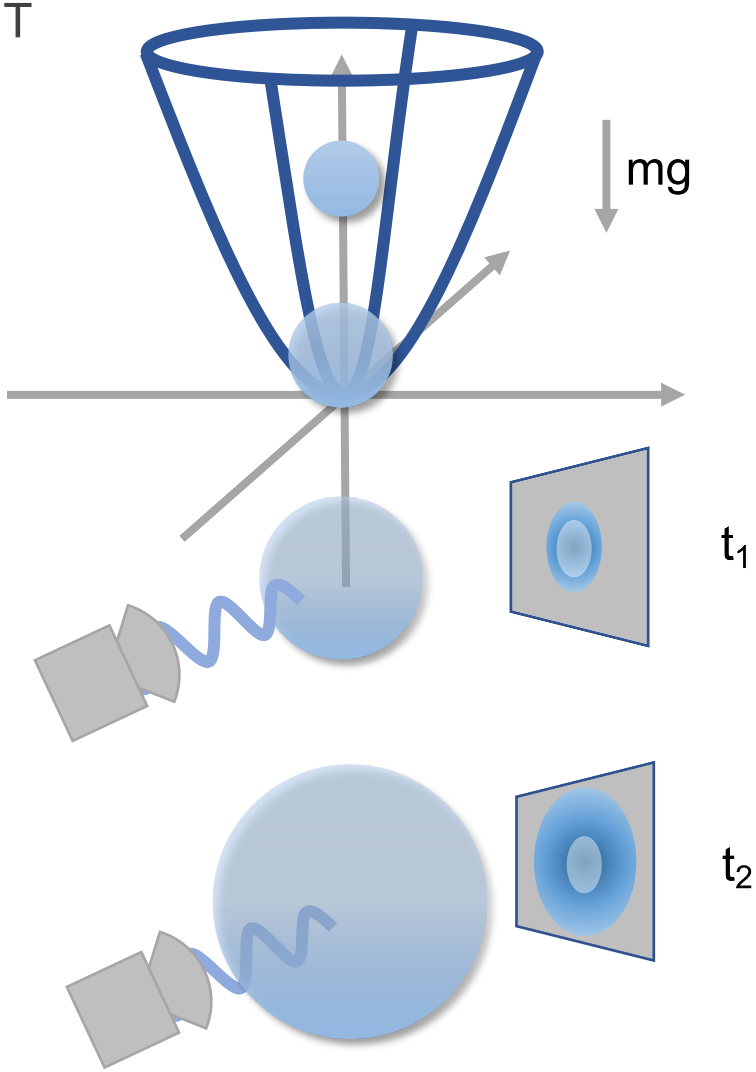
**Velocity mixture** in Maxwell-Boltzmann leads to a T-dep. expansion of free-falling gas cloud

**BEC momentum profile**




$$\propto e^{-\frac{m}{k_B T}(v_x^2+v_y^2+v_z^2)}$$

$$P_{\rm TOF}(t) \propto exp\left[-\frac{g(t_0-t)^2}{2\frac{k_B T}{m}}\right]$$




System of interest is **destroyed** during the process!

Only non-condensed atoms relevant, signal may be **shadowed** by condensated peak

# Probes as precision enhancer

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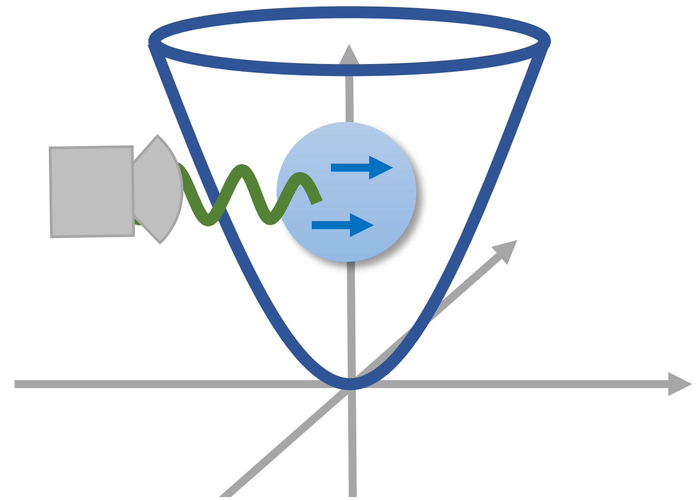
"**Too cold to touch**" – impurity probes must be embedded in the ensemble (thermal equilibrium)

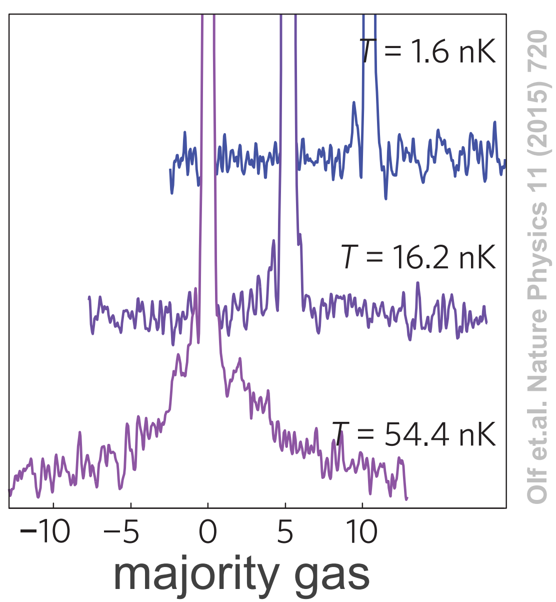


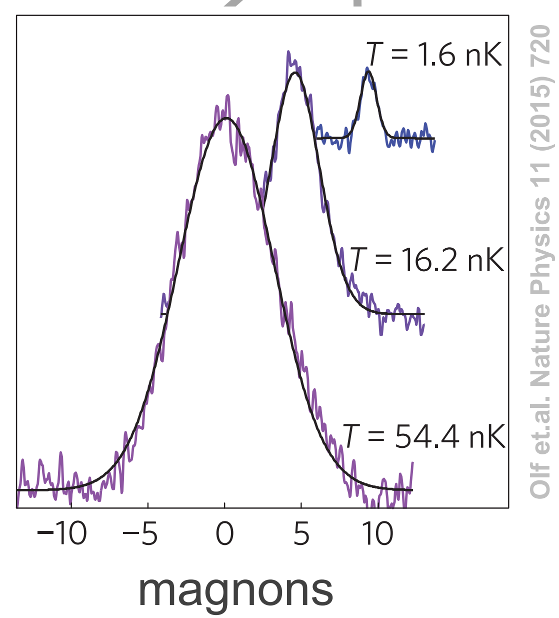
**majority gas |F=1, $m_F$=-1>**, minority of states via RF pulse into **|F=1, $m_F$=0>** quasiparticle impurities **spinwaves**

Quasiparticle magnons quickly thermalize but kept dilute enough not to condensate

**TOF**

**$m_F$≠0 particles weeded out by microwave**



# Non-destructive precision thermometry

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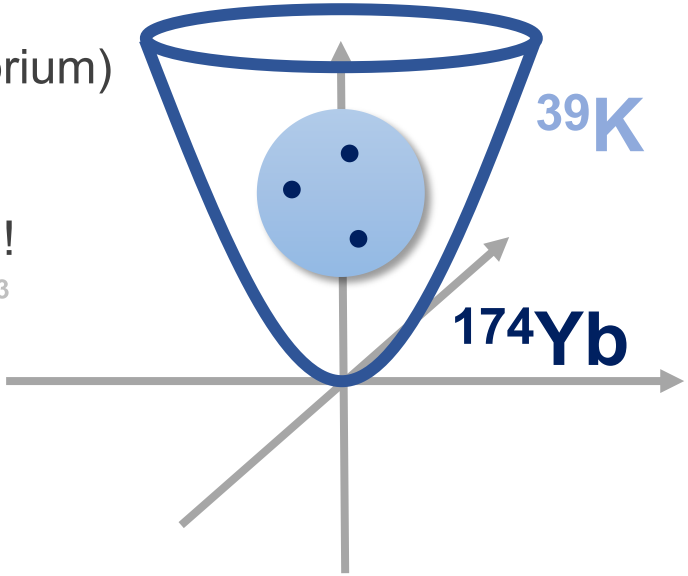
"**Too cold to touch**" – impurities of different atom species (polarons) embedded in the ensemble (equilibrium)




Laser cool **majority gas $^{39}$K** together with **minority gas $^{174}$Yb**. (Quantum) Brownian motion!

see e.g. M. Mehboudi et.al. PRL122 (2019) 030403

What properties of the probe allow us to obtain most precise measurement (sensitivity)?

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

error in T measurement — spread in impurity property

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2\rangle - \langle \hat{O}\rangle^2}{\sqrt{N\chi_T^2[\hat{O}]}} \quad \text{classically random var.} \sim \frac{1}{\sqrt{N}} \quad \text{quantum Cramer-Rao bound} \geq \frac{1}{\sqrt{N\mathcal{F}[T]}}$$

# of measurements — how sensitive is O to T — Fisher information

Optimum sensitivity $\chi_T[\hat{O}] = \partial_\xi \mathrm{Tr}[\rho_{\rm probe}(\xi)\hat{O}]_{\xi=T}$ reached via unique quantity $\hat{\Lambda}_T$ :

$$\hat{\Lambda}_T\hat{\rho}_{\rm probe} + \hat{\rho}_{\rm probe}\hat{\Lambda}_T = 2\partial_T\hat{\rho}_{\rm probe} \quad \Rightarrow \quad \hat{\Lambda}_T^{\rm QBM} = C_x(\hat{x}^2 - \langle\hat{x}^2\rangle) + C_p(\hat{p}^2 - \langle\hat{p}^2\rangle)$$

"symmetric logarithmic derivative" — optimal observable from $\rho_{\rm probe}$

# Quantum Brownian Motion

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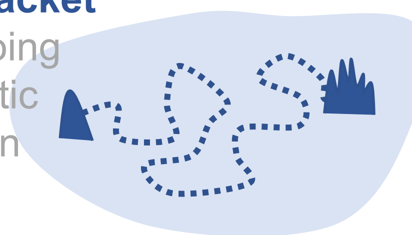
Iconic model of an Open Quantum System: **heavy point-like impurity** in a medium of harmonic oscillators

see e.g. Breuer, Pettrucione The theory of Open Quantum Systems

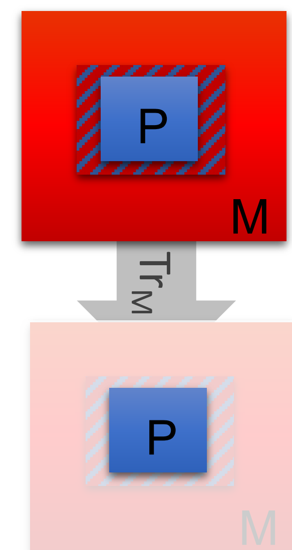
**wave packet** undergoing stochastic evolution



$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int}$$

$g_k$: all details M-P interaction

$$\frac{\hat{p}^2}{2m_p} + \sum_k E_k \hat{b}_k^\dagger \hat{b}_k + \sum_k \hbar g_k \hat{x}(\hat{b}_k + \hat{b}_k^\dagger)$$




how to derive an evolution equation for the probe particle?

$$\frac{d}{dt}\rho_{\rm probe}(t) = -i{\rm Tr}_{\rm M}\Big[[H_{\rm tot}, \rho_{\rm probe} \otimes \rho_{\rm M}]\Big]$$

Born approximation

$$\tau_{\rm rel} > \tau_{\rm M}$$

Markovian approximation

$$\tau_{\rm P} > \tau_{\rm M}$$

Brownian motion limit

yields the **Caldeira-Leggett** master equation "damping rate" $\gamma(g_k)$ explicitly computable

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}] - \frac{\gamma}{8m_p k_B T}[\hat{p}, [\hat{p}, \hat{\rho}_{\rm probe}]]$$

coherent dynamics — fluctuations — dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587 & Annals of Phys. 149 (1983) 374

# The Lindblad equation



A **general mathematical structure** is hiding in the Quantum-Brownian motion e.o.m.

Summarize medium effects on probe by **one operator** and **one damping rate**

$$\hat{\mathcal{L}} = \sqrt{\frac{4 m_p k_B T}{\hbar^2}}\hat{x} + i\sqrt{\frac{1}{4 m_p k_B T}}\hat{p}$$

**Lindblad operator** for QBM

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] + \sum_k \gamma_k \Big( \hat{\mathcal{L}}_k \hat{\rho}_{\rm probe} \hat{\mathcal{L}}_k^\dagger - \frac{1}{2}[\hat{\mathcal{L}}_k^\dagger \hat{\mathcal{L}}_k, \hat{\rho}_{\rm probe}] \Big)$$

**Linblad equation** for **Markovian** open quantum systems

G. Lindblad Commun. Math. Phys. 48 (1976) 119,
V. Gorini, et.al. J. Math. Phys. 17 (1976) 821

prover

$\langle r$

sanity

$\frac{d}{dt}\langle\hat{x}\rangle$

$\frac{d}{dt}\langle\hat{p}\rangle = -2\gamma\langle p\rangle \quad \langle p(t)\rangle = e \quad \langle p(0)\rangle$

$\infty \Rightarrow \frac{k_B T}{m_p \gamma} t$

r
to classical Brownian motion

All ingredients are ready to compute predictions for precision low T thermometry using Quantum Brownian probe particles

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2\rangle - \langle \hat{O}\rangle^2}{\sqrt{N\chi_T^2[\hat{O}]}} \qquad \hat{\Lambda}_T^{\rm QBM} = C_x(\hat{x}^2 - \langle\hat{x}^2\rangle) + C_p(\hat{p}^2 - \langle\hat{p}^2\rangle)$$

caveat: neglected possible memory effects etc. - simplest case study

# The hottest matter in the universe

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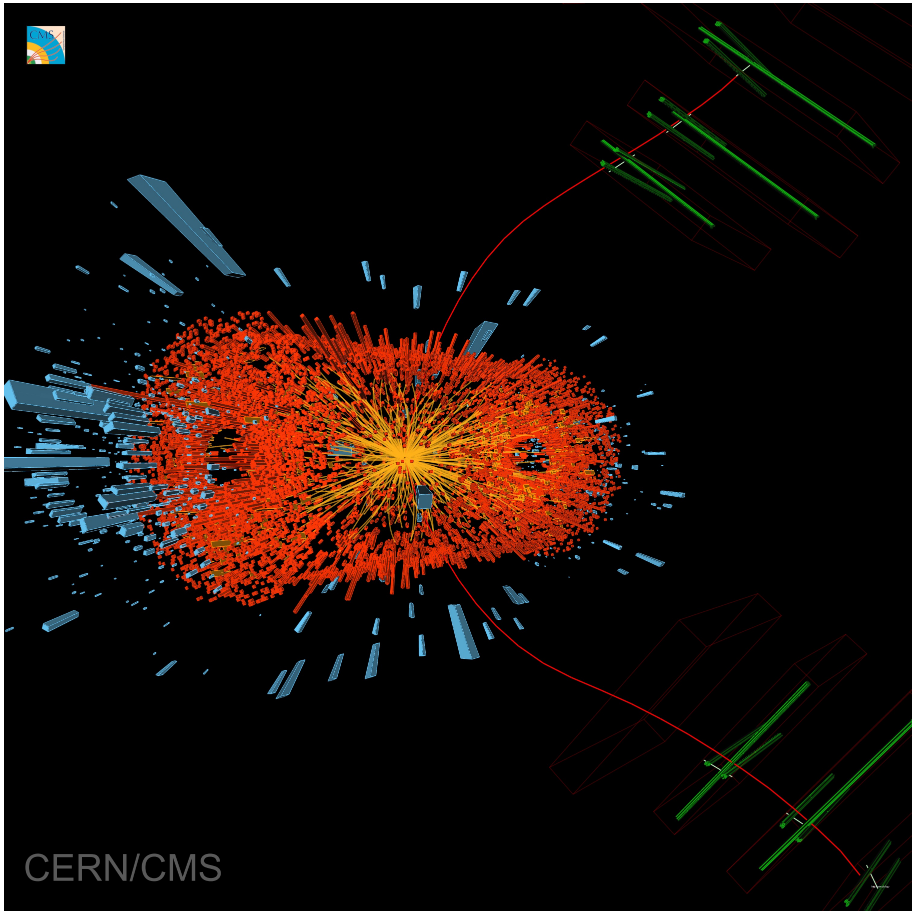


Quark-Gluon Plasma (QGP) from relativistic heavy-ion collisions

# Quark-Gluon Plasma

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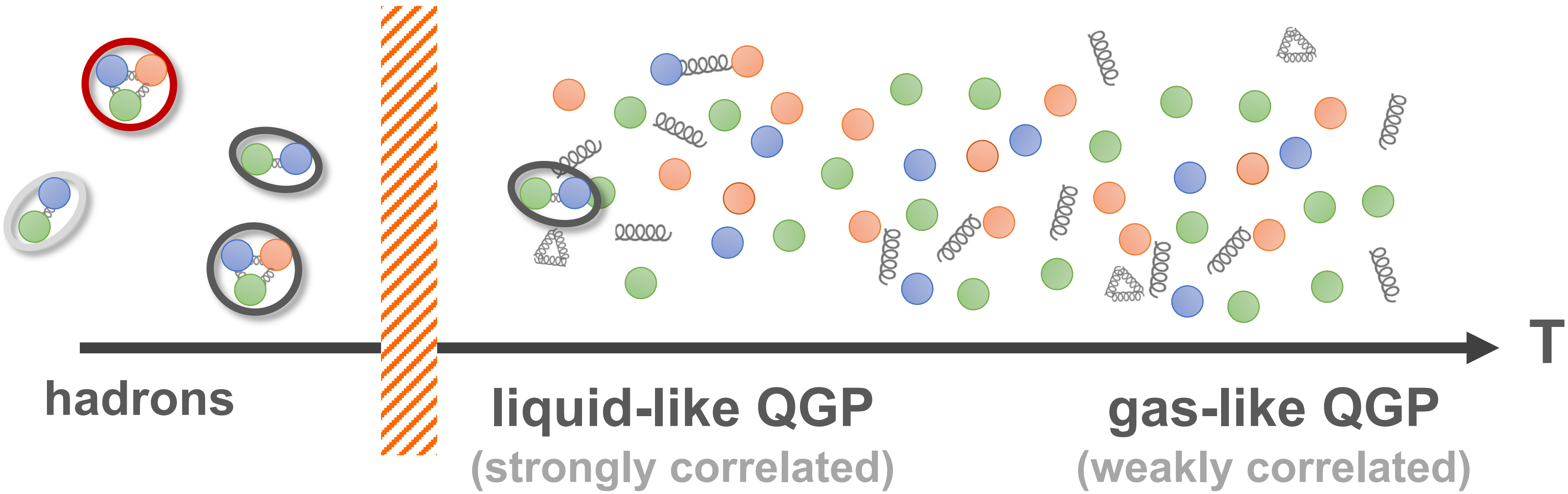
In 1965 Hagedorn hinted at existence of a **new phase** of nuclear matter at high T: **QGP**

R. Hagedorn Nuovo Cimento, Suppl. 3, no. CERN-TH-520 (1965): 147

nuclear matter at T=276K: strongly-interacting quarks and gluons **confined** into hadrons

for a review see Hatsuda and Fukushima Rept.Prog.Phys. 74 (2011) 014001




**crossover**

$T_C$ **= 155MeV – 1.8x10$^{12}$K**

due to
asymptotic freedom

QGP filled the universe shortly after the **Big Bang** – possibly exists inside **neutron stars**

Hatsuda et. al. "QGP: From Big Bang to little Bangs" Cambridge

Annala et.al. Nature Physics 16 (2020) 907

Theory basis: **Quantum ChromoDynamics (QCD)**, the field theory of quarks & gluons endowed with a triple valued **charge called color** (r,g,b)

# Relativistic heavy-ion collisions

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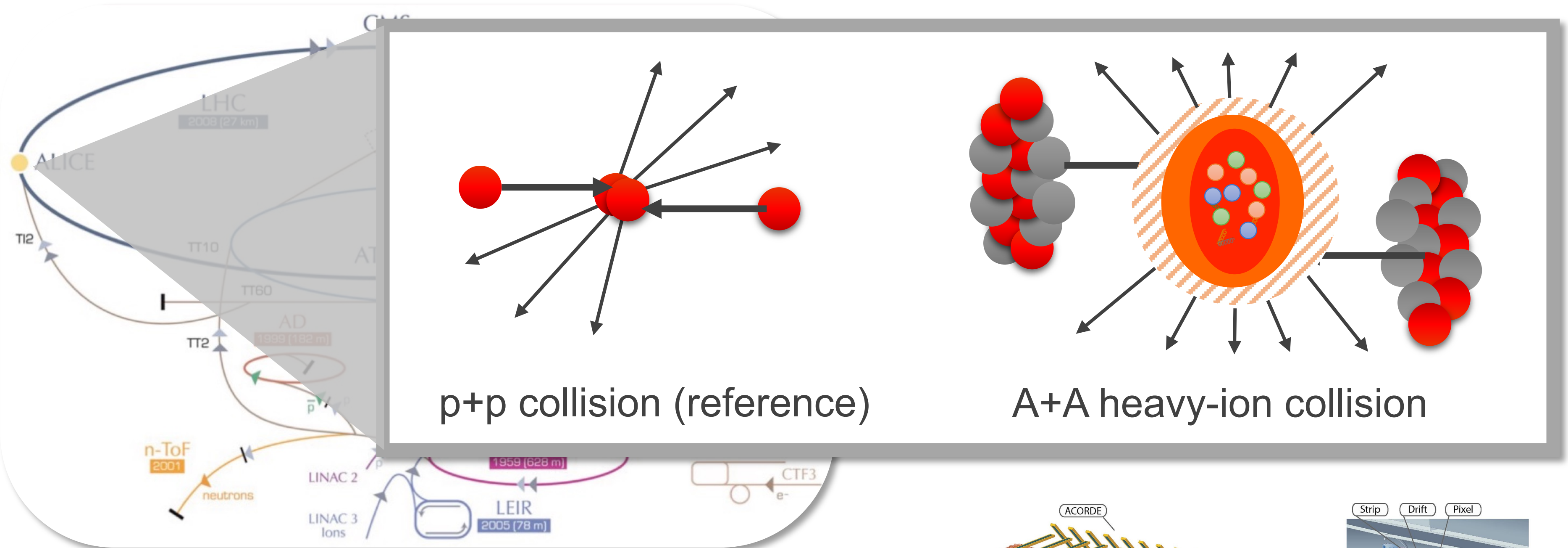
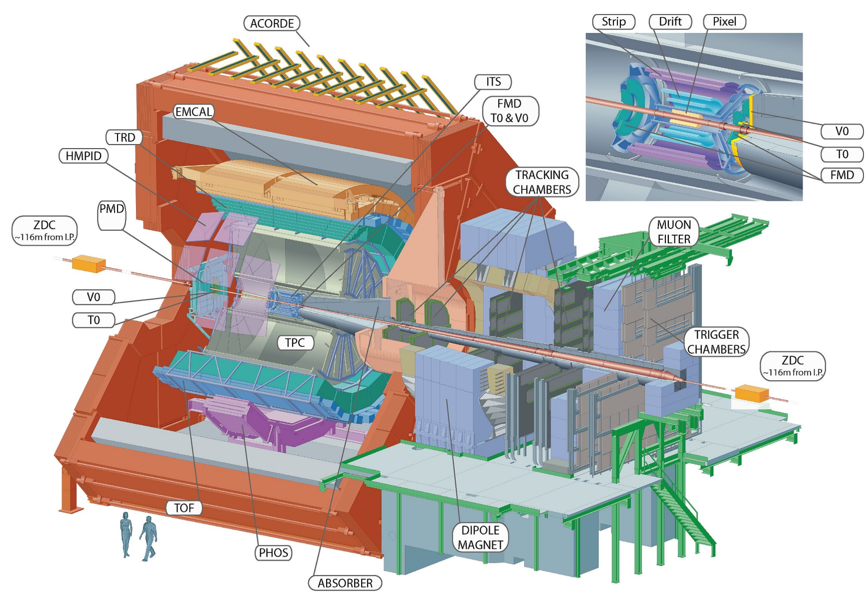



- Vast amount of kinetic energy transformed into new quarks and gluons
- In RHICs: high enough number density of quarks to form a locally thermal QGP
- Compare p+p result vs. A+A results to infer presence and properties of medium



# Radiation Thermometry

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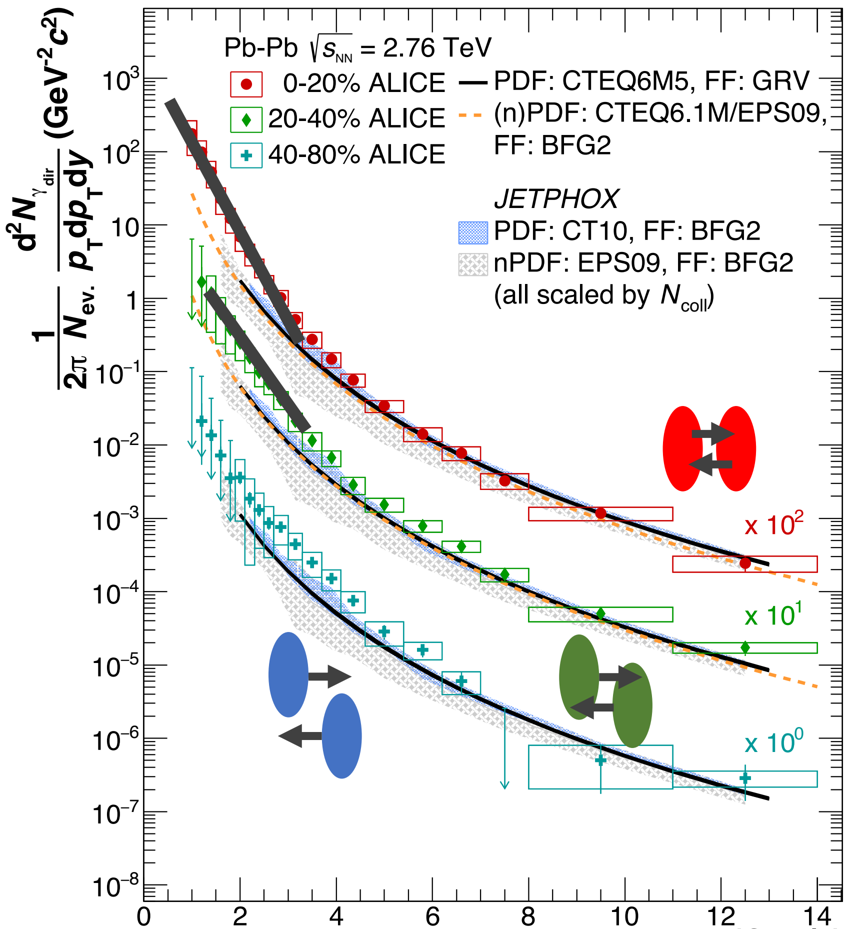
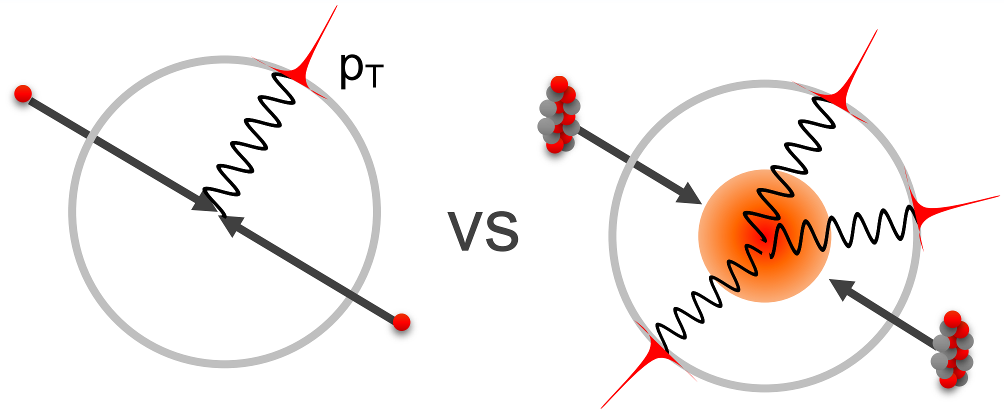
Photon spectrum emitted by a RHIC




[ALICE collaboration] Phys.Lett.B 754 (2016) 235




passive imaging

see e.g. K. Reygers CERN Courier 55 (2015) 22

- **At high $p_T$**: photons from highly energetic quark collisions before formation of QGP. similar to multiple p+p collisions added.
- **At low $p_T$**, expect photon emission from the QGP to dominate: **excess** over rescaled p+p
- Fitting excess with Maxwell-Boltzmann: $300 < T < 400$ MeV $\gg T_C$
- Challenge: photon source not obvious – really from the QGP?

# Heavy Quarkonium

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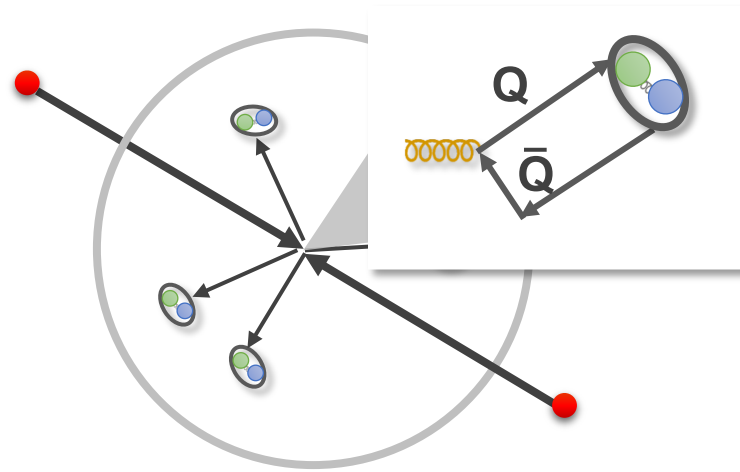
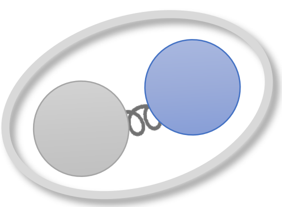
"**Too fast to watch**" – need to use collision remnants as probe particles.

Bound states of heavy quarks **$c\bar{c}$** (charmonium) **$b\bar{b}$** (bottomonium)

for a review see Brambilla et.al. Eur.Phys.J.C 71 (2011) 1534

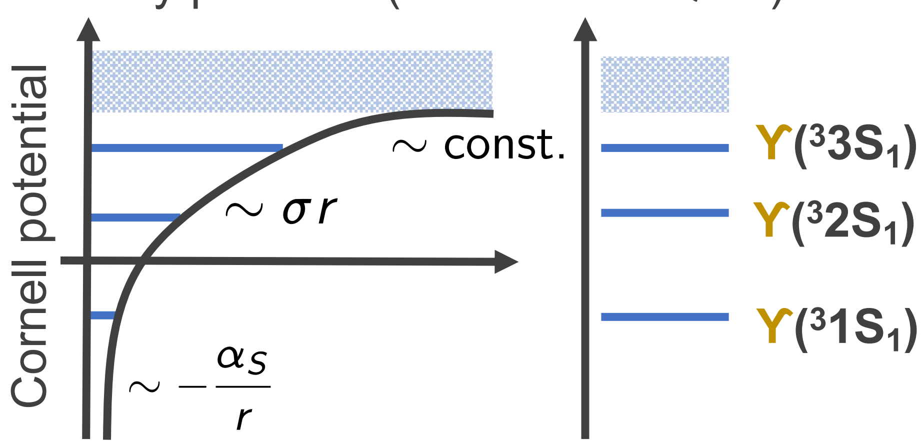
$Q\bar{Q}$ pairs either color singlet (attractive) or octet (repulsive)

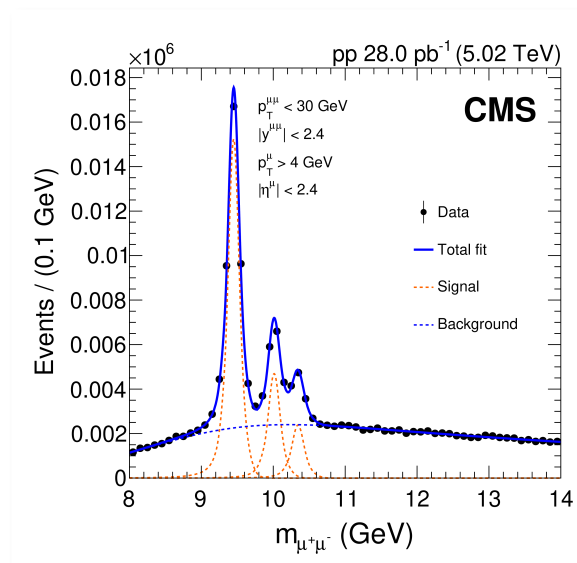


- decay into hadrons forbidden (OZI rule): keV widths
- significant decay into dileptons: clean experimental signals

- non-relativistic since $\Lambda_{QCD}/m_Q$<<1 & described by a simple 2-body potential (known from QCD)




[CMS collaboration] PLB 790 (2019) 270

# Quarkonium in heavy-ion collisions

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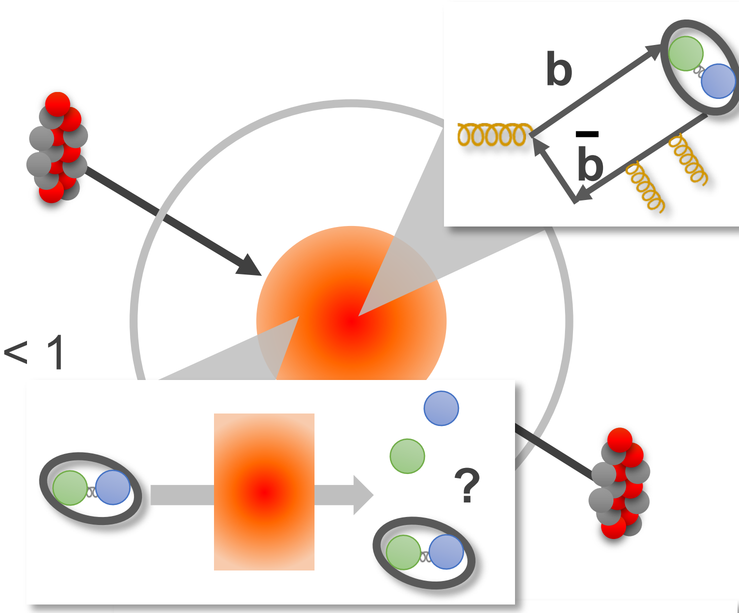
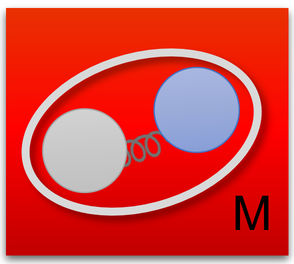
"**Too fast to watch**" – need to use collision remnants as probe particles.

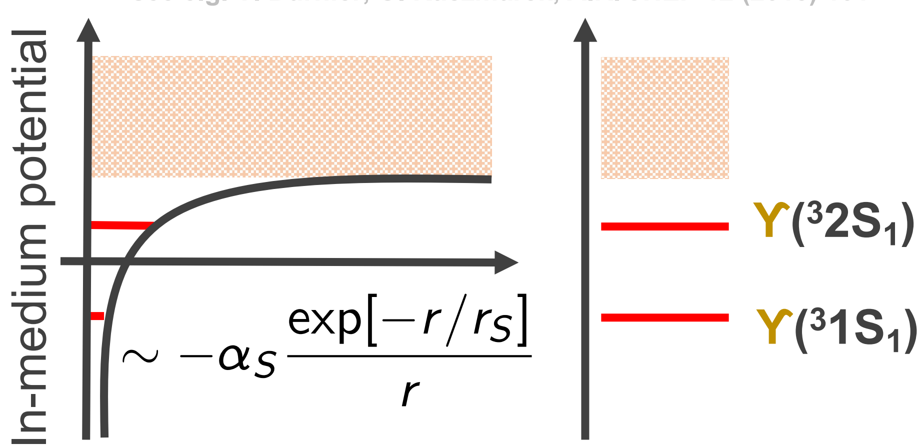
Bound states of heavy quarks **$c\bar{c}$** (charmonium) **$b\bar{b}$** (bottomonium)

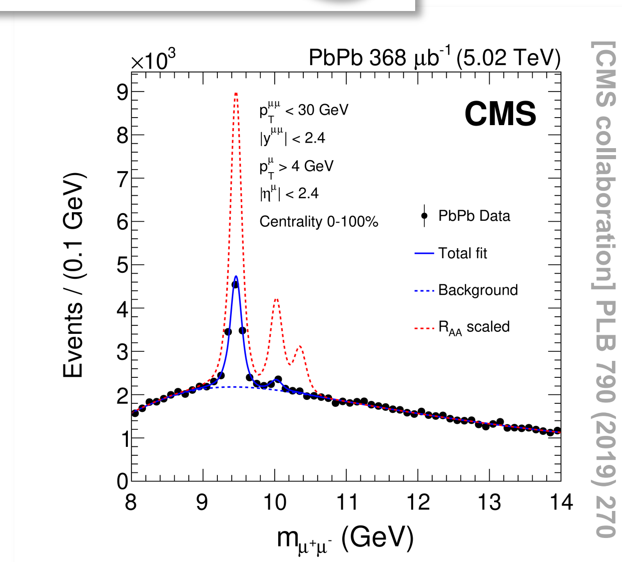



- still non-relativistic since $T_C/m_Q << 1$ Quantum Brownian Motion !
- presence of QGP modifies the interaction potential

see e.g. Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101




Early realization: screening by medium will weaken quarkonium and modifies production yields Matsui & Satz Phys.Lett.B 178 (1986) 416

# A Lindblad equation for quarkonium

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**Caldeira Leggett not applicable** as probe has finite extent and carries color charge.
**Progress**: QBM Lindblad equation from QCD at high T (**weakly coupled** regime)

Y. Akamatsu PRD91 (2015) 056002

$$\frac{d}{dt}\hat{\rho}_{Q\bar{Q}} = -\frac{i}{\hbar}[\hat{H}_{Q\bar{Q}}, \hat{\rho}_{Q\bar{Q}}] + \sum_k \left( \hat{\mathcal{L}}_k \hat{\rho}_{Q\bar{Q}} \hat{\mathcal{L}}_k^\dagger - \frac{1}{2}[\hat{\mathcal{L}}_k^\dagger \hat{\mathcal{L}}_k, \hat{\rho}_{Q\bar{Q}}] \right)$$

In-medium Hamiltonian exhibits a **screened potential:**

$$H_{Q\bar{Q}} = \frac{\mathbf{p}_{\rm rel}^2}{m_Q} + V_{Q\bar{Q}}(\mathbf{r}) \qquad V_{Q\bar{Q}}(\mathbf{r}) = -\alpha_S \frac{\exp[-r/r_S(T)]}{r}$$

$r_S$(T) explicitly known from QCD

Behind the scenes: In QCD, quarkonium interacts with medium via gluon exchange which induces **momentum transfer** & **color rotation**

D(r,T) explicitly known from QCD

**fluctuations** **dissipation**

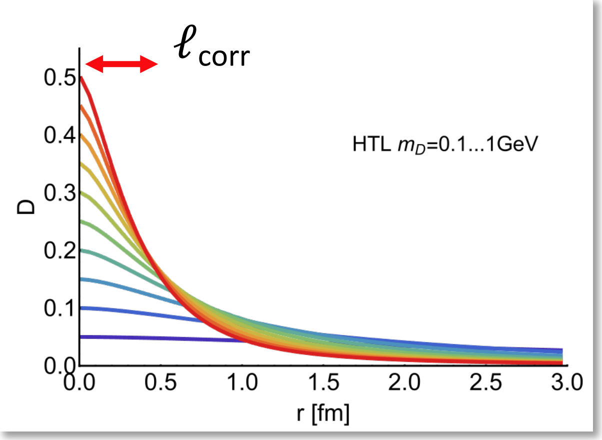
$$\mathcal{L}_{\mathbf{k},a} = \sqrt{\frac{D(\mathbf{k})}{2}} \left[ 1 - \frac{\mathbf{k}}{4 m_Q T} \cdot \left( \frac{1}{2}\mathbf{P}_{\rm CM} + \mathbf{p}_{\rm rel} \right) \right] e^{i\mathbf{k}\cdot\mathbf{r}/2} (T^a \otimes 1)$$

medium acting on the quark

$T^a$ 3x3 matrix in color space

$$- \sqrt{\frac{D(\mathbf{k})}{2}} \left[ 1 - \frac{\mathbf{k}}{4 m_Q T} \cdot \left( \frac{1}{2}\mathbf{P}_{\rm CM} + \mathbf{p}_{\rm rel} \right) \right] e^{i\mathbf{k}\cdot\mathbf{r}/2} (1 \otimes T^a)$$

medium acting on the anti-quark




see T. Miura, Y. Akamatsu, M. Asakawa, A.R. PRD101 (2020) 034011

# Screening and Decoherence

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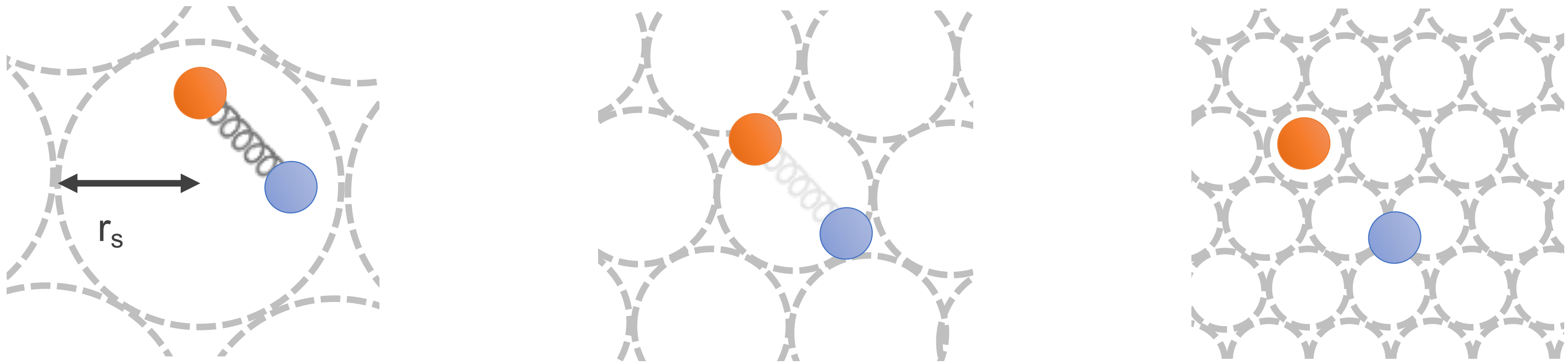
## screening

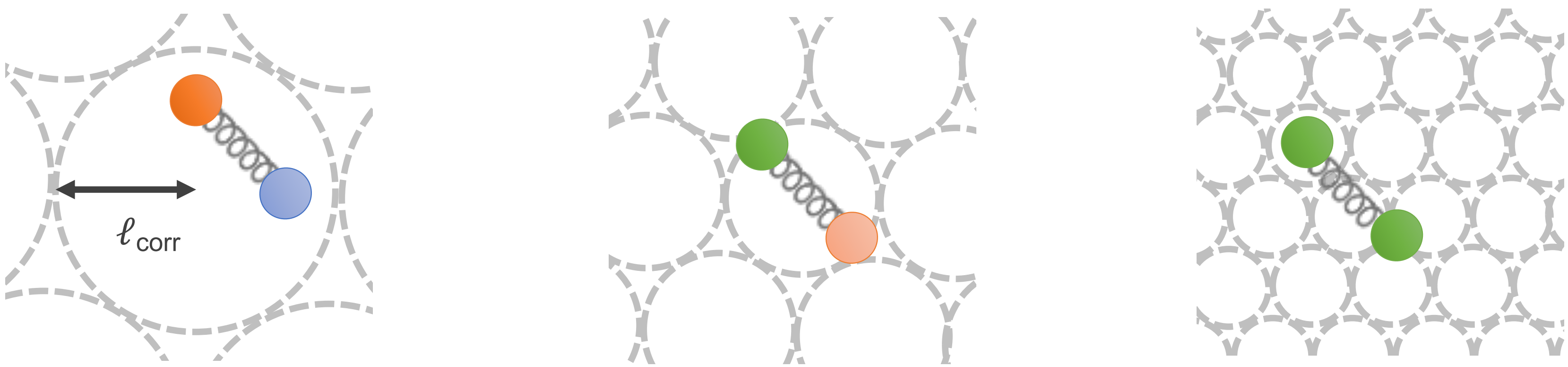



binding force acts efficiently among Q and $\bar{Q}$

medium impedes gluons mediating force

increasing temperature

## decoherence




color rotation acts coherently on Q and $\bar{Q}$

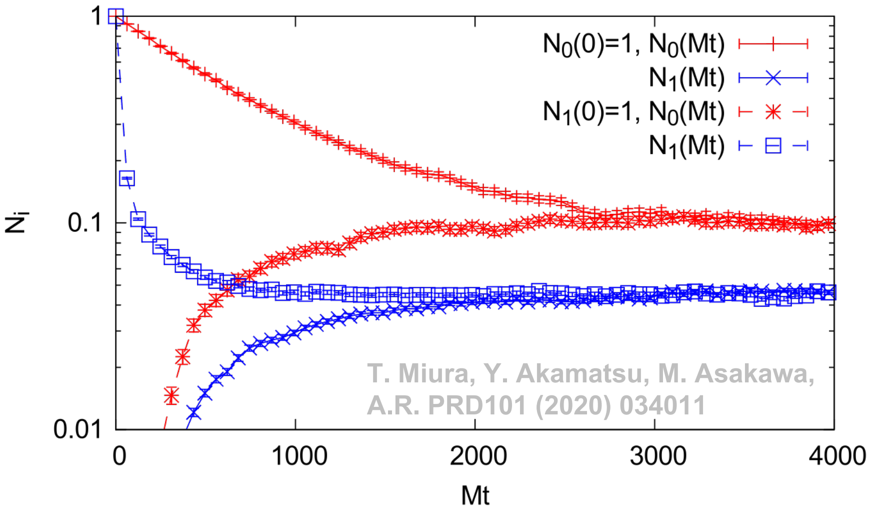
color rotation acts individually on Q and $\bar{Q}$

see discussion in S. Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018), 014003

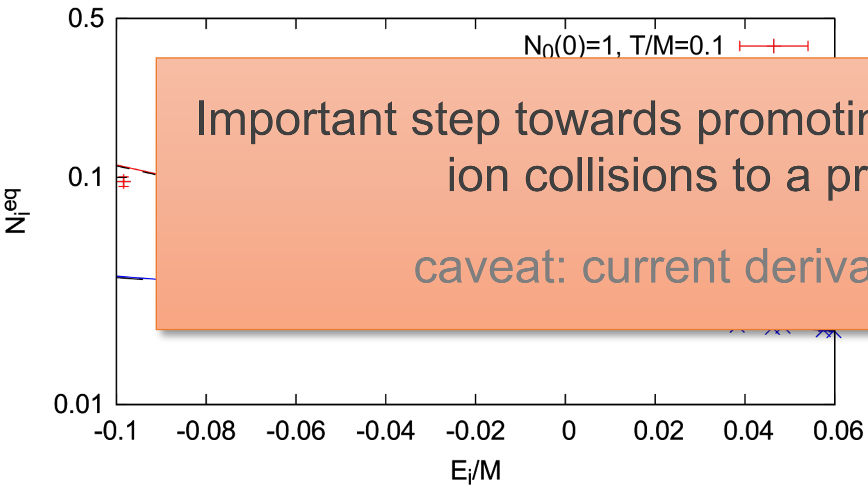
# Solving the 1d Lindblad equation

University of Stavanger

- For the first time: possible to **thermalize** quarkonium in a **fully quantum** fashion




- Start either with single ground state or single excited state and monitor survival probability
- Late-time results independent of initial conditions: steady-state




Important step towards promoting the study of quarkonium in heavy-ion collisions to a precision thermometry tool

caveat: current derivation only for weak coupling

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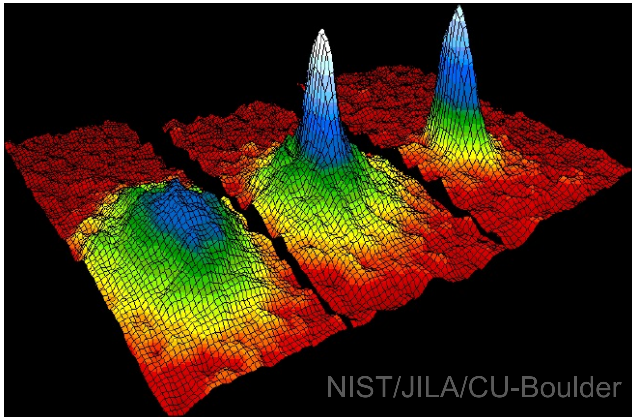
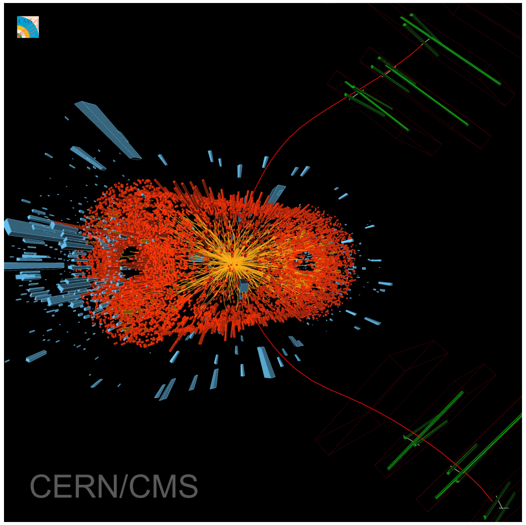
# Summary

- **Open Quantum Systems**: **impurities as quantum probes** play an essential role in **precision thermometry** at vastly different energy scales




Bose Einstein Condensate (BEC) from trapped ultracold atoms




Quark-Gluon Plasma (QGP) from relativistic heavy-ion collisions

- **Theory framework** developed to describe Open Quantum Systems (e.g. Lindblad equation) provides a **common language** among formerly disparate research fields

- Exciting prospect for **collaboration** between ultracold atoms and heavy-ion collision community on **impurity physics** (quarkonium, jets, hadronization,….)