Non-linear structure formation in cosmologies with non-trivial time- and scale-dependence

Based on M. Garny and **P. Taule** 2008.00013
T. Baldauf, M. Garny, **P. Taule** and T. Steele 2110.13930
M. Garny and **P. Taule** in preparation
M. Escudero, M. Garny and **P. Taule** in preparation

Petter Taule

Technical University of Munich

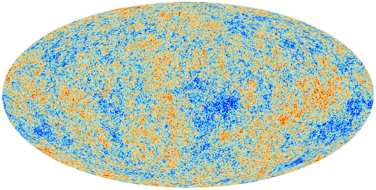
Astro&Theory Seminar, NTNU Trondheim

SFB 1258 Neutrinos Dark Matter Messengers

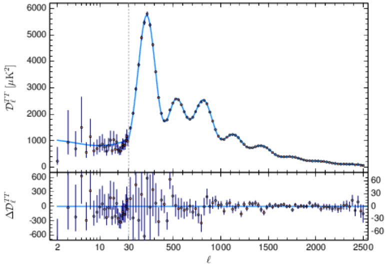
Outline

1. Intro: Large-scale structure as a probe of fundamental physics
2. Eulerian perturbation theory $\rightarrow$ effective field theory of LSS
3. Extension to capture time- and scale-dependence
4. Application: Structure formation in the presence of massive neutrinos
5. Two-loop bispectrum
6. Summary

ΛCDM



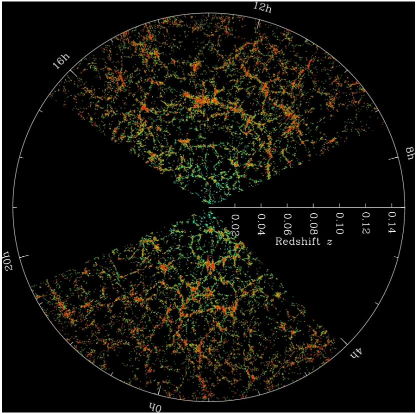
- Baryons ($+e^-$) 5%
- Cold dark matter (CDM) 27%
- Cosmological constant Λ 68%
- Thermal photon+neutrino background
- Flat FLRW geometry
- Gaussian, adiabatic initial conditions



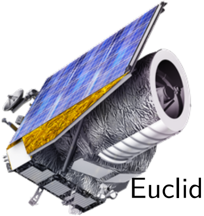
$$
\begin{aligned}
\Omega_b h^2 &= 0.02237 \pm 0.0001 \\
\Omega_{\rm cmb} h^2 &= 0.1197 \pm 0.001 \\
n_s &= 0.9649 \pm 0.0042 \\
\ln(10^{10} A_s) &= 3.044 \pm 0.014 \\
100\theta &= 1.0411 \pm 0.0003 \\
\tau &= 0.0544 \pm 0.0073
\end{aligned}
$$

Planck collaboration 1807.06209

Large-scale structure



SDSS-III



Power spectrum

Density contrast:

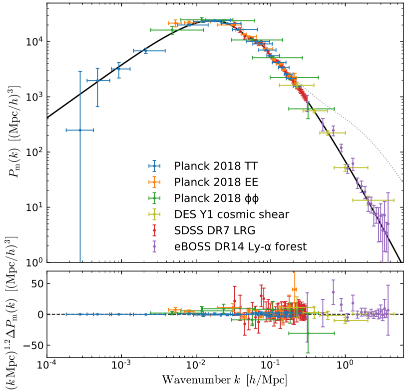
$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \overline{\rho}}{\overline{\rho}}$$

Two-point correlation

$$\langle \delta(\mathbf{x} + \mathbf{r}) \delta(\mathbf{x}) \rangle = \xi(r)$$

Power spectrum $P(k)$ in fourier space

$$\langle \delta(\mathbf{k}) \delta(\mathbf{k}') \rangle = \delta_D^{(3)}(\mathbf{k} - \mathbf{k}') P(k)$$



S. Chabanier et. al. 1905.08103

Power spectrum

Density contrast:

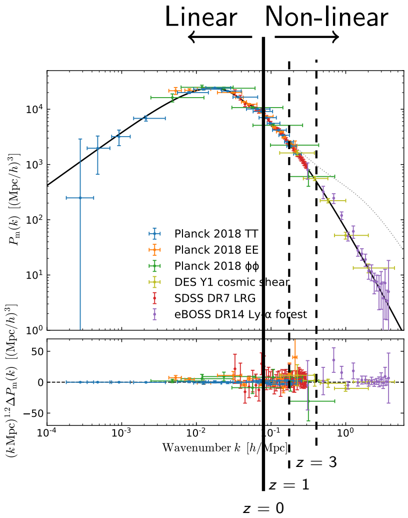
$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \overline{\rho}}{\overline{\rho}}$$

Two-point correlation

$$\langle \delta(\mathbf{x}+\mathbf{r})\delta(\mathbf{x}) \rangle = \xi(r)$$

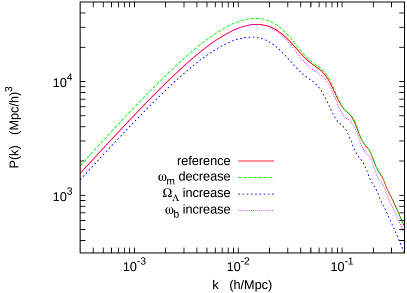
Power spectrum $P(k)$ in fourier space

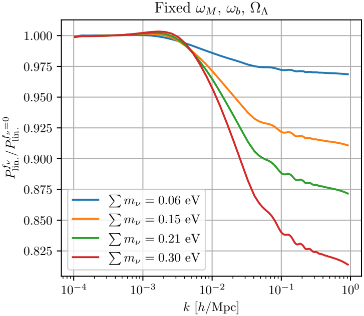
$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = \delta_D^{(3)}(\mathbf{k}-\mathbf{k}')P(k)$$



S. Chabanier et. al. 1905.08103

Power spectrum





J. Lesgourgues et. al. astro-ph/0603494

Vlasov-Poisson system

- Number of particles $\mathrm{d}N = f(\mathbf{x}, \mathbf{p}, \tau)\, \mathrm{d}^3x\, \mathrm{d}^3p$
- Vlasov equation (conformal time $\mathrm{d}\tau = \mathrm{d}t/a$)

$$\left[\frac{\partial}{\partial \tau} + \frac{\mathbf{p}}{am}\frac{\partial}{\partial \mathbf{x}} - am\nabla\phi\,\frac{\partial}{\partial \mathbf{p}}\right] f(\mathbf{x}, \mathbf{p}, \tau) = 0$$

- Poisson-equation

$$\nabla^2\phi(\mathbf{x}, \tau) = \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(\mathbf{x}, \tau)$$

- Solution strategies: N-body, Kinetic field theory[1], **Eulerian**/Lagrangian perturbation theory,...
- Moments

$$\rho = \overline{\rho}(1+\delta) \int \mathrm{d}^3p\, f\,, \qquad \rho\mathbf{u} = m \int \mathrm{d}^3p\, \frac{\mathbf{p}}{am} f\,, \qquad \ldots$$

[1] M. Bartelmann et. al. 1411.0806

Eulerian perturbation theory

- Fluid eqs in fourier space, $\theta = \partial_i \mathbf{u}^i$ (neglecting vorticity)

$$\delta'(\mathbf{k}) + \theta(\mathbf{k}) = -\int_{\mathbf{q}} \alpha(\mathbf{q}, \mathbf{k}-\mathbf{q})\theta(\mathbf{q})\delta(\mathbf{k}-\mathbf{q})$$

$$\theta'(\mathbf{k}) + \mathcal{H}\theta(\mathbf{k}) + \frac{3}{2}\Omega_\mathrm{m}\mathcal{H}^2\delta(\mathbf{k}) = -\int_{\mathbf{q}} \beta(\mathbf{q}, \mathbf{k}-\mathbf{q})\theta(\mathbf{q})\theta(\mathbf{k}-\mathbf{q}) - \tau_\theta(\mathbf{k})$$

- Define $\psi = (\delta, -\theta/\mathcal{H}f)$; rescaled time $\eta = \log D$

$$\partial_\eta \psi_a(\mathbf{k}) + \Omega_{ab}\,\psi_b(\mathbf{k}) = \int_{\mathbf{q}} \gamma_{abc}(\mathbf{q}, \mathbf{k}-\mathbf{q})\,\psi_b(\mathbf{q})\psi_c(\mathbf{k}-\mathbf{q})$$

$$\Omega_{ab} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f^2} & \frac{3}{2}\frac{\Omega_m}{f^2} - 1 \end{pmatrix}$$

$$\gamma_{121}(\mathbf{q}_1, \mathbf{q}_2) = 1 + \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} \qquad \gamma_{222}(\mathbf{q}_1, \mathbf{q}_2) = 1 + \frac{(\mathbf{q}_1 + \mathbf{q}_2)^2(\mathbf{q}_1 \cdot \mathbf{q}_2)}{2q_1^2 q_2^2}$$

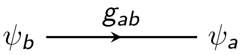
Bernardeau et.al. astro-ph/0112551

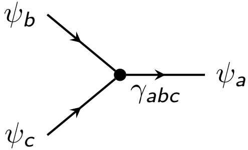
Eulerian perturbation theory

- Perturbative solution

$$\psi_a(\mathbf{k},\eta) = \sum_{n=1}^{\infty} \int_{\mathbf{q}_1,\ldots,\mathbf{q}_n} \delta_D(\mathbf{k}-\mathbf{q}_{1\cdots n})\, F_a^{(n)}(\mathbf{q}_1,\ldots,\mathbf{q}_n) \prod_{j=1}^{n} \delta_0(\mathbf{q}_j,\eta_{\text{ini}})$$

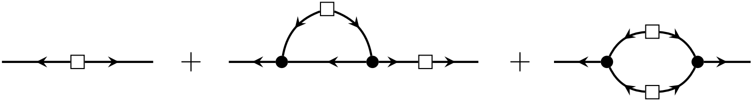
- "Feynman diagrams"





- E.g. matter power spectrum $P_m = \langle \delta\delta \rangle = \langle \psi_1 \psi_1 \rangle$ at 1-loop

$P_m =$

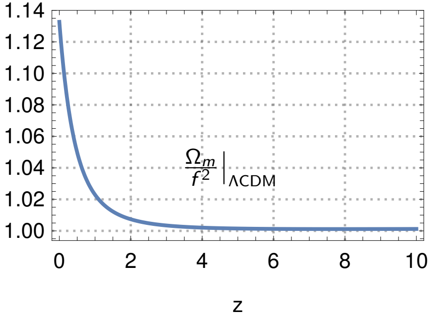


Einstein-de–Sitter approximation

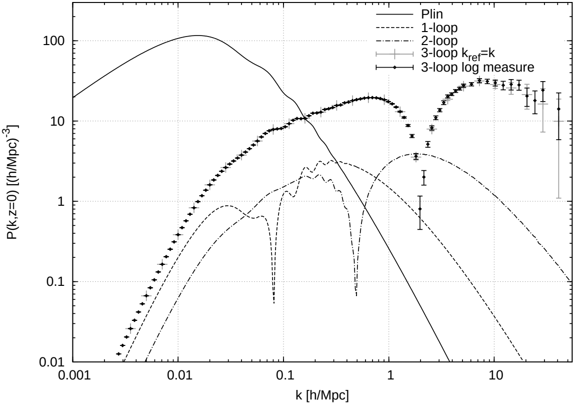
- Einstein-de-Sitter (EdS) universe: $\Omega_m = 1$

$$\Omega_{ab} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f^2} & \frac{3}{2}\frac{\Omega_m}{f^2} - 1 \end{pmatrix} \xrightarrow{EdS} \begin{pmatrix} 0 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

→ Analytic solutions



Standard perturbation theory (SPT)



D. Blas et. al. 1309.3308

- No well-defined expansion parameter
- Cutoff-dependence

Effective field theory

- Perfect fluid approximation inconsistent
- Consider coarse-grained fields:

$$f_l(\mathbf{x},\mathbf{p},\tau) = \int \mathrm{d}^3x' \, W_\Lambda(\mathbf{x}-\mathbf{x}') f(\mathbf{x}',\mathbf{p},\tau)$$

- EoM

$$\delta_l'(\mathbf{k}) + \theta_l(\mathbf{k}) = -\int^\Lambda \mathrm{d}^3q \, \alpha(\mathbf{q},\mathbf{k}-\mathbf{q})\theta_l(\mathbf{q})\delta_l(\mathbf{k}-\mathbf{q})$$

$$\theta_l'(\mathbf{k}) + \mathcal{H}\theta_l(\mathbf{k}) + \frac{3}{2}\Omega_\mathrm{m}\mathcal{H}^2\delta_l(\mathbf{k}) = -\int^\Lambda \mathrm{d}^3q \, \beta(\mathbf{q},\mathbf{k}-\mathbf{q})\theta_l(\mathbf{q})\theta_l(\mathbf{k}-\mathbf{q}) - \tau_\theta(\mathbf{k})$$

- Effective stress tensor τ_θ
  - Expectation value of short scales (integrate out UV d.o.f.) + stochastic contribution

D. Baumann et. al. 1004.2488
J. J. M. Carrasco et. al. 1206.2926

Effective field theory

- Treat τ_θ as functional of smoothed fields
- Write down operators allowed by symmetry, gradient expansion (k/k_{NL})

$$\tau_\theta\big|_{\mathrm{LO}} = -\gamma_1 \Delta \delta_1$$
$$\tau_\theta\big|_{\mathrm{NLO}} = -\gamma_1 \Delta \delta_2 - e_1 \Delta \delta_1^2 - e_2 \Delta s^2 - e_3 \partial_i \left[s^{ij} \partial_j \delta_1 \right]$$

s^{ij} = tidal tensor. (Neglecting stochastic contributions)

- Sound speed $\gamma_1 = c_s^2$
- Cutoff-dependence absorbed by counterterms

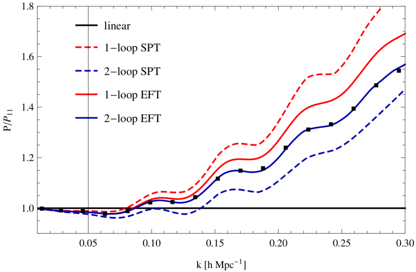
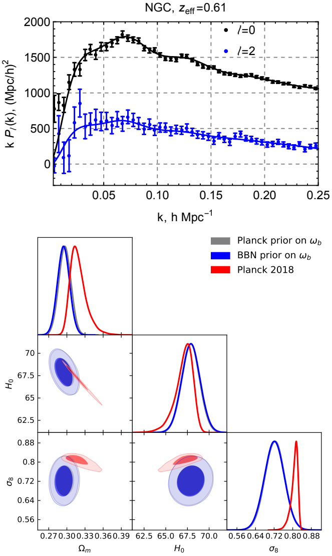


Figure: T. Baldauf

Full-shape analysis

- IR resummation (impact of large bulk flows)
- Biased tracers (e.g. galaxies)
- Redshift space distorsions
- Fast evaluation for MCMC analysis
- 2019: analysis of BOSS full-shape data[1,2,3]



[1]

[1] M. Ivanov et. al. 1909.05277
[2] G. d'Amico et. al. 1909.05271
[3] T. Tröster et. al. 1909.11006

Improvements

- **Two-loop correction**
- **Relax EdS approximation**
- Effect of baryons
- **Neutrinos perturbations beyond the linear order**
- Relativistic effects close to Hubble scale
- Anisotropic stress from first principles
  $\rightarrow$ decrease size of counterterms
- Extended cosmological models: clustering DE, modified gravity

Extension beyond SPT/EFTofLSS & massive neutrinos

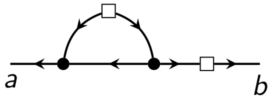
Extension

Multiple fields $\psi_a = (\delta, -\theta/\mathcal{H}f, \ \ldots\ )$

$$\partial_\eta \psi_a(\mathbf{k}) + \Omega_{ab}(\mathbf{k},\eta)\, \psi_b(\mathbf{k}) = \int_{\mathbf{q}} \gamma_{abc}(\mathbf{q},\mathbf{k}-\mathbf{q})\psi_b(\mathbf{q})\psi_c(\mathbf{k}-\mathbf{q}) - \tau_\theta(\mathbf{k})$$

$$\psi_a(\mathbf{k},\eta) = \sum_{n=1}^{\infty} \int_{\mathbf{q}_1,\ldots,\mathbf{q}_n} \delta_D(\mathbf{k}-\mathbf{q}_{1\cdots n})\, F_a^{(n)}(\mathbf{q}_1,\ldots,\mathbf{q}_n,\eta) \prod_{j=1}^{n} \delta_0(\mathbf{q}_j,\eta_{\text{ini}})$$

→ no analytic solution in general
→ can describe cosmologies with generic time/scale-dependence



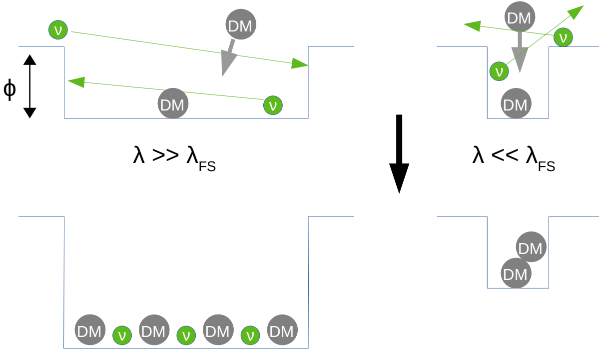
$$= P_0(\mathbf{k}) \int^{\Lambda} \mathrm{d}^3 q\, F_a^{(3)}(\mathbf{k},\mathbf{q},-\mathbf{q}) F_b^{(1)}(\mathbf{k}) P_0(\mathbf{q})$$

Numerical solution for F's for every Monte Carlo integration point

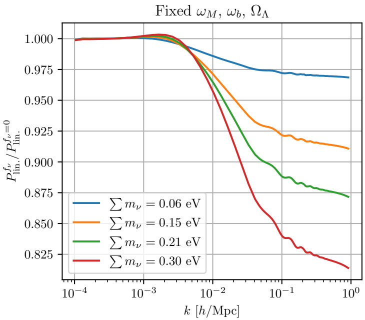
M. Garny, PT 2008.00013

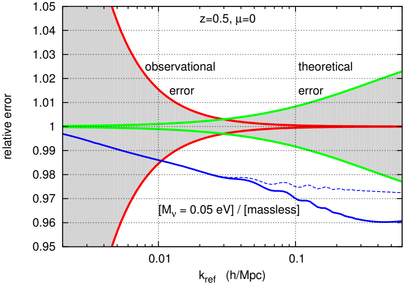
Neutrinos in cosmology

- Neutrino masses are direct evidence of BSM physics
- Cosmic neutrino background $\Omega_\nu = \frac{\sum m_\nu}{93.14 h^2 \text{eV}} = 0.0024 \left(\frac{\sum m_\nu}{0.1 \text{ eV}} \right)$
- Non-relativistic transition at $z_{\text{nr}} \simeq 189 \left(\frac{m_\nu}{0.1 \text{ eV}} \right)$
- Free-streaming $\rightarrow$ scale-dependent suppression



Neutrinos in cosmology





Euclid forecast B. Audren et. al. 1210.2194

Three degenerate neutrinos [1]

- Planck18: $\sum m_\nu < 0.26$ eV Planck 1807.06209
- Planck18+BAO: $\sum m_\nu < 0.12$ eV Planck 1807.06209
- Planck18+BOSS $P(k)$: $\sum m_\nu < 0.16$ eV M. Ivanov et.al. 1912.08208
- Euclid forecast: $\sigma(\sum m_\nu) = 0.028$ eV M. Ivanov et.al. 1907.06666

[1] M. Archidiacono et.al. 2003.03354

Two-component fluid

- CDM+baryons: imperfect fluid (EFT), one joint component
- Neutrinos: $(z_{\text{match}} = 25 \ll z_{\text{nr}})$ [1]
  - $z > z_{\text{match}}$: Full Boltzmann hierarchy
  - $z < z_{\text{match}}$: Fluid description
- Fluid perturbations ψ_a [2]

$$\psi_1 = \delta_{\text{cb}}\,, \quad \psi_2 = -\frac{\theta_{\text{cb}}}{\mathcal{H}f}\,, \quad \psi_3 = \delta_\nu\,, \quad \psi_4 = -\frac{\theta_\nu}{\mathcal{H}f}\,,$$

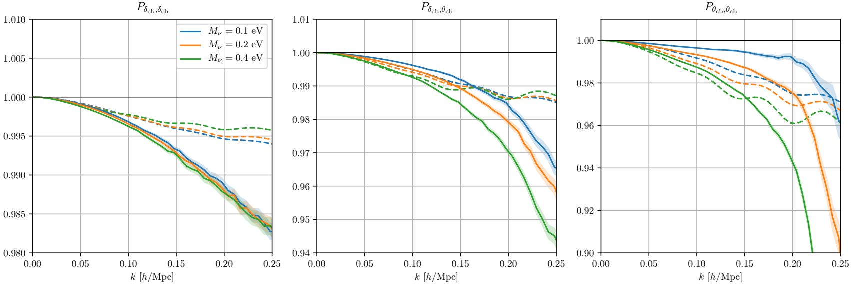
$$\Omega_{ab}(k,\eta) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -\frac{3}{2}\frac{\Omega_m}{f^2}(1-f_\nu) & \frac{3}{2}\frac{\Omega_m}{f^2} - 1 & -\frac{3}{2}\frac{\Omega_m}{f^2} f_\nu & 0 \\ 0 & 0 & 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f^2}(1-f_\nu) & 0 & -\frac{3}{2}\frac{\Omega_m}{f^2}[f_\nu - k^2 c_s^2(k,\eta)] & \frac{3}{2}\frac{\Omega_m}{f^2} - 1 \end{pmatrix}$$

with effective neutrino sound speed c_s^2.

[1] D. Blas et.al. 1408.2995
[2] M.Garny, PT 2008.00013

Comparison

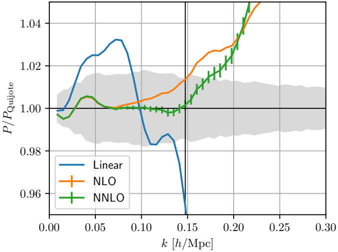
- Comparison to simplified treatment: only linear neutrino perturbations and EdS approx.
- Dashed lines: linear+1-loop. Solid lines: linear+1-loop+2-loop

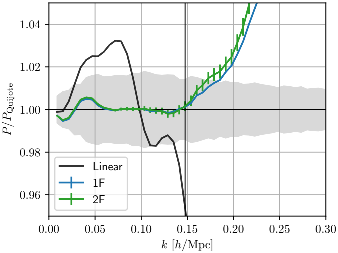


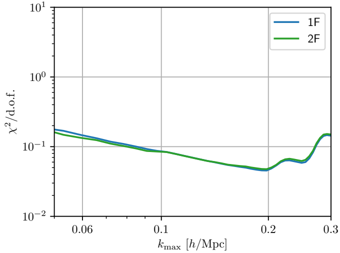
M.Garny, PT in prep.

Comparison to N-body

- Including EFT counterterms, fitted to Quijote[1] N-body results







M.Garny, PT in prep.

[1] F. Villaescusa-Navarro et.al. 1909.0573

Two-loop bispectrum

The bispectrum in LSS

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

- Leading non-Gaussian statistic
- Shape sensitive to gravitational instability
- Break parameter degeneracy between galaxy bias and cosmological parameters
- Measure initial non-Gaussianity
- Power spectrum + bispectrum analysis of BOSS at 1-loop [1]

$$f_{\rm NL}^{\rm equi.} = 2 \pm 112$$
$$f_{\rm NL}^{\rm orth.} = 126 \pm 72$$
$$f_{\rm NL}^{\rm loc.} = -30 \pm 29$$

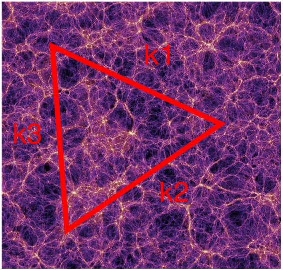


Image: V. Springel et. al. '05

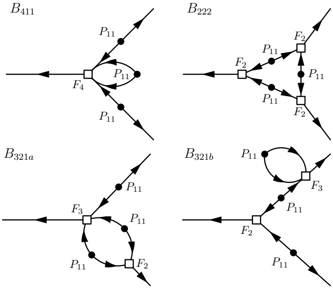
[1] G. d'Amico et.al. 2201.11518

Effective stress tensor

$$\tau_\theta\big|_{\mathrm{LO}} = -\gamma_1 \Delta\delta_1$$

$$\tau_\theta\big|_{\mathrm{NLO}} = -\gamma_1 \Delta\delta_2 - e_1 \Delta\delta_1^2 - e_2 \Delta s^2 - e_3 \partial_i \left[s^{ij} \partial_j \delta_1 \right]$$

$$B_{1L} = B^s_{411} + B^s_{321a} + B^s_{321b} + B_{222}$$



Dominant UV sensitivity

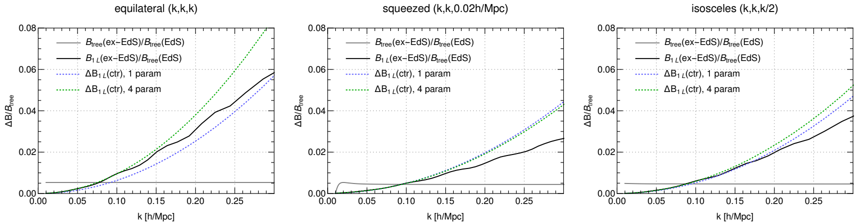
$$k^2 \int^{\Lambda} \mathrm{d}q \, P(q) \equiv k^2 \sigma_d^2$$

Absorbed by EFT operators $\{\gamma_1, e_1, e_2, e_3\}$ [1]

[1] T. Baldauf et. al. 1406.4135
R. Angulo et. al. 1406.4143

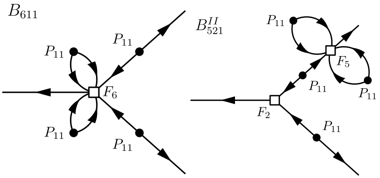
Exact time-dependent dynamics

Can the departure from EdS be absorbed into the counterterms?



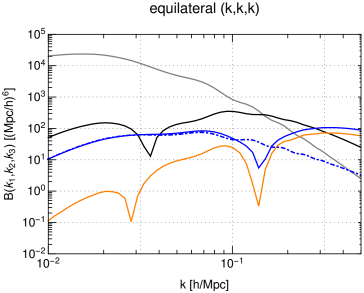
Two-loop bispectrum

- 11 diagrams + perm.
- Double-hard limit $\mathbf{q}_1, \mathbf{q}_2 \to \infty$
  - B_{611} and B_{521}^{II} dominant k^2 UV sensitivity
  - Analytic limit of $F^{(6)}$
  - Absorbed by EFT operators $\{\gamma_1, e_1, e_2, e_3\}$



. . .

$$B_{2L}^{\text{sub}} = B_{2L} - B_{2L}^{hh}$$



Two-loop bispectrum

- Single-hard limit: $\mathbf{q}_1$ (or $\mathbf{q}_2$) $\to \infty$
- Numerical evaluation

$$b^h_{2L}(k_1, k_2, k_3) = \int^{\Lambda} d^3 q_2 \, d\Omega_{q_1} \left[\lim_{q_1 \to \infty} q_1^2 \, b_{2L}(k_1, k_2, k_3, \vec{q}_1, \vec{q}_2) \right] P_{11}(q_2)$$

$$B^h_{2L}(k_1, k_2, k_3) \propto \sigma_d^2 \, b^h_{2L}(k_1, k_2, k_3)$$

- Subtract degenerate part

$$b^{h,\text{sub}}_{2L} = b^h_{2L} - b^{hh}_{2L}$$

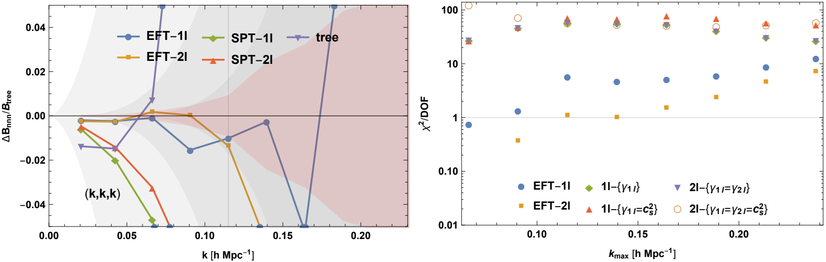
- Counterterm

$$B^{\text{ctr}}_{2L} \propto \gamma_2 \, b^{h,\text{sub}}_{2L}$$

- Similar prescription for 2-loop powerspectrum:
  T. Baldauf et. al. 1507.02256

Results

- Realization-based GridPT
- EFT-1L: $\{\gamma_1, e_1, e_2, e_3\}$
- EFT-2L: $\{\gamma_1, e_1, e_2, e_3, \gamma_2\}$

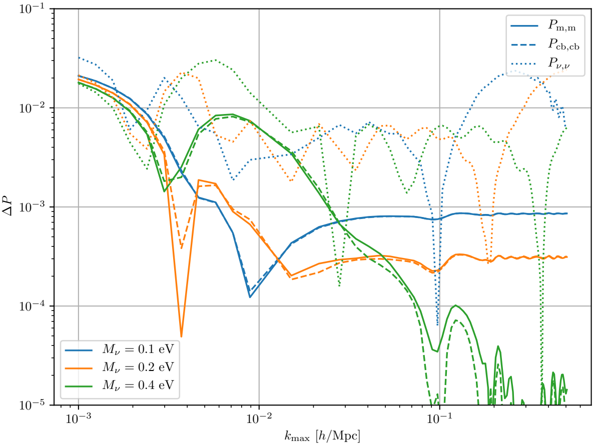


Summary

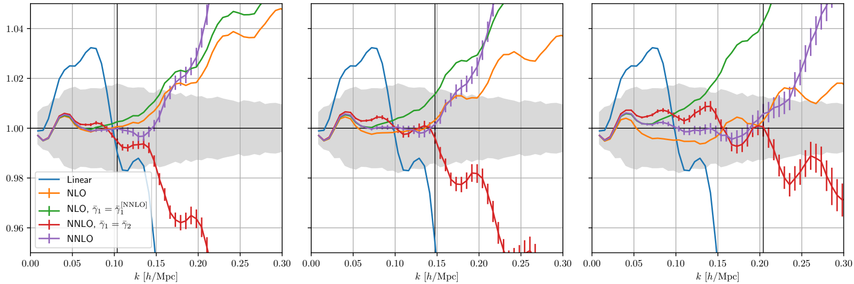
- Large-scale structure is a leading probe in precision cosmology
- EFTofLSS systematically parametrizes the effect of small-scales on perturbative scales
- Extension that can capture general time/scale-dependence
  - Scrutinize approximations
  - Extended cosmological models
- Presence of massive neutrinos in structure formation
- Bispectrum
  - Adding the two-loop extends wavenumbers with 1σ agreement to $k \simeq 0.15\ h\ \mathrm{Mpc}^{-1}$
  - Departure from EdS can largely be absorbed in counterterms

Extra slides

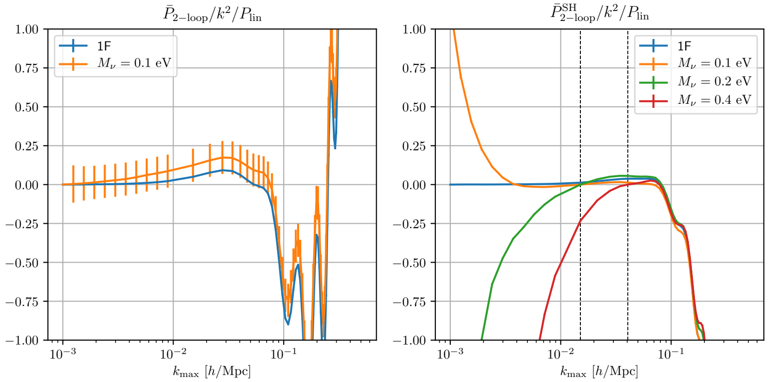
Linear two-fluid evolution



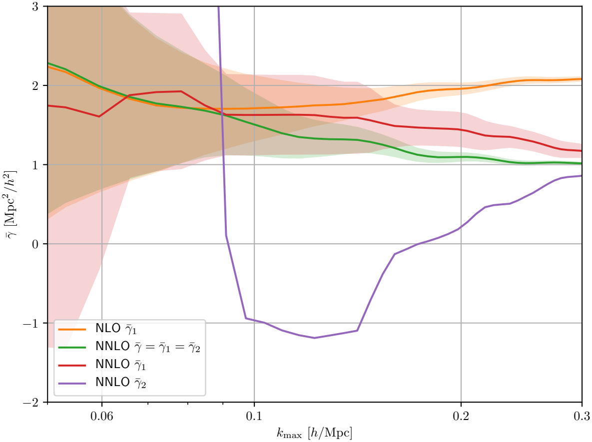
Order/parameter comparison



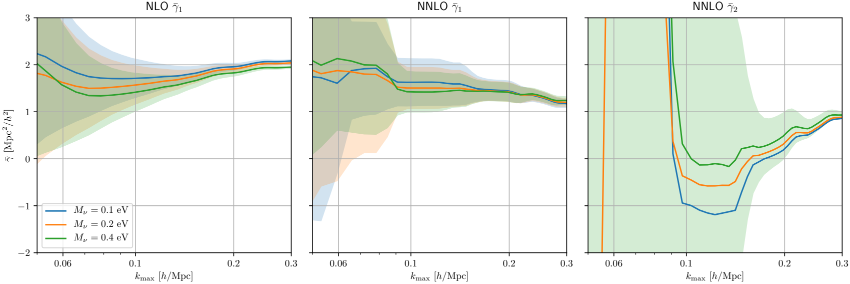
Two-loop subtraction



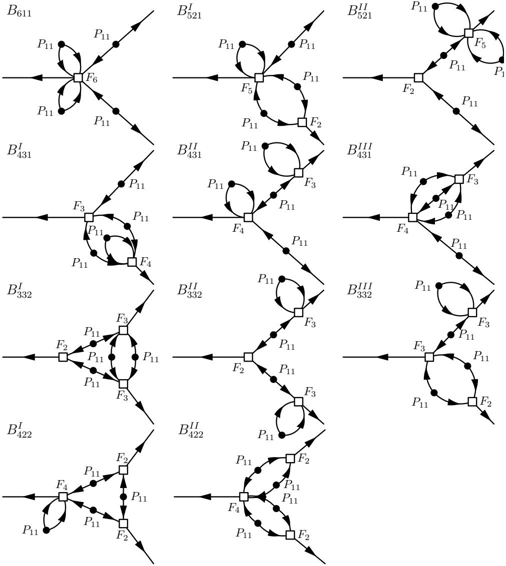
EFT parameters



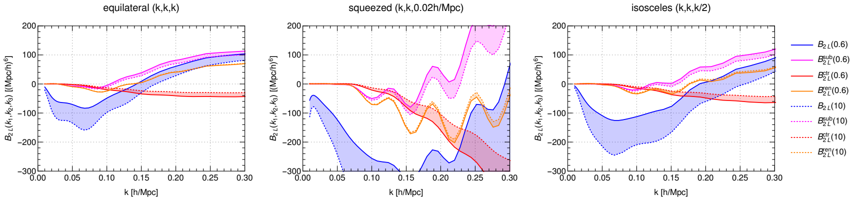
EFT parameters



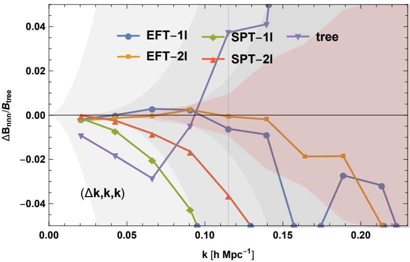
All two-loop diagrams

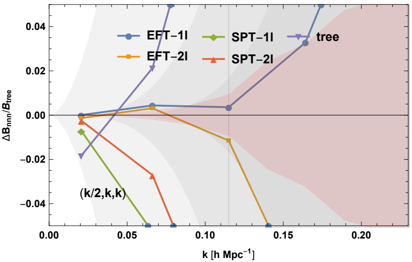


Cutoff dependence

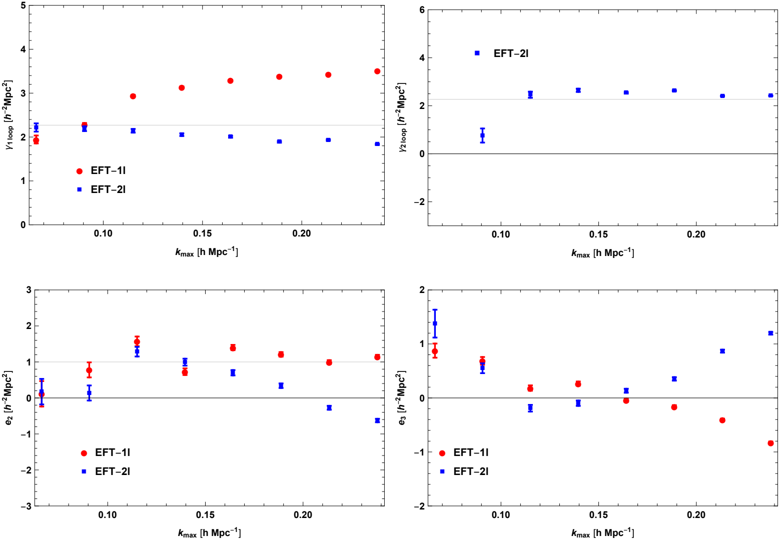


Squeezed and isosceles configurations

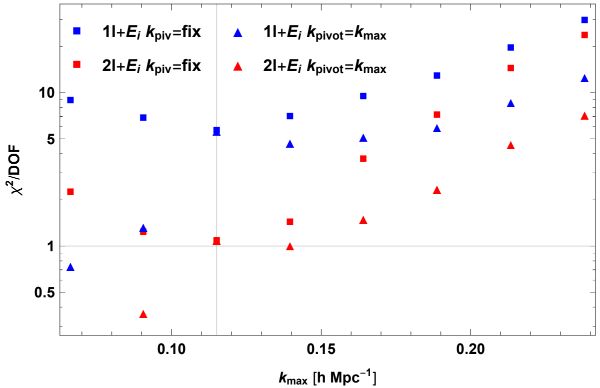




EFT parameters



χ^2 for fixed/running parameter constraints



$(c_s^2 k^2)^2$ terms