χPT at finite Isospin

Quark, Pion and Axial Condensates

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Introduction: QCD phase diagram 1

- QCD phase diagram:

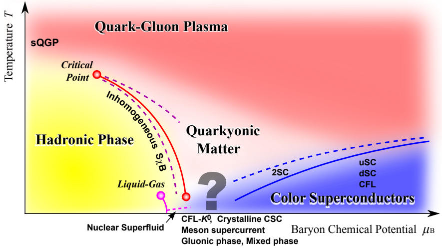


Figure: Fukushima and Hatsuda '11.

- Only a small part of the diagram is available with ab. initio. methods due to asymptotic freedom and the fermion sign problem.
- Most results are model dependent.

LQCD and the Sign Problem

$$\mathcal{Z}_{\rm QCD} = \int DAD\bar{q}Dqe^{-S}, \qquad S = S_{\rm YM} + \int d^4x\bar{q}Mq, \tag{1}$$

$$\mathcal{Z} = \int DAe^{-S_{\rm YM}}\,\mathrm{det}[M(\mu_B, \mu_I, \mu_S)], \tag{2}$$

$$\rho(A) \sim e^{-S_{\rm YM}}\,\mathrm{det}[M(\mu_B, \mu_I, \mu_S)], \tag{3}$$

$$[\mathrm{det}\, M(\mu_B)]^* = \mathrm{det}\, M(-\mu_B^*) \in C. \tag{4}$$

- No sign problem at finite isospin chemical potential $\mu_I \neq 0$, with $\mu_B = \mu_S = 0$.
- Possible to compare model calculations with first principle calculations!

Introduction: QCD phase diagram 2

- Deconfinement transition at high T.
- BEC of pions at low T and intermediate μ_I.
- BEC-BSC crossover at large μ_I.

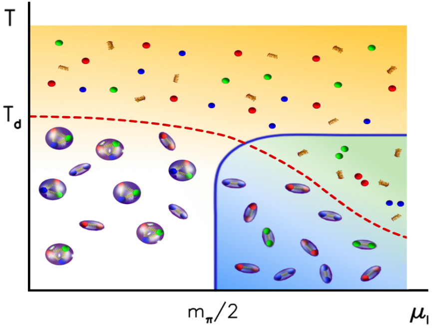


Figure: Brandt '18.

This talk

- Pion Condensation at $T = 0$ using two and three flavor χPT.
- Importance of next-to-leading-order corrections in ChiPT when comparing with LQCD data.

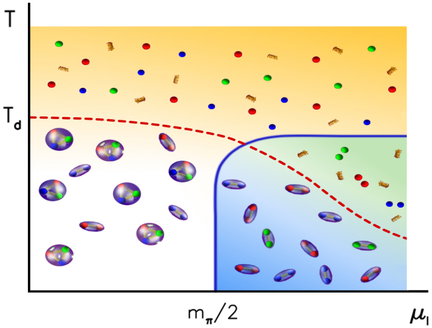


Figure: Brandt '18.

Symmetry

- χPT is uniquely determined by the global symmetries and symmetry-breaking patterns of QCD.
- Global $SU(N)_L \times SU(N)_R \times U(1)_B$, N is the number of quark flavors. (Exact/approximate for massless/massive quarks).
- SSB $SU(N)_L \times SU(N)_R \times U(1)_B \to SU(N)_V \times U(1)_B$.
- Goldstone's theorem tells us that there are $N^2 - 1$ Goldstone bosons. For $N = 2$ we get the three pions. For $N = 3$: pions, kaons, eta. These are the DoF in χPT.
- The Goldstone bosons parametrize the vacuum manifold $SU(N)_L \times SU(N)_R / SU(N)_V$ (more on this in a minute).
- (For $N = 2$) the appearance of $\mu_I > 0$ reduces the symmetry to $SU(2)_V \times U(1)_B \to U(1)_{I_3} \times U(1)_B$.

From symmetries to Lagrangians

- Weinberg's principles tells you to include all terms that are allowed by symmetry.
- Leads to an infinite number of terms... Predictive power out the window?
- No! Power counting scheme: $m = \mathcal{O}(p)$, $f = \mathcal{O}(p)$, $\tilde{\partial}^{\mu} = p^{\mu}$.
- Systematic expansion in powers of momenta (p): $\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + ...$
- One loop costs p^2.
- Non-renormalizable in the "old sense of the word". Higher order in loops $\implies$ more couplings l_i must be determined from experiments.
- No problem, as long as one is content with finite precision, which is the essence of an EFT.

LO Lagrangian and GS

- Lagrangian:

$$\mathcal{L}_2 = \frac{f^2}{4}\mathrm{Tr}\left[\nabla_\mu\Sigma^\dagger\nabla^\mu\Sigma\right] + \frac{f^2}{4}\mathrm{Tr}\left[\chi^\dagger\Sigma + \Sigma^\dagger\chi\right], \tag{5}$$

$$\chi = 2B_0M + 2iB_0j_1\tau_1 + 2iB_0j_2\tau_2, \tag{6}$$

$$\nabla_\mu\Sigma \equiv \partial_\mu\Sigma - i\left[v_\mu, \Sigma\right] - i\{a_\mu, \Sigma\} \tag{7}$$

$$v_\mu = v_0^a\frac{\lambda_a}{2}\delta_\mu^{\ 0},\ \ v_0^3 = \mu_I, a_\mu = a_0^a\frac{\lambda_a}{2}\delta_\mu^{\ 0}\ , \tag{8}$$

$$\tag{9}$$

- Parametrize GS by parameter α:

$$\Sigma_\alpha \equiv A_\alpha^2 \equiv e^{i\alpha(\hat{\phi}_1\tau_1+\hat{\phi}_2\tau_2)}, \qquad \hat{\phi}_1^2 + \hat{\phi}_2^2 = 1. \tag{10}$$

Plug this ansatz into $\mathcal{H}_2$ and minimize the energy to determine α:

$$\cos\alpha = 1, \quad \mu_I < m, \qquad \cos\alpha = \frac{m^2}{\mu_I^2}, \quad \mu_I \geq m. \tag{11}$$

Parametrizing fluctuations.

- Parametrize fluctuations around the (condensed) ground state,

$$\Sigma = L_\alpha \Sigma_\alpha R_\alpha^\dagger, \tag{12}$$

$$L_\alpha = A_\alpha U A_\alpha^\dagger, \tag{13}$$

$$R_\alpha = A_\alpha^\dagger U^\dagger A_\alpha, \tag{14}$$

$$U = e^{i\frac{\phi_i \tau_i}{2f}}. \tag{15}$$

- Can be nice to think of a less abstract example: $SO(3) \to SO(2)$.

Renormalization

- NLO terms relevant for our calculations:

$$\mathcal{L}_4 = \frac{1}{4} l_1 \left(\mathrm{Tr}\left[\nabla \mu \Sigma^\dagger \nabla^\mu \Sigma \right] \right)^2 + \frac{1}{4} l_2 \mathrm{Tr}\left[\nabla_\mu \Sigma^\dagger \nabla_\nu \Sigma \right] \mathrm{Tr}\left[\nabla^\mu \Sigma^\dagger \nabla^\nu \Sigma \right]$$
$$+ \frac{1}{16}(l_3 + l_4)(\mathrm{Tr}[\chi^\dagger \Sigma + \Sigma^\dagger \chi])^2 + \frac{1}{8} l_4 \mathrm{Tr}\left[\nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \right] \mathrm{Tr}[\chi^\dagger \Sigma + \Sigma^\dagger \chi]$$
$$+ \frac{1}{2} h_1 \mathrm{Tr}[\chi^\dagger \chi] \,. \quad (16)$$

$$l_i = l_i^r(\Lambda) - \frac{\gamma_i \Lambda^{-2\epsilon}}{2(4\pi)^2} \left[\frac{1}{\epsilon} + 1 \right] \,, \quad (17)$$

$$l_i^r(\Lambda) = \frac{\gamma_i}{2(4\pi)^2} \left[\bar{l}_i + \log \frac{M^2}{\Lambda^2} \right] \,, \quad (18)$$

Three-flavor Effective Potential...

$$V_{\text{eff}} = -f^2B_0(2m_j+m_s) - \frac{1}{2}f^2(\mu_I\sin\alpha + a_0^1\cos\alpha)^2 - [64L_6^r + 16L_8^r$$
$$+8H_2^r + \frac{1}{(4\pi)^2}\left(\frac{37}{18} + \log\frac{\Lambda^2}{\tilde{m}_1^2} + 2\log\frac{\Lambda^2}{m_3^2} + \log\frac{\Lambda^2}{\tilde{m}_4^2} + \frac{1}{9}\log\frac{\Lambda^2}{m_8^2}\right)\Big] B_0^2m_j^2$$
$$-\left[64L_6^r + \frac{1}{(4\pi)^2}\left(\frac{11}{9} + 2\log\frac{\Lambda^2}{\tilde{m}_4^2} + \frac{4}{9}\log\frac{\Lambda^2}{m_8^2}\right)\right] \times B_0^2m_jm_s$$
$$-\left[16L_6^r + 8L_8^r + 4H_2^r + \frac{1}{(4\pi)^2}\left(\frac{13}{18} + \log\frac{\Lambda^2}{\tilde{m}_4^2} + \frac{4}{9}\log\frac{\Lambda^2}{m_8^2}\right)\right] B_0^2m_s^2$$
$$-\left[8L_4^r + \frac{1}{2(4\pi)^2}\left(\frac{1}{2} + \log\frac{\Lambda^2}{\tilde{m}_4^2}\right)\right] \times B_0(2m_j+m_s)(\mu_I\sin\alpha + a_0^1\cos\alpha)^2$$
$$-\left[8L_5^r + \frac{1}{2(4\pi)^2}\left(\frac{3}{2} + 4\log\frac{\Lambda^2}{m_3^2} - \log\frac{\Lambda^2}{\tilde{m}_4^2}\right)\right] \times B_0m_j(\mu_I\sin\alpha + a_0^1\cos\alpha)^2 +$$
$$-\left[4L_1^r + 4L_2^r + 2L_3^r + \frac{1}{16(4\pi)^2}\left(\frac{9}{2} + 8\log\frac{\Lambda^2}{m_3^2} + \log\frac{\Lambda^2}{\tilde{m}_4^2}\right)\right](\mu_I\sin\alpha + a_0^1\cos$$

And here comes the observables!

$$\langle\bar{\psi}\psi\rangle \equiv \langle\bar{u}u\rangle = \langle\bar{d}d\rangle \overset{a=0}{=} \frac{1}{2}\frac{\partial V_{\text{eff}}}{\partial m}, \tag{20}$$

$$\langle\bar{s}s\rangle \overset{a=0}{=} \frac{\partial V_{\text{eff}}}{\partial m_s}, \tag{21}$$

$$\langle\pi^+\rangle \overset{a=0}{=} \frac{1}{2}\frac{\partial V_{\text{eff}}}{\partial j}, \tag{22}$$

$$\langle\bar{\psi}\tfrac{\tau_2}{2}\gamma^0\gamma_5\psi\rangle \overset{a=0}{=} \frac{\partial V_{\text{eff}}}{\partial a_0^1}. \tag{23}$$

Parameter Fixing (in two-flavor χPT)

- Experimentalists have determined the LECs:

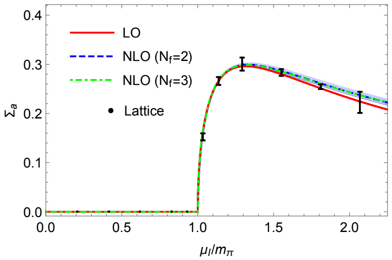
$$\bar{l}_1 = -0.4 \pm 0.6 \,, \qquad \bar{l}_2 = 4.3 \pm 0.1 \,, \tag{24}$$
$$\bar{l}_3 = 2.9 \pm 2.4 \,, \qquad \bar{l}_4 = 4.4 \pm 0.2 \tag{25}$$
$$\bar{h}_1 = -1.5 \pm 0.2 \,. \tag{26}$$

- Our LQCD friends have provided data that we use to fix f and m through the following NLO relations:

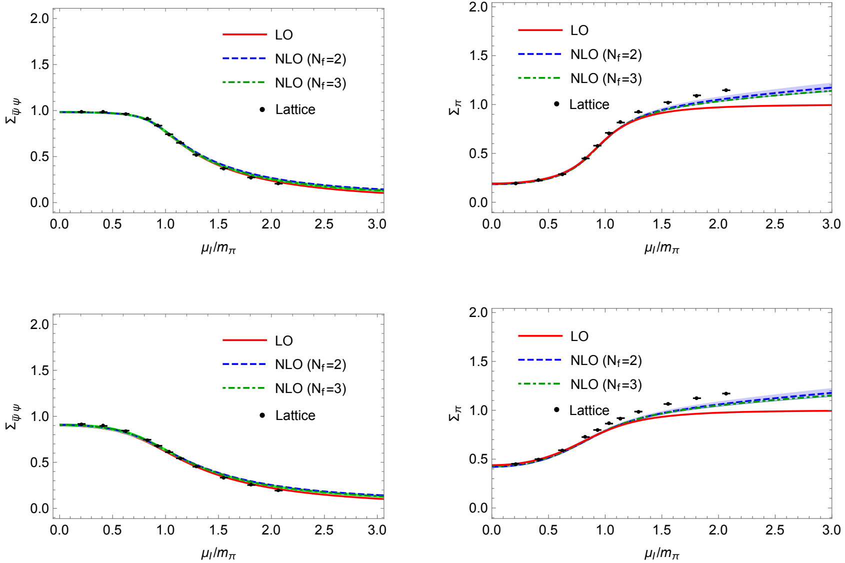
$$m_\pi^2 = 2B_0 m \left[1 - \frac{B_0 m}{(4\pi)^2 f^2} \bar{l}_3\right] = 131 \pm 3 MeV \,, \tag{27}$$
$$f_\pi^2 = f^2 \left[1 + \frac{4B_0 m}{(4\pi)^2 f^2} \bar{l}_4\right] = \frac{128 \pm 3}{\sqrt{2}} MeV \,. \tag{28}$$

(Numerical) Results

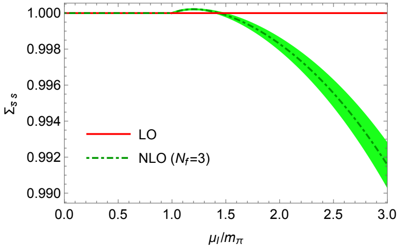
(Numerically) minimize V_{eff} w.r.t GS parameter α for given values of μ_I and source fields, and then generate condensates by taking derivatives of V_{eff} w.r.t sources.



(Numerical) Results



(Numerical) Results...



Conclusions

- This work completes a series of papers on calculations of observables to next-to-leading order in two- and three flavor χPT in the pion-condensed phase.
- Good agreement with LQCD at $T = 0$ and low temperatures. First precision tests of χPT at NLO with finite n_I.
- Related topic: Pions in a magnetic field. Something my collaborators have been working on more recently.

Questions