NTNU

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TFE4120 Electromagnetics - Crash course

Lecture 7: Summary

Friday August 19th, 9-12 am.

In this crash course we started out from Maxwell's equations for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}, \tag{4}$$

where ρ is the total charge density, and $\mathbf{J}$ the total current density. We represented the bound charges by a polarization $\mathbf{P}$, and the bound currents by a magnetization $\mathbf{M}$, and defined

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \tag{5}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \tag{6}$$

to obtain Maxwell's equations in a medium:

$$\nabla \cdot \mathbf{D} = \rho, \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{8}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{9}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}, \tag{10}$$

where ρ is the *free* charge density, and $\mathbf{J}$ the *free* current density. In linear media we have

$$\mathbf{D} = \epsilon \mathbf{E}, \tag{11}$$

$$\mathbf{B} = \mu \mathbf{H}. \tag{12}$$

The electric permittivity ϵ describes the electric properties, and the magnetic permeability μ describes the magnetic properties of the medium.

For situations with a degree of symmetry, Maxwell's equations were used to find $\mathbf{D}$ from a given ρ (Gauss' law), and $\mathbf{H}$ from a given $\mathbf{J}$ (Ampere's law).

The scalar potential V and vector potential $\mathbf{A}$ were defined such that

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \tag{13}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{14}$$

and were shown to be useful.

Maxwell's equations were shown to be compatible with charge conservation

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \tag{15}$$

From equation (3) Faraday's law of induction was derived:

$$e = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}, \tag{16}$$

where the emf e was defined as

$$e = \oint_C \frac{\mathbf{F}_{\text{tot}}}{q} \cdot \mathrm{d}\mathbf{l}. \tag{17}$$

The sum of all emf in a circuit drives the current through the circult, as described by

$$\sum \text{emf} = RI. \tag{18}$$

Some other relations:

- Poisson's equation $\nabla^2 V = -\frac{\rho}{\epsilon_0}$.
- The Lorentz force acting on a charge q in an electromagnetic field: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.
- The energy density (energy per unit volume) in an electric field: $w_{\text{e}} = \frac{1}{2}\mathbf{D} \cdot \mathbf{E} = \frac{1}{2}\epsilon E^2$.
- Boundary conditions for the electric and magnetic fields:

$$E_{1\text{t}} = E_{2\text{t}}, \tag{19}$$

$$D_{1\text{n}} - D_{2\text{n}} = \rho_S, \tag{20}$$

$$H_{1\text{t}} - H_{2\text{t}} = J_S, \tag{21}$$

$$B_{1\text{n}} = B_{2\text{n}}. \tag{22}$$

- The wave equation for the electric field in a linear, homogeneous, isotropic medium without sources: $\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$, where $c = 1/\sqrt{\epsilon\mu}$ is the speed of light.

Finally the antenna equations

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(\mathbf{r}', t - \frac{R}{c})}{R} \mathrm{d}V', \tag{23}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}', t - \frac{R}{c})\mathrm{d}V'}{R}, \tag{24}$$

were derived, which gives the potentials V and $\mathbf{A}$ from a given charge distribution $\rho(\mathbf{r}, t)$ and current distribution $\mathbf{J}(\mathbf{r}, t)$. During the derivation it was argued that the potentials are not uniquely defined, and may be shiftet through Gauge transformations

$$\mathbf{A}' = \mathbf{A} + \nabla f, \tag{25}$$

$$V' = V - \frac{\partial f}{\partial t}. \tag{26}$$

for an arbitrary scalar function $f(x, y, z; t)$ without altering the physical quantities $\mathbf{E}$ and $\mathbf{B}$. In our derivation we used the Lorentz' Gauge, where $\nabla \cdot \mathbf{A} + \epsilon\mu\frac{\partial V}{\partial t} = 0$.