The Perceived Value of Proving in Learning Engineering Mathematics and its Dependence on Motivation and Study Habits

T. Tossavainen[1] and E. Burtseva[1*]

[1] Luleå University of Technology, Sweden

*Corresponding author. Email: evgeniya.burtseva@ltu.se

Abstract: This study reports on engineering students (N=369) from two Swedish universities and focuses on their perceived value of proving in learning engineering mathematics and some factors that may explain the observed variation in the perceived value. Our findings show that there is no significant difference in the perceived value between female and male students. In general, proving is not highly valued, and students are not confident in their skills in proving, except for proving by mathematical induction. However, students' motivation in mathematics correlates with the perceived value and certain study habits are more regular among those students who appreciate proving as a suitable method for learning mathematics. Examples of such study habits include actively communicating with mathematics course teachers and reading the course textbook both before and after lectures.

1 Introduction

A well-known fact is that the mathematics courses are the most challenging part of an engineering education for many students. This is partly due to the challenges related to the transition from secondary education to studying at university (e.g., Bengmark et al., 2017; Selden, 2012). However, the fact that engineering students' views of mathematics differ to some degree from those of mathematics students may also play a role. For example, compared to ordinary mathematics students, engineering students seem to value the applicability of mathematical concepts more than understanding how they are related to one another (e.g., Maull & Barry, 2000).

DOI: 10.5324/njsteme.v8i2.5070

Proof and proving are content areas in mathematics that are especially challenging for the students of mathematics in general, not only for engineering students. On the other hand, if this fact is acknowledged in the design of mathematics courses, first-year students can significantly improve their skills in proving and their proof-related self-efficacy can increase (Häsä et al., 2023). Not surprisingly, motivation plays a central role in this (ibid).

At the same time, previous research has also shown that epistemological beliefs about the nature of mathematics, especially those concerning exact reasoning, are related to task performance in mathematics (e.g., Tossavainen et al., 2021a). This suggests that engineering students' perceived value of proving is a relevant factor to explain their mathematical behaviour. Specifically, we may hypothesize that acknowledging the value of proving is beneficial also for engineering students and their achievement in mathematics, even though the goals of engineering mathematics education naturally emphasize applying mathematics in engineering tasks.

To address these considerations, we have formulated the following research questions.

1. *How do engineering students perceive their proving skills and the value of proving for their learning in engineering mathematics?*
2. *How does the perceived value of proving correlate with engineering students' motivation in mathematics?*
3. *How is the perceived value of proving related to engineering students' study habits in mathematics?*

The present study can be seen as a sequel to the study by Tossavainen et al. (2022). In their study, the authors focused on investigating how engineering students' mathematical self-concept relates to their study habits and views of mathematics, but they did not specifically examine the relationships between the perceived value of proving and other factors, although their questionnaire included some items on this issue. We were granted a permission to use their data and explore these relationships.

Furthermore, we highlight the novelty of the present study for the following reasons. Firstly, to the best of our knowledge, there are no recent Nordic studies that have addressed proving as a learning method in mathematics, although proof and proving have been considered from other perspectives. Secondly, university students' study habits (not to be confused with approaches to learning, as discussed in Section 2) in mathematics have been rarely investigated in recent years, as outlined in Section 3.

2 Theoretical framework

2.1 Expectancy–value theory

There are several theories that describe an individual's motivation to learn (Eccles & Wigfield, 2002). In particular, the distinction between intrinsic and extrinsic motivation play an important role in theories explaining students' engagement (Sansone & Harackiewicz, 2000). Intrinsic motivation is closely connected to a student's engagement in an activity because of the student's interest and enjoyment in this activity, whereas extrinsic motivation is rooted in instrumental or other similar reasons, such as receiving a reward (Eccles & Wigfield, 2002).

In this study, we consider the relationship between engineering students' views of proving and their motivation in mathematics within the framework of the Expectancy–Value Theory (Eccles et al., 1983; Wigfield & Eccles, 2000; Eccles & Wigfield, 2020). According to this theory, an individual's beliefs about how well she will perform in a task and how much she values that activity can explain her choices, persistence, and performance in the task. Expectancies for an individual's success in a task, defined by Eccles et al. (1983) represent her beliefs about how well she will do in the task. These beliefs reflect her perception of her present capacity, i.e., they are based on her estimation of that how well she can succeed in the task she is facing.

An individual's motivation for carrying a task is a latent construct, but it is reflected in her subjective task values. In the Expectancy–Value Theory, the number of such values has varied slightly over time, but most often and in the recent version (Eccles & Wigfield, 2020), an individual's motivation is operationalized with the aid of four values: interest and enjoyment value, attainment value, utility value, and relative costs.

Interest and enjoyment value represent how much an individual is interested in and enjoys carrying out the task. Clearly, it reflects inner motivation and is sometimes also referred to as *intrinsic* value. Attainment value indicates how important it is for an individual to perform well in the task, and therefore, it is also called *importance* value. Utility value, or usefulness, depicts an individual's understanding of how succeeding in the task aligns with her goals for future plans, such as further education or work. Finally, relative costs describe how much time and effort an individual is willing to invest in the task.

Although the Expectancy–Value Theory is more than 40 years old, there are still aspects that can be improved, developed, and expanded (Eccles & Wigfield, 2020). For example, the relationship between attainment value and personal identity remains partly unclarified. Similarly, when measuring utility value, all possible personal goals should be taken into account, some of which are closely related to intrinsic motivation. However, in many cases, utility value can be connected only to extrinsic motivation, such as a student's need to take math courses to proceed with further studies or for their future job as an engineer (Winberg & Palm, 2021).

A practical challenge in measuring relative costs is that they can be perceived both as positive and negative values. Nevertheless, despite these deficiencies, the Expectancy-Value Theory remains one of the most common theories of achievement choice and has been used in several thousand educational studies.

2.2 Study habits and approaches to learning

In this section, we define study habits and briefly discuss a few factors that affect them. Study habits in this study refer to various observable ways of participating in studying. We specifically focus on a student's engagement in activities such as attending lectures, preparing for lectures in advance, and reviewing lecture content afterward. Additionally, we consider the social aspect of participation in lectures, including how actively students engage in discussions about course content and exercises with teachers or on social media platforms.

Furthermore, our framework includes the dimension of using textbooks and internet resources as part of study habits. For example, questions like 'how actively do you participate in lectures and practicals?' and 'how often do you discuss your solutions with other students and your teacher?' are indicators of study habits. They can be answered using a numeric scale without additional operationalization. In other words, they are relevant variables to categorize students' study habits along dimensions such as 'passive–active' and 'asocial–social'.

It's important to note that study habits should not be confused with approaches to learning, which refer to a student's mental processes before engaging in a learning activity (Marton and Booth, 1997). Approaches to learning are rooted in phenomenography and focus on a learner's personal experience. A central dimension in approaches to learning is the surface approach–deep approach dichotomy. The former involves memorization, while the latter emphasizes understanding the content deeply. There's also a strategic approach aimed at achieving high grades through effective resource organization and time management (Zakariya et al., 2023).

Study habits play a crucial role in an individual's learning process (Nonis & Hudson, 2010). Evidence suggests that these habits are related to factors such as motivation, study environment, and views on the nature of mathematics (Tossavainen et al., 2020). However, in our study, we focus on understanding the connections between students' study habits and their perception of the value of proving as a method of learning mathematics, rather than delving into how study habits are formed in detail.

3 A review of literature

Proving is a significant part of academic mathematics and one of the main ways to learn and to understand what mathematics is (Weber et al., 2020). However, students do not always grasp the importance of proving and they may not see a compelling reason for developing proofs (Zaslavsky et al., 2012). Many research studies indicate that although a majority of engineering students, after some time in university, recognize the relevance of mathematics to their future career, they tend to view mathematics as an applicable tool within their main subject area rather than focusing on the conceptual understanding of mathematics (e.g., Flegg et al., 2012; Raveh et al., 2017).

According to Tossavainen et al. (2021a, 2022), perceiving mathematics solely as a "toolbox" does not enhance learning and can even hinder it. Similar outcomes arise when students view mathematics as a set of problems to be solved through memorization and application of procedures (cf. Zakariya et al., 2023). Conversely, students who prioritize understanding mathematical concepts and are keen on developing their mathematical thinking tend to perform better academically and have a higher self-concept compared to those who view mathematics primarily as a "toolbox" (e.g., Cardella, 2008; Rensaa & Tossavainen, 2022; Tossavainen et al., 2022).

While Flegg et al. (2012) suggest that engineering students should be taught to use mathematics as a tool for real-world problem-solving, they also stress the importance of adopting proper mathematical terminology and notation to ensure mathematical rigor. These examples highlight the importance of including proving and mathematical reasoning in engineering students' mathematical courses for their overall benefit. Moreover, learning to prove does not have to be an overwhelming challenge, even for first-year students. Häsä et al. (2023) studied 267 first-year mathematics students' skills and attitudes towards proving in an introductory course. Although students were somewhat unsure in the beginning of the course, their proof-related self-efficacy increased significantly as the course progressed. Moreover, high motivation at the outset of the course predicted good performance in the final project of the course.

Becoming proficient in proving is not easy. Students may not fully grasp what it means to present a mathematical proof in general (Weber et al., 2020; Zaslavsky et al., 2012). An indication of this is their tendency to replace a general proof with a concrete example (Stavrou, 2014). When Viholainen et al. (2019) investigated 29 Swedish and Finnish third-year students' self-efficacy beliefs about proofs, they found that although students were highly motivated to learn and construct proofs, they were uncertain about their proving skills. Many students doubted their ability to construct acceptable proofs and feared making mistakes. This uncertainty does not always stem from not knowing the validity of the claim to be proved but rather from uncertainty about what is sufficient to establish a proof in a given situation.

The use of proving as a method to teach problem-solving and critical thinking skills has been suggested in numerous previous research studies (e.g., Hemmi, 2006; Zaslavsky et al., 2012). If an introductory course in proving is not provided as a part of study programme, it is obviously challenging to convince engineering students on the importance of proving, especially if students do not perceive the relevance of mathematics to their engineering programme (Flegg et al., 2012; Harris et al., 2015).

One approach to engaging students in proving is to find an appropriate level of accuracy of proving (Recio & Godino, 2001). For example, informal proofs should not be dismissed as completely incorrect, especially at the beginning of university studies. This idea is supported also by Hersh (1993), who emphasizes the importance of recognizing the various goals of a mathematical proof; see also de Villiers (1990), where five different functions of proof in mathematics are described. While "proving as convincing" is more suitable for research in academic mathematics, the primary role of proving in education is to provide an explanation. Weber et al. (2020) considered "proving as convincing" and investigated students' capability of justification in mathematics in the framework of self-efficacy and motivation. Their results show that students appreciate proving more when they understand why they need to prove and when they feel confident in constructing necessary proofs. However, students' motivations for proving can vary qualitatively. Some students are genuinely interested in verifying statements, not only in mathematics but also in real-life scenarios, while others may be motivated solely by the desire to pass the course (Weber et al., 2020).

When it comes to relevant previous research on study habits, we acknowledge Randahl's (2012) study on the role of mathematics textbooks in engineering education. Her findings show that first-year students mostly ignore the textbook. In practice, they use it mainly as a source of tasks. One of the explanations for this could be that concepts introduced in the textbook use formal definitions, which are very difficult for students to understand, and therefore students prefer to study the theoretical content using their lecture notes. On the other hand, students' use of textbooks can develop over the years. Tossavainen et al. (2020) found that older students use the textbook more often than younger ones. Older students are also more open to ask teachers some questions while younger students rather communicate with their peers.

Viholainen et al. (2023) conducted a survey on the relations between university mathematics students' epistemological beliefs of the nature of mathematics, motivational values, and study habits. They observed that certain beliefs and motivational values predict study habits to some extent. For instance, students with a higher attainment value tend to value independent study habits more, taking personal responsibility for their learning by preparing oneself before lectures, among other practices. Similarly, students with a strong interest in mathematics have more often mathematical hobbies that are not directly related to study programmes (e.g., programming). Interestingly, the study also found a statistically significant positive relation between willingness to ask questions during lessons and a view of mathematics that emphasizes its practical applications.

In general, Nordic first-year engineering students show high motivation to study mathematics (Bengmark et al., 2017; Tossavainen et al., 2021b; Rensaa & Tossavainen, 2022). However, student motivation does not always directly correlate with their task performance. Bengmark et al. (2017) discovered that motivation is not a strong predictor of Swedish engineering students' performance at the beginning of their studies, but its influence increases along with self-efficacy by the end of their first year, becoming a more significant predictor of performance.

Similarly, Rensaa and Tossavainen (2022) investigated the link between motivation and task performance among Norwegian first-year engineering students. They found no notable direct connection between motivation and task performance early in students' studies. Conversely, Tossavainen et al. (2021b) observed a positive covariance between motivation and task performance among Swedish first-year engineering students at the start of their studies. However, they also discovered that a strong extrinsic motivation could have a negative impact on performance in mathematical tasks (Tossavainen et al., 2021a).

When it comes to gender differences, both male and female students demonstrate low mathematical self-concept in the domain of proof (cf. Tossavainen et al., 2022). However, gender differences in motivation and self-efficacy values vary from case to case and may depend on cultural differences between countries. For instance, Häsä et al. (2023) reported that male students have significantly higher motivation and self-efficacy than female students. Moreover, the authors hypothesized that such a gap exists also with respect to proving, but they could not detect this in their studies. Contrary to this, Tossavainen et al. (2021a) found that female engineering students performed a little better than male students. In addition, female first-year engineering students have approximately the same level of motivation as male students and higher self-efficacy in certain aspects.

In summary, previous research literature indicates that the appreciation of exact reasoning in mathematics and intrinsic and achievement motivational values are interconnected. Furthermore, some study habits are more prevalent among those students with intrinsic motivation for learning mathematics compared to other students.

4 Method

4.1 Participants and data collection

The data for this study were collected from two Swedish universities which provide engineering education. An online questionnaire was sent out to all active students in the engineering programmes at these universities (altogether 16 programmes). The data consist of 112 responses from female students, 249 responses from male students, and eight responses from students who did not want to reveal their gender. In other words, 369 students participated in this study. The participation was voluntary and no reward was promised. However, all invited students were encouraged to participate by telling them that their responses help these universities to improve the teaching of engineering mathematics. The data collection was conducted following the ethical guidelines of the Swedish Research Council.

4.2 Questionnaire

The questionnaire consists of 64 questions, divided into four sections, surveying the students' background, their mathematical self-concept in general, orientations to mathematics and study habits when studying engineering mathematics courses, and finally, their mathematical self-concept in the domains of discrete mathematics and calculus. The latent variables contained in the questionnaire were operationalized using five-point Likert-scales (1= strongly disagree, 2= disagree, 3 = neutral view, 4 = agree, 5= strongly agree). We interpret them being interval scales. The statements concerning the general mathematical self-concept, study habits, and orientations are revised versions of those statements designed, tested, and used by Felbrich et al. (2008), Viholainen et al. (2019) and Tossavainen et al. (2020, 2021, 2022). In addition to Likert-scales, each respondent was asked to choose from four metaphors which one best corresponds to his/her view of mathematics.

The statements that were used for measuring students' domain-specific mathematical self-concept were designed taking into account two levels:Level I is labelled as task-specific as it focuses on abilities related to (a) interpreting and manipulating mathematical expressions, (b) choosing and performing mathematical calculations and procedures, (c) argumentation and proving. Level II is labelled as course-specific; the items at this level deal with abilities specifically related to courses in (a) calculus of several variables and (b) discrete mathematics. The number of items in this section is altogether 34 and they have been published in the appendix of Tossavainen et al. (2022). Altogether eight scales were designed for measuring the perceived value of proving and proving skills, eight scales for measuring study habits, and four scales for measuring motivational values. The questionnaire was originally in Swedish.

4.3 Analyses

Data were analysed using SPSS version 28. The following procedures were applied: Student's t-tests, One-way ANOVA with Bonferroni's post hoc test, Pearson correlations, Cohen's d for estimating the effect size, and the standard methods for computing descriptive statistic measures such as mean and standard deviation.

4.4 Limitations of study

A clear limitation of the study is that statements in Table 2 do not cover a complete view of students' proving abilities. Similarly, there are more different study habits than those covered in our questionnaire. Nevertheless, our data enable us to answer our research questions at a level that can be useful for designing engineering mathematics courses. For example, if some study habits are typical for those students who value proving, instructors can try to promote these study habits in the design of their courses.

5 Results

Our first research question concerns engineering students' perceived proving skills and how students value proving with respect to their learning in engineering mathematics. The first part of our answer is based on the students' replies to two statements which are shown in Table 1.

***Table 1**. Engineering students' perceived value of proving (1= strongly disagree, 2= disagree, 3 = neutral view, 4 = agree, 5= strongly agree).*

	$\bar{x}_{female}$	s_{female}	$\bar{x}_{male}$	s_{male}	$\bar{x}_{total}$	s_{total}
1a. I learn mathematics by studying mathematical proving. (N=359)	2.47	1.27	2.37	1.13	2.40	1.18
1b. I am interested in mathematical proving. (N=360)	2.21	1.22	2.36	1.31	2.32	1.29

The main result in Table 1 is that, on average, engineering students do not find proving especially useful for their learning in mathematics. In this issue, female and male students are equal; in Student's independent samples t-test, the mean differences between genders are insignificant in both 1a and 1b. However, in our data, there are also students who strongly agreed with at least one of these claims. Now, the Pearson correlation between 1a and 1b is r = 64 (p<.001), which shows that those students who are interested in proving also find proving very useful for their learning – and vice versa. In order to understand the results in Table 1 a little deeper, we investigated how students perceive their ability to prove and argue for their solutions in some typical situations. The summary of their responses is shown in Table 2.

Table 2 shows differences in the perceived proving competence both between some content areas and gender. In 2d, the mean difference is significant (t (357) = 2.01, p<.05) so that male students consider themselves more competent in this task. The mean difference in 2a is, in practice, equally significant in favour of male students since t (359) = 1.01 and p=.06. However, these differences do not indicate a remarkable effect size; a proper conclusion is that female and male students are quite alike when it comes to their perceived proving competence. Indeed, more significant differences are found when mean differences between different statements are concerned. For example, the mean difference between 2b and 2e is highly significant (t (94) = -4.58, p<.001) and the effect size d=.47 is almost medium large. In other words, students estimated that their ability to prove and argue for mathematical claims and results depends on the topic. They are most sure about applying induction, otherwise they estimate their proving skills as rather modest. The lower value of N in 2e and 2f is because these questions were presented only to those students who had taken a course in discrete mathematics.

***Table 2**. Engineering students' views of their proving skills.*

	$\bar{x}_{female}$	s_{female}	$\bar{x}_{male}$	s_{male}	$\bar{x}_{total}$	s_{total}
2a. I am sure that I can prove trigonometric formulas. (N=360)	3.03	1.20	3.29	1.24	3.20	1.24
2b. I can prove whether or not an ordinary differential equation is linear. (N=360)	2.74	1.29	2.81	1.28	2.77	1.29
2c. I can determine which improper integrals are convergent and which are divergent. (N=359)	2.71	1.24	2.73	1.34	2.72	1.31
2d. I am good at motivating mathematically my solution to a real-world problem. (N=359)	2.93	1.13	3.20	1.19	3.11	1.18
2e. I can apply induction in order to prove the summation formulas. (N=94)	3.89	1.45	3.58	1.31	3.65	1.34
2f. I can prove the combination formulas. (N=94)	3.72	1.32	3.64	1.13	3.67	1.16

Table 3 shows the Pearson correlations between the proof-related items. The coefficient between 1a–b and 2a–f varies between 0.23 and 0.64 and is significant in each case. In other words, the more self-confident one is in proving, the more one values proving as a learning method and the more one is interested in proving. Another interesting observation is that the perceived skills to construct proofs in some areas of advanced calculus (2c) and in the context of real-world problems (2d) are only weakly, if at all, related to the perceived proving skills in discrete mathematics (2e–f), although discrete mathematics is the content area where students feel themselves most capable in proving.

***Table 3**. The Pearson correlations between the perceived value of proving and perceived proving skills.*

	1a	1b	2b	2c	2d	2e.	2f.
1a	1	.64***	.33***	.32***	.30***	.41***	.23*
1b		1	.31***	.32***	.28***	.46***	.30**
2a			.43***	.42**	.41***	.40***	.41***
2b			1	.52***	.40***	.37***	.31**
2c				1	.46***	.04	.08
2d.					1	.04	.20
2e						1	.55***
2f							1

* = p<.05, ** = p<.01, *** = p<.001

Our second research question focuses on the relationship between the perceived value of proving students' motivation in mathematics. Table 4 shows which statements were used to measure motivation and summarises our findings. It indicates that intrinsic (3a) and attainment (3c) values are the most significant motivational variables that affect how engineering students perceive the value of proving for their learning in engineering mathematics. These variables also correlate with a remarkable effect size (r=.43). The relationship is especially clear between interest in proving and interest in mathematics in general. Both of these variables represent inner motivation. On the other hand, utility value (3b) is only weakly related to the perceived value of proving. Moreover, the relationship between interest in proving (1b) and getting motivated in mathematics for its usefulness (3b) is negative. In other words, if one appreciates mathematics only for its usefulness for other purposes, it does not increase interest in proving or encourage learning mathematics by proving; if there is an effect, it is merely in the opposite direction.

Table 4*. The Pearson correlations between the perceived value of proving and motivation (N=365–368).*

	1a.	1b.	3a. I really like studying mathematics.	3b. I am motivated to study mathematics mostly because it is useful to my other studies.	3c. I want to succeed as well as possible in my mathematics studies.	3d. I would be ready to suspend my hobbies in order to have enough time to prepare myself for exams in mathematics.
1a	1	.64***	.28***	.07	.24***	.15**
1b		1	.43***	-.15**	.29***	.14**
3a			1	-.13*	.43***	.28***
3b				1	.05	.03
3c					1	.35***
3d						1

* = p<.05, ** = p<.01, *** = p<.001

Concerning our last research question, Table 5 summarizes the Pearson correlation coefficients between the variables measuring the perceived value of proving and investigated study habits. The asterisks indicate the same thing as in Tables 3 and 4.

The overview of Table 5 is that there are only a few correlations with a moderate effect size; most statistically significant correlations are rather weak. The highest correlations are related to 4h and 4d. Both of them represent a student's activity to communicate with teachers. Also, 4e is related to a student's tendency to search for more information and take an active role in her learning. The same can be said for 4b and 4c. Interestingly, the activity to participate in lectures (4a) does not correlate with the perceived value of or interest in learning by proving.

***Table 5**. The Pearson correlations between the perceived value of proving and study habits. (N=363–368)*

	1a	1b
4a. I participate in lectures.	.05	-.03
4b. I familiarize myself with the content of a forthcoming lecture beforehand.	.12*	.09
4c. I study the material discussed during the lectures also afterwards.	.14*	.09
4d. If I do not understand something during the lecture, I ask the lecturer to explain the issue.	.18***	.18***
4e. If I do not understand something, I try to find a solution to my problem by reading the textbook used in the course.	.16**	.17***
4f. If I do not understand something, I try to find a solution to my problem in internet.	-.09	-.03
4g. I discuss the exercises and other content of the course in social media.	.10	.04
4h. I discuss my solutions to the given exercises with the teacher of the course.	.28**	.27***

6 Discussion and conclusions

6.1 Discussion of findings

Table 1 shows that engineering students, on average, are not particularly interested in mathematical proving and they do not find proving useful for learning mathematics. It is noteworthy that there is no gender difference in this issue. These findings align with Harris et al. (2015) and Tossavainen et al. (2021a), who have demonstrated that engineering students often have a “toolbox” view. They prefer concrete examples that demonstrate how to apply mathematical results in the real world instead of seeking a deep understanding of how those results were derived. However, as the variation in the measured variables indicate, there are also several students who are interested in proving and recognize its potential for learning.

Tables 2 and 3 summarize students’ views of their proving skills separately for male and female respondents. We found some gender differences in the perceived proving skills, but these differences are smaller than differences between investigated content areas. It is not surprising that students perceive themselves more competent in proving by induction than in constructing proofs for claims in calculus. A plausible reason for this is that, at least, some students may have encountered examples of the former method already during upper secondary school. Similar findings were reported already twenty years ago by Tossavainen and Luostarinen (2004) and, more recently, by Viholainen et al. (2019). Participants in each mentioned study felt most secure with proving by induction but generally expressed that proving is challenging for them.

Table 4 shows that students who have intrinsic motivation to study mathematics also appreciate proving as a learning method. In light of previous studies (e.g., Cardella, 2008; Tossavainen et al., 2022), this finding is not very surprising. Another question arises: how strong was this relationship expected to be? The highest correlation in Table 4 can be interpreted to represent a rather large effect size. On the other hand, other correlations are weaker, yet several of them are statistically highly significant. Anyway, a comparison of Tables 4 and 5 shows that the relationship between the perceived value of proving and motivation is clearly stronger than the relationship between the perceived value of proving and study habits. This difference may be explained by the fact that proving does not today play so central a role in the participating students' mathematical education that it would require, for example, studying proofs in textbooks before lectures. Moreover, the findings of Weber et al. (2020) and Zaslavsky et al. (2012) may also play some role; if many students do not have a clear idea of what a proof is, then they tend to avoid proving regardless of their study habits.

An interesting detail related to Table 4 is column 3b. It seems to verify the results of Winberg and Palm (2021); if utility value is connected only to extrinsic motivation and students do not perceive a personal interest in proving, or they do not believe that it has high value for their learning, it is logical that the coefficients in this table are negative (though not highly significantly).

Table 5 clearly indicates that a higher perceived value of proving correlates with more active communication with the teachers of the mathematics courses. This behaviour is often associated with high self-efficacy and intrinsic motivation for mathematics, cf. Bengmark et al. (2017), Tossavainen et al. (2021a, 2022), and Viholainen et al. (2023). Our results also demonstrate that students who perceive the value of proving are also more active in reading textbooks. Consequently, they differ from typical students in this aspect as well, cf. Randahl (2012). A likely explanation for this is that they seek a deeper understanding of the concepts and formal definitions provided in the course textbook.

It is somewhat surprising that we did not find any correlation between the activity of participating in lectures and the perceived value of proving. This result is not easily explained based on previous research. One partial explanation is that students, on average, indicated high activity in participating in lectures (mean value is 4.38). However, there is variation also in this variable (standard deviation is 0.92), so this is only a partial explanation. Another notable missing relationship is between the perceived value of proving and use of internet resources. One might expect that students who acknowledge the value of proving were also more active in using those resources. Again, a partial explanation is that, on average, students actively to use those resources (mean value is 4.25).

6.2 Conclusions

Engineering students generally do not appreciate proving as a learning method, which aligns with their primary motivation to study mathematics due to its utility in their other studies – or merely because it is obligatory. Most students expressed that proving is challenging for them and they feel insecure in constructing proofs. However, including more proving in their learning processes in mathematics could help them to develop their mathematical self-concept and, as a result, to perform better in their studies. Therefore, our conclusion related to the first research question is that we strongly encourage engineering educators to find proper ways of including proving in the mathematics curricula for engineering students.

Our second research question focuses on the correlation between students' motivation and their perceived value of proving. Some engineering students already have intrinsic motivation to study mathematics, and according to the results in Table 4, these students appreciate proving as a learning method. Increasing the role of proving in engineering mathematics courses would be especially beneficial for these students as it would support their motivation and help them develop their mathematical thinking. How this can be done in practice is a complicated question. One way to address this issue could be by introducing a special assignment in proving, which could serve as an alternative way to pass (a part of) an engineering mathematics course.

The relationship between study habits and students' perceived value of proving is not as strong as the correlation between students' motivation and their perceived value of proving. Table 5 illustrates that students who have a high perceived value of proving tend to prefer active communication with teachers in mathematics courses and use course textbooks. These findings are interesting, particularly in relation to learning approaches. Hence, we conclude that there is a need for further investigation of the relationship between students' study habits, learning approaches, and their perceived value of proving.

Acknowledgements

We thank Peter Wall and Marcus Sundhäll for their valuable help in collecting the data.

References

Bengmark, S., Thunberg, H., & Winberg, T. M. (2017). Success-factors in transition to university mathematics. *International Journal of Mathematical Education in Science and Technology, 48*(7), 988-1001. https://doi.org/10.1080/0020739X.2017.1310311

Capuno, R., Necesario, R., Etcuban, J. O., Espina, R., Padillo, G., & Manguilimotan, R. (2019). Attitudes, study habits, and academic performance of junior high school students in mathematics. *International Electronic Journal of Mathematics Education, 14*(3), 547-561. https://doi.org/10.29333/iejme/5768

Cardella, M. (2008). Which mathematics should we teach engineering students? An empirically grounded case for a broad notion of mathematical thinking. *Teaching Mathematics and Its Applications, 27*, 150-159. http://dx.doi.org/10.1093/teamat/hrn008

de Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras 24*(24), 17–24.

Eccles, J. S., Adler T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J., & Midgley, C. (1983). Expectancies, values and academic behaviours. In J. T. Spence (Ed.), *Achievement and Achievement Motives*, (pp. 75-146). San Francisco, CA: W. H. Freeman.

Eccles, J. S., & Wigfield, A. (2002). Motivation beliefs, values, and goals. *Annual Review of Psychology, 53*(1), 109-132. https://doi.org/10.1146/annurev.psych.53.100901.135153

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology, 61*, Article 101859. https://doi.org/10.1016/j.cedpsych.2020.101859

Felbrich, A., Müller, C., & Blömeke, S. (2008). Epistemological beliefs concerning the nature of mathematics among teacher educators and teacher education students in mathematics. *ZDM Mathematics Education, 40*, 763-776. https://doi.org/10.1007/S11858-008-0153-5

Flegg, J., Mallet, D., & Lupton, M. (2012). Students' perceptions of the relevance of mathematics in engineering. *International Journal of Mathematical Education in Science and Technology, 43*(6), 717-732. https://doi.org/10.1080/0020739X.2011.644333

Harris, D., Black, L., Hernandez-Martinez, P., Pepin, B., & Williams, J. (2015). Mathematics and its value for engineering students: what are the implications for teaching? *International Journal of Mathematical Education in Science and Technology, Vol. 46*(3), 321-336. https://doi.org/10.1080/0020739X.2014.979893

Hemmi, K. (2006). *Approaching proof in a community of mathematical practice*. [Doctoral thesis, Stockholm University]. DIVA. https://su.diva-portal.org/smash/get/diva2:189608/FULLTEXT01.pdf

Hersh, R. (1993). Proving is convincing and explaining. *Educational Studies in Mathematics, 24,* 389-399. https://doi.org/10.1007/BF01273372

Häsä, J., Westlin, L., & Rämö, J. (2023). Undergraduate students' attitudes towards mathematical proving in an introduction to proof course. *Educational Studies in Mathematics, 114*, 393-415. https://doi.org/10.1007/s10649-023-10239-8

Marton, F., & Booth, S. A. (1997). Learning and awareness. Lawrence Erlbaum.

Maull, W., & Berry, J. (2000). A questionnaire to elicit the mathematical concept images of engineering students. *International Journal of Mathematical Education in Science and Technology, 31*(6), 899-917. https://doi.org/10.1080/00207390050203388

Nonis, S. A., & Hudson, G. I. (2010). Performance of college students: impact of study time and study habits. *Journal of Education for Business, 85*(4), 229-238. https://doi.org/10.1080/08832320903449550

Randahl, M. (2012). First-year engineering students' use of their mathematics textbook: opportunities and constraints. *Mathematics Education Research Journal, 24*(3), 239-256. https://doi.org/10.1007/s13394-012-0040-9

Raveh, I., Trotskovsky, E., & Sabag, N. (2017). Mathematical understanding vs. engineering understanding: engineering students' perceptions. *International Research in Higher Education, 2*(2), 15-24. https://doi.org/10.5430/irhe.v2n2p15

Recio, A. M., & Godino, J. D. (2001). Institutional and personal meanings of mathematical proof. *Educational Studies in Mathematics, 48*(1), 83-99. https://doi.org/10.1023/A:1015553100103

Rensaa, R. J., & Tossavainen, T. (2022). Norwegian freshman engineering students' self-efficacy, motivation, and view of mathematics in light of task performance. *Nordic Journal of STEM Education, 6*(1). https://doi.org/10.5324/njsteme.v4i2.3927

Sansone, C., & Harackiewicz J. M. (Eds.). (2000). *Intrinsic and Extrinsic Motivation: The Search for Optimal Motivation and Performance.* New York: Academic Press.

Selden, H. (2012). Transitions and proof and proving at tertiary level. In G. Hanna & M. de Villiers (Eds.), Proof and proving in mathematics education: The ICMI Study (pp. 391–420). New York: Springer

Stavrou, S. G. (2014). Common Errors and Misconceptions in Mathematical Proving by Education Undergraduates. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*, 1-8.

Tossavainen, T., Gröhn, J., Heikkinen, L., Kaasinen, A., & Viholainen, A. (2020). University mathematics students' study habits and use of learning materials. *LUMAT: International Journal on Math, Science and Technology Education, 8*(1), 252-270. https://doi.org/10.31129/LUMAT.8.1.1317

Tossavainen, T., Rensaa, R. J., Haukkanen, P., Mattila, M., & Johansson, M. (2021a). First-year engineering students' mathematics task performance and its relation to their motivational values and views about mathematics. *European Journal of Engineering Education, 46*(4), 604-617. https://doi.org/10.1080/03043797.2020.1849032

Tossavainen, T., Rensaa, R. J., & Johansson, M. (2021b). Swedish first-year engineering students' views of mathematics, self-efficacy and motivation and their effect on task performance. *International Journal of Mathematical Education in Science and Technology, 51*(1), 23-38. https://doi.org/10.1080/0020739X.2019.1656827

Tossavainen, T., Wall, P., & Sundhäll, M. (2022). Engineering students' mathematical self-concept and its dependence on their study habits and views about mathematics. *International Journal of Engineering Education, 38*(5A), 1354-1365.

Viholainen, A., Tossavainen, T., Viitala, H., & Johansson, M. (2019). University mathematics students' self-efficacy beliefs about proof and proving. *LUMAT: International Journal on Math, Science and Technology Education, 7*(1), 148-164. https://doi.org/10.31129/LUMAT.7.1.406

Viholainen, A., Tossavainen, T., & Gröhn, J. (2023). Epistemological beliefs, motivational values, and study habits among students of tertiary mathematics. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(5), em2269. https://doi.org/10.29333/ejmste/13163

Weber, K., Lew, K., & Mejía-Ramos, J. P. (2020). Using expectancy value theory to account for individuals' mathematical justifications. *Cognition and Instruction, 38*, 27-56. https://doi.org/10.1080/07370008.2019.1636796

Wigfield, A., & Eccles, J. S. (2000). Expectancy-Value Theory of Achievement Motivation. *Contemporary Educational Psychology, 25*(1), 68-81. https://doi.org/10.1006/ceps.1999.1015

Winberg, M., & Palm, T. (2021). Antecedents and Relative Importance of Student Motivation for Science and Mathematics Achievement in TIMSS. *Frontiers in Education, 6*(97), Article-id 575926. https://doi.org/10.3389/feduc.2021.575926

Zakariya , Y. F., Nilsen , H. K., Bjørkestøl, K., & Goodchild, S. (2021). Analysis of relationships between prior knowledge, approaches to learning, and mathematics performance among engineering students. *International Journal of Mathematical Education in Science and Technology*, *54*(6), 1015-1033. doi:10.1080/0020739X.2021.1984596

Zaslavsky, O., Nickerson, S. D., Stylianides, A. J., Kidron, I., & Winicki-Landman, G. (2012). The need for proof and proving: mathematical and pedagogical perspectives. In G. Hanna & M. de Villiers (Eds.), *Proof and Proving in Mathematics Education* (pp. 215-229). New ICMI Study Series, Springer. https://doi.org/10.1007/978-94-007-2129-6_9