Differences in Swedish and Norwegian pre-service teachers' explanations of solutions of linear equations

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Abstract: Solving linear equations is a cornerstone in the learning of algebra. There are two main strategies for solving a linear equation, 'swap sides swap signs' (SSSS) and 'do the same to both sides' (DSBS). While SSSS can often be more efficient for solving equations, DSBS has been shown to better promote the learning of algebra. Thus, the preference of SSSS or DSBS might depend on the purpose of solving equations. Since both approaches are common, mathematics teachers, and thus also pre-service teachers (PSTs), must be familiar with both SSSS and DSBS. This study draws on data from 161 Swedish and 146 Norwegian PSTs. They were given a correct but short and unannotated solution to the linear equation $x + 5 = 4x - 1$. The PSTs were invited to explain the provided solution for a fictive friend. Of the Norwegian PSTs, 2/3 explained the additive steps in the solution by SSSS, while only 1/3 of the Swedish PSTs applied SSSS. Consequently, DSBS was more frequent among the Swedish PSTs regarding the additive steps. However, in the final, multiplicative step, 3/4 of the Norwegian PSTs invoked DSBS. On the contrary, among the Swedish PSTs, the proportion applying DSBS for the multiplicative step decreased, and it was common to provide an incomplete explanation of the final operation. We also analysed how mathematics textbooks for secondary school presented how to solve linear equations. In Sweden, all textbooks utilised DSBS through the whole solution for all years in secondary school. This also applied for Norwegian textbooks for the first two years of lower secondary school. However, in last year of lower secondary school, they changed their approach and promoted an SSSS strategy in additive steps, while DSBS was still suggested for multiplicative steps. This might explain the differences between the two countries regarding the PSTs' preferences of solution strategies. We suggest that these results can be useful for teacher education, since increased awareness of PSTs' pre-knowledge is beneficial to support their development of teaching linear equations.

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1 Introduction

Algebra, including linear equations, has been referred to as the 'gatekeeper' to further studies (Blanton et al., 2015; Ladson-Billings, 1997; Moses & Cobb Jr, 2001), since it is an essential prerequisite for other parts of the subject of mathematics (Cai et al., 2010). Furthermore, learning to solve linear equations plays an important role for the development of algebraic thinking – specifically, working with structures and relationships rather than numbers (Blanton et al., 2015). Hence, it is not surprising that the topic of linear equations is a part of school mathematics all over the world (Andrews & Sayers, 2012; 2012; Houang et al., 2004).

Algebraic competence, as measured by international evaluations like Trends In Mathematics and Science Study (TIMSS), differs between countries. In Sweden and Norway, students have significantly lower results in algebra compared to other mathematical topics (Bråting et al., 2019; Pedersen, 2015), while other countries, such as Singapore, have had high results in algebra over a long period of time (Lessani et al., 2014). Thus, it seems like the cultural context matters for the development of algebraic competence.

We find it reasonable to assume that pre-service teachers' (PSTs') algebraic competence, including how to solve linear equations, is also influenced by the cultural context. Most likely, their knowledge of linear equations originates primarily from their school mathematics. Swedish and Norwegian PSTs have met algebra in compulsory and upper secondary school (Norwegian Directorate for Education and Training, 2013; Swedish National Agency for Education, 2011, 2012). According to current curricula, they will be expected to teach algebra, including linear equations, to their students (Norwegian Directorate for Education and Training, 2020; Swedish National Agency for Education, 2022). Hence, it is interesting to investigate how PSTs' prior education has affected their knowledge of this topic when they commence teacher education. Since textbooks have a large influence on the teaching and learning of mathematics in Sweden and Norway (Johansson, 2006; Larson, 2014; Kongelf, 2015), it is also relevant to explore how the topic of linear equations is presented in secondary school textbooks. The study will benefit by a comparison between the two countries, since cross-cultural comparisons have proven to facilitate a deeper understanding of one's own cultural context (Hiebert, 2003, p. 3).

This leads us to the main purpose of this paper, which is to compare how PSTs in Sweden and Norway explain the solution of a linear equation, and to explore how potential differences in their explanations can be connected to differences in secondary school textbooks. We particularly focus on the parts of the explanations connected to the operational steps of the solution. The study is framed by two research questions:

> How do Swedish and Norwegian pre-service teachers explain how to solve a linear equation, and what similarities and differences can be found between the two countries?
>
> What connections can be identified between Swedish and Norwegian pre-service teachers' explanations of how to solve linear equations and how the topic is presented in secondary school textbooks?

2 Background

Basic linear equations with a single unknown can roughly be divided into equations with the unknown on one side of the equals sign , for example $4x - 7 = 12$, and equations with unknown terms on both sides, for example $x + 5 = 4x - 1$ (Andrews, 2020). There exist several suggestions of how to label these two categories, for example "arithmetical" and "non-arithmetical" (Filloy & Rojano, 1989), "procedural" and "structural" (Kieran, 1992), and "arithmetic" and "algebraic" (Andrews & Sayers, 2012). All these suggestions expose that the former category can be solved by inverse arithmetic operations or informal methods as 'cover the unknown', while to solve the latter it is "necessary to operate on what is represented" (Filloy & Rojano, 1989, p. 20). This indicates that equations with the unknown on both sides of the equals sign tend to require a more structured solution strategy than equations with the unknown on one side only.

A well-known part of the procedure to solve an equation with the unknown on both sides is to collect all unknowns on one side of the equals sign. This can be accomplished by two different strategies, 'do the same on both sides' (DSBS) and 'swap sides swap signs' (SSSS) (Larson, 2024; de Lima & Tall, 2008; Tall, 2017). Utilising DSBS would be, e.g., to subtract x from both sides while using SSSS would involve a move of x to the other side of the equals sign and a change of the sign to $-x$. The reasoning behind DSBS is often referred to as the balance property, which requires a relational view of the equals sign (Otten et al., 2020).

Two important factors that affect the choice of DSBS or SSSS are cultural influences and the purpose of solving linear equations. A cultural aspect is that the DSBS strategy tends to be promoted in western countries, while SSSS often is preferred in many Asian countries (Larson, 2024; Ngu et al., 2015). Regarding benefits and drawbacks, there are several arguments for and against the two strategies. DSBS elucidates the balance property of an equation (Ngu et al., 2015) and supports the learning of algebraic structures (Otten et al., 2020; Wasserman, 2014). However, it is regarded to be less efficient than SSSS, which is thought not to burden the working memory as much as DSBS (Ngu et al., 2015). In line with this, students employing SSSS have been found to outperform those who apply DSBS when solving equations (Ngu & Phan, 2016). On the other hand, SSSS can be seen as a 'black box' that hides the structure and relations, thus not promoting algebraic thinking (de Lima & Tall, 2008; Star & Seifert, 2006), which may be an obstacle for further mathematics studies (Capraro & Joffrion, 2006). This suggests that if the purpose of the equation solving is to find the solution, SSSS might be more efficient, while DSBS might be favourable if the aim is to improve algebraic understanding.

Moreover, there are two possibilities to construe the transfer of terms over the equals sign in SSSS. The literal meaning of swapping sides and swapping signs naturally relates to the additive operations, because to swap signs of a number implies you change plus to minus or vice versa. It is difficult to in the same way connect the signs of multiplication and division to a number, and that swapping signs then means to change multiplication to division, or the opposite. However, another interpretation of the SSSS approach is that swapping sides yields the inverse operation (Ngu et al., 2015). Adopting this view facilitates a solution by moving over the equals sign also for multiplicative operations, and hence to utilise SSSS for the whole solving procedure.

The aspects of DSBS and SSSS have been explored in previous studies on pre-service teachers (PSTs) in Cyprus (Andrews & Xenofontos, 2017; Xenofontos & Andrews, 2017), Sweden (Andrews, 2020) and Norway (Larson, 2024), and on upper secondary school students in Sweden (Andrews & Öhman, 2017). This paper draws on data from the same task as in these studies, where a correct solution of the linear equation $x + 5 = 4x - 1$ was presented, though with no explanations of the steps of the solution. The participants were invited to explain this solution to a friend. Results from previous studies utilising this task show that among PSTs in Cyprus all relevant explanations invoked an SSSS approach for the operational steps of the solution (Andrews & Xenofontos, 2017). This contrasts with Swedish PSTs and upper secondary school students, who both showed a preference for DSBS (Andrews, 2020; Andrews & Öhman, 2017). Interestingly, Norwegian PSTs demonstrated a mix of these approaches, favouring SSSS for the additive steps and DSBS for the multiplicative step (Larson, 2024). Even though no definite conclusions can be drawn from a few studies, the contrast between the Cypriot and the Swedish participants' preferences for DSBS or SSSS suggests a cultural influence on teaching (cf. Andrews & Larson, 2017).

Solution strategies like SSSS and DSBS might be encouraged in mathematics textbooks as well. Since it is fair to assume that the participants in this study on several occasions had met linear equations in school, it is also likely that secondary school mathematics has had a significant impact on their understanding of the topic. Textbooks play a central role in mathematics teaching in schools in Sweden (e.g., Johansson, 2006; Larson, 2014) and Norway (e.g., Kongelf, 2015), and thus textbooks probably affect students' understanding, either when students themselves use the book or because the teaching is adjusted to how the topic is presented in the textbook. Hence, possible similarities or differences between Swedish and Norwegian PSTs' understanding of linear equations might be connected to the textbooks used.

This paper partly builds on the same data as reported by Andrews (2020) and Larson (2024). While Andrews' (2020) study focused on Swedish students only, and Larson's (2024) study focused on Norwegian students only, this paper contributes to the research field by conducting a comparison between the countries. Furthermore, this paper contributes by providing examples that disclose common solution strategies from each country. In addition, to enable a potential explanation of the similarities and differences between the countries, this paper also draws on new data from a textbook analysis. Including PSTs' examples and a textbook analysis also confirms that this paper is a development of the paper presented at the MNT-conference 2021 (Larson & Larsson, 2021).

3 Method

The first part of the results of this paper draws on data collected from PSTs, and the second part on an analysis of mathematics textbooks for lower and upper secondary school.

The participants for the first part were PSTs at one university in Sweden and one in Norway, following their first course in mathematics during teacher education. The data collections were made before the topic of equations was treated in the courses. In Sweden and Norway, compulsory school starts at the age of 6 and lasts for ten years. In

Sweden this is grades F–9 and in Norway 1–10. The 161 Swedish participants followed the teacher education for primary schoolteachers (F–3 or 4–6), while the 146 Norwegian participants followed a programme for primary ($N = 83$) or lower secondary ($N = 63$) school, that is grades 1–7 or 5–10 respectively. Mathematics was mandatory for all Swedish PSTs and for the Norwegian 1–7 group, while the Norwegian PSTs for grades 5–10 had voluntarily chosen mathematics as one of the subjects in their education. Data were collected anonymously, and the participation was voluntary.

The participants were, on the lower half of a sheet of paper, invited to explain to a friend how the equation $x + 5 = 4x - 1$ was solved. On the upper half the solution of the equation was presented in four lines:

$x + 5 = 4x - 1$; $5 = 3x - 1$; $6 = 3x$; $2 = x$

The equation was chosen for the following reasons. This is an equation that does not require complicated calculations, which might disturb the focus on the solution strategy (Larson, 2024); it has the unknown on both sides of the equals sign, which requires operations on the variable terms (Filloy & Rojano, 1989); and finally it ends up with the unknown on the right-hand side of the equals sign, which challenges the relational aspect of the equals sign (Kieran, 1981; Stephens et al., 2013).

The scripts were analysed by a set of 16 low-inference codes, of which eleven codes deal with the operational steps of the solution. Of these, five are connected to SSSS and five to DSBS. The last operational code is “Unspecified operation on the coefficient”, which will be clarified in the results section. The five remaining codes capture other aspects of the explanation of the solution and were not focused on in this paper (see Larson (2024) for a description of those).

Here, we describe the five codes for SSSS. The claim ‘you can move any term to the other side of the equals sign if you change the sign’ would be the code “SSSS general”, since it neither refers to the equation under scrutiny nor a specific operation. ‘You can move any term to the other side of the equals sign if you change + to − or vice versa’ is “SSSS general additive”. ‘If you move x to the right-hand side, it becomes $-x$’ is “SSSS particular additive”, since it refers to the equation under scrutiny. ‘You can move a factor over the equals sign if you change multiplication to division’ is “SSSS general multiplicative”, while ‘if you move 3 in $6 = 3x$ to the left side it becomes division by 3’ is “SSSS particular multiplicative”. A remark is that ‘swapping signs’ naturally refers to changing + to − or vice versa, at least in the Scandinavian languages. This implies the two codes for SSSS multiplicative will be rarely used, at least if the operation is interpreted as swapping signs. However, if SSSS is interpreted as applying the inverse operation (Ngu et al., 2015), SSSS multiplicative is likely to be more frequently invoked. The corresponding five codes for DSBS work analogously to the five codes for SSSS. For each script, every code can be registered together with any other code. If a code appears more than one time in a script, it is still registered just once.

The first author coded the Norwegian scripts, while the second author coded the Swedish. This was followed by a crosscheck coding of 20 scripts from each stack chosen by random, in which we concluded Cohen’s kappa to be 0.89 (cf. Larson, 2024), implying the consistency between the authors’ coding was excellent.

To explore how the topic of linear equations is treated in mathematics textbooks, we scrutinised textbooks used at secondary school in Sweden and Norway. Although students meet equations in primary school, we chose to focus on the three years of lower secondary school and the first mathematics course at upper secondary, since the solving procedure for linear equations is formalised and established mainly during these years. The sample of books consisted in Sweden of four textbook series for lower and upper secondary school respectively. For Norway, the corresponding numbers were three for each level. There exist no official statistics of which textbooks are used in schools, but after getting information from publishers, combined with our own and colleagues' experiences, we claim that we hereby have included the book series used by a vast majority of the schools during the 2010s. We utilised the same set of 16 codes to analyse 'theory text', 'examples', 'rule boxes' and 'summaries' in each book.

In the results section, we provide some examples of how the analytic tool was operationalised. There, we also present the frequencies of codes connected to the operational steps of the solution. We decided to omit the codes 'SSSS general multiplicative' and 'SSSS particular multiplicative', which were not present in the textbooks and uncommon in the PSTs' scripts. We also merge the two codes 'SSSS general' and 'SSSS general additive' since there were few instances of the former and the distinction between them was insignificant in these data. Thus, we include eight codes, all connected to the operational steps of the solution, which is in line with our purpose of this paper.

4 Results

In this section, we first provide three typical examples of how the solution of the equation was explained. Then, we present results from the data obtained by the coding of the scripts and highlight the differences between the countries. Finally, we explore if and how these differences can be explained by textbooks used in secondary school.

4.1 The pre-service teachers' approaches to the solving procedure

We start by presenting three participants' scripts. These scripts demonstrate how the analytic tool was operationalised as well as highlight some typical explanations from the two countries regarding the operational steps in the solution. To make the analyses easier to follow, we here utilise the full set of 16 codes, although codes not connected to the operational steps are omitted later, since they do not contribute to the purpose of this paper. Remember, even if a code was identified repeatedly in a script, it was registered only once. The first script comes from a Swedish PST, who explained both the additive and the multiplicative steps by doing the same to both sides (DSBS). This was more frequent among Swedish PSTs (see Table 4 below).

Figure 1. *Participant invoking DSBS for both additive and multiplicative steps***.**

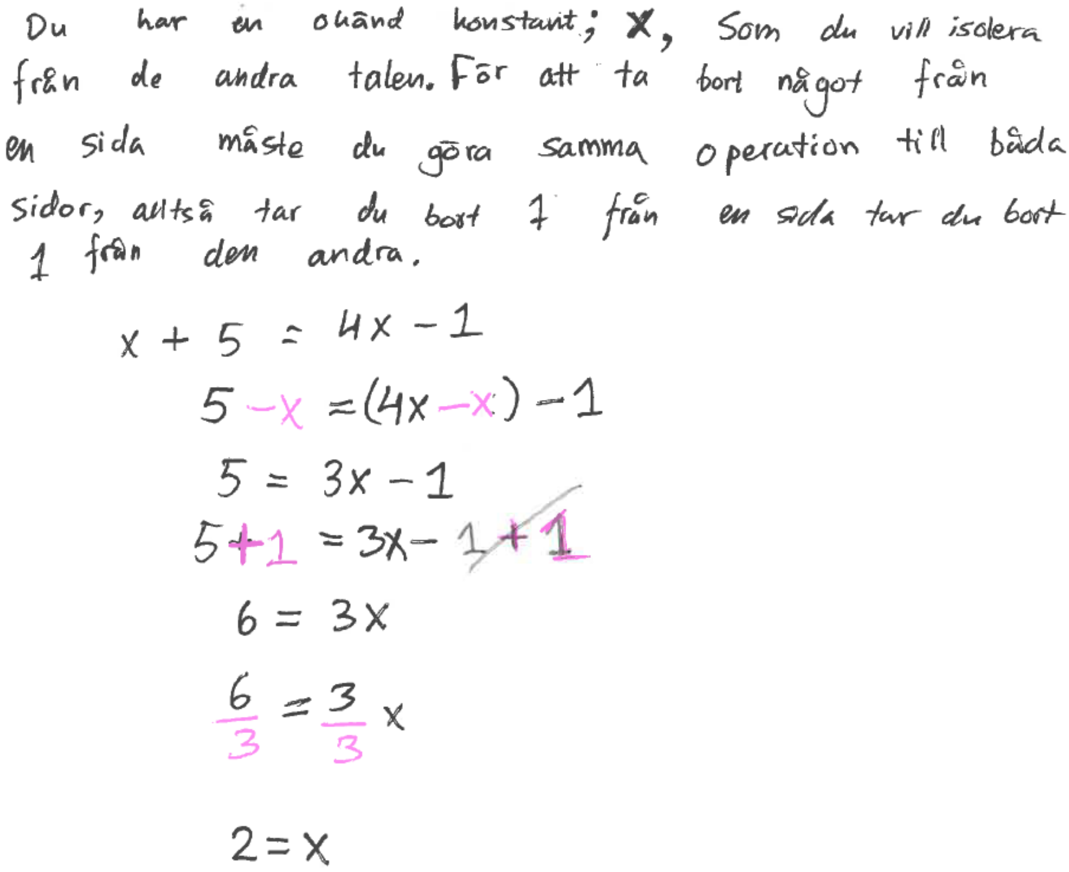

Table 1. *The codes present in the script presented in Figure 1..*

Swedish	English	Assigned code
Du har en okänd konstant; *x*	You have an unknown constant; *x*	Discusses the nature of *x* (*x* is an unknown)
som du vill isolera från de andra talen	which you want to isolate from the other numbers	Conceptual objective (we want to find the value of *x*)
För att ta bort något från en sida måste du göra samma operation till båda sidor	To take something away from one side you must do the same operation to both sides	DSBS general additive (subtracts from both sides without reference to the equation under scrutiny)
alltså tar du bort 1 från en sida tar du bort 1 från den andra	that is, if you take away 1 from one side you take away 1 from the other	DSBS particular additive (subtracts from both sides in the equation under scrutiny)
$x + 5 = 4x - 1$	$x + 5 = 4x - 1$	No code (just copy of the equation)
$5 - x = (4x - x) - 1$	$5 - x = (4x - x) - 1$	DSBS particular additive (subtracts *x* from both sides)
$5 = 3x - 1$	$5 = 3x - 1$	No code
$5 + 1 = 3x - 1 + 1$	$5 + 1 = 3x - 1 + 1$	DSBS particular additive (adds 1 to both sides)
$6 = 3x$	$6 = 3x$	No code
$6/3 = 3x/3$	$6/3 = 3x/3$	DSBS particular multiplicative (divides both sides by 3)
$2 = x$	$2 = x$	No code

The second script also comes from a Swedish PST and demonstrates the code "Unspecified operation on the coefficient". Even though this script is rather clear in the additive steps, it is not clear that you must divide both sides of the equation by 3 in the last, multiplicative step. This was rather frequent among Swedish PSTs, but rare among the Norwegian.

Figure 2. *Participant invoking DSBS for the additive steps but is unclear in the multiplicative step.*

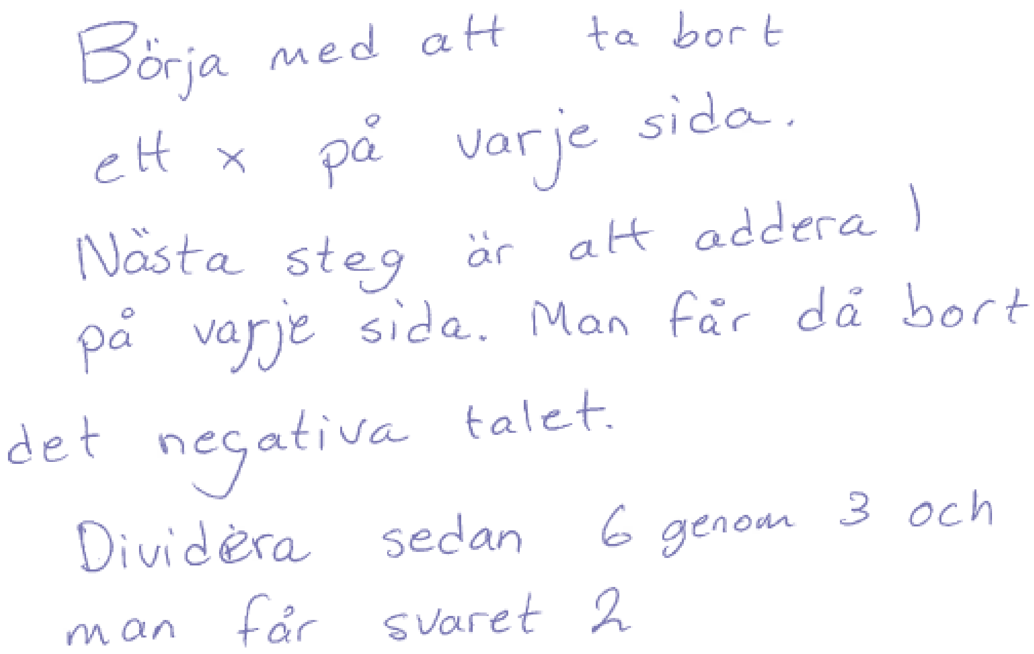

Table 2. *The codes present in the script presented in Figure 2..*

Swedish	**English**	**Assigned code**
Börja med att ta bort ett *x* på varje sida.	Start with taking away one *x* on each side.	DSBS particular additive (subtracts *x* on both sides)
Nästa steg är att addera 1 på varje sida. Man får då bort det negativa talet.	Next step is to add 1 on each side. Then, you get rid of the negative number.	DSBS particular additive (adds 1 to both sides)
Dividera sedan 6 genom 3 och man får svaret 2.	Then, divide 6 by 3 and you get the answer 2.	Unspecified operation on the coefficient (the coefficient 3 of 3*x* is present but it is not expressed that you divide on both sides or explained why you divide by 3)

The last script comes from a Norwegian PST. It shows a way of explaining the operative steps that was common among the Norwegian participants. The additive steps were explained by SSSS, while the multiplicative step was explained by DSBS.

Figure 3. *Participant invoking SSSS for the additive steps, but DSBS for the multiplicative step.*

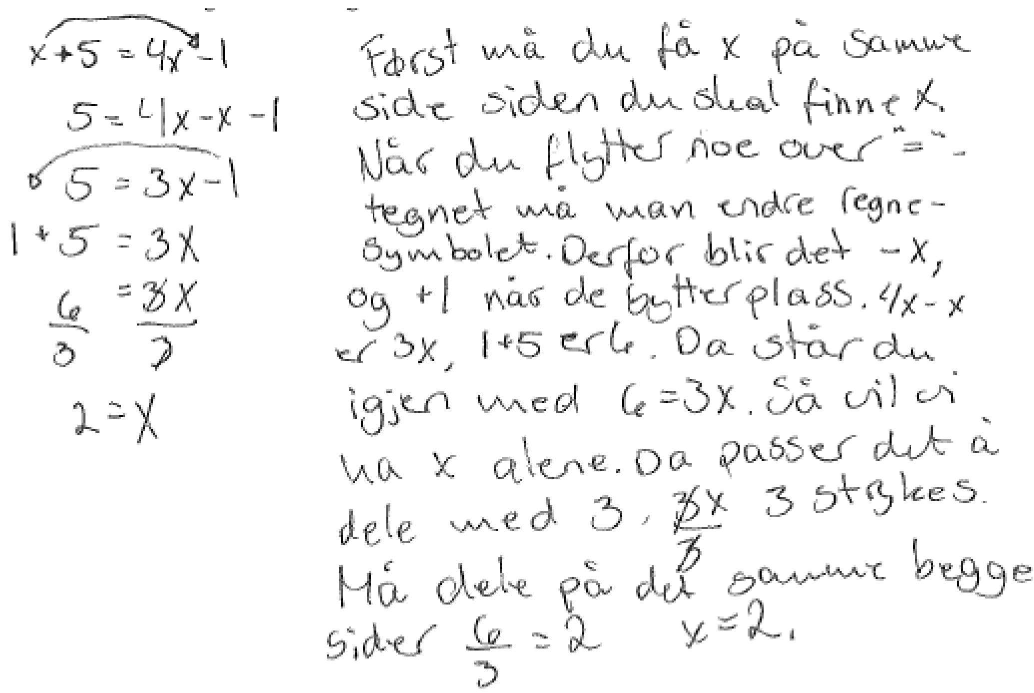

Table 3. *The codes present in the script presented in Figure 3..*

Norwegian	English	Assigned code
Først må du få *x* på samme side	First you must get *x* on the same side	Procedural objective (we want to separate unknowns from knowns)
siden du skal finne *x*.	because you shall find *x*.	Conceptual objective (we want to find the value of *x*)
Når du flytter noe over "="-tegnet må man endre regnesymbolet.	When you move something over the "="-sign you must change the calculation sign.	SSSS general (moving terms and changing signs without reference to the task)
Derfor blir det $-x$ og +1 når de bytter plass.	Hence, it becomes $-x$ and +1 when they change places.	SSSS particular additive (moving and changing signs of a term in the current equation)
$4x - x = 3x$, $1 + 5 = 6$. Da står du igjen med $6 = 3x$.	$4x - x = 3x$, $1 + 5 = 6$. Then you have $6 = 3x$ left.	No code (just the results of operations)
Så vil vi ha *x* alene.	Then, we want to have *x* alone.	Procedural objective (we want to isolate the unknown)
Da passer det å dele med 3. 3 strykes. Må dele på det samme begge sider.	Then, it is appropriate to divide by 3. 3 is cancelled. Have to divide by the same both sides.	DSBS particular multiplicative (dividing both sides of the equation in the task)

As illustrated in the scripts above, some clear differences between the countries were identified in the PSTs' utilisation of SSSS or DSBS to explain the operational steps of the solution. Table 4 presents the eight codes concerning operational steps in the explanations. Since the Norwegian data included PSTs for both primary and secondary school, we also split them into two groups, henceforth called Nor 1–7 and Nor 5–10, when they are discussed separately. The motive for splitting is that the Swedish participants all were PSTs for primary school, and thus a separate comparison to Nor 1–7 is relevant. However, Nor 5–10 intersects with the Swedish group (Norwegian 5–7 is the same as Swedish 4–6), which entails that a comparison to Nor 5–10 is relevant.

Table 4. *Relative frequencies of codes identified in the PSTs' explanations of the operational steps of the solution.*

		SSSS general (merged)	SSSS particular additive	DSBS general	DSBS general additive	DSBS particular additive	DSBS general multiplicative	DSBS particular multiplicative	Unspecified operation on the coefficient
Sweden (*N* = 161)	(%)	11.2	35.4	13.0	8.1	44.7	1.9	34.8	31.7
Norway (*N* = 146)	(%)	52.1	64.4	16.4	2.7	21.2	15.8	76.0	7.5
Nor 1–7 (*N* = 83)	(%)	62.7	71.1	12.0	2.4	12.0	18.1	69.9	9.6
Nor 5–10 (*N* = 63)	(%)	38.1	55.6	22.2	3.2	33.3	12.7	84.1	4.8

Regarding the additive steps, the SSSS approach dominated among the Norwegian PSTs, especially for Nor 1–7. The Swedish proportion invoking 'SSSS additive' was clearly lower. This also applies to the code 'SSSS general (merged)'. Hence, it was not surprising that the Swedish PSTs were much more frequent in utilising DSBS to explain the additive steps of the solution than the Norwegian PSTs were, particularly compared to Nor 1–7.

Considering these results about the Norwegian PSTs' preference for SSSS in the additive steps, it is noteworthy that a clear majority of the Norwegians chose to explain the final, multiplicative step by DSBS. That means many Norwegian PSTs changed their approach from SSSS to DSBS as the solution went from additive to multiplicative operations. Among the Swedish PSTs, the use of DSBS instead decreased for the multiplicative step. That did not mean they changed to an SSSS approach. Instead, almost the same number of PSTs gave an explanation coded as "Unspecified operation on the coefficient" as the number that invoked DSBS. Furthermore, the 'unspecified operation' code was almost absent among the Norwegian PSTs. In the following subsection, we will connect these results to textbooks used in secondary schools in both countries.

4.2 How linear equations are introduced in textbooks

Essentially, the structure of the content in the Swedish and Norwegian textbooks were similar. A typical subsubsection contained an introduction to the topic in regular body text, sometimes a summary in a 'rule box', solved examples, and finally included some tasks to solve. We identified 221 units to analyse in the Swedish books and 111 units in the Norwegian, distributed as in Table 5 below. A section introducing theory in body text was counted as one unit independent of its length, if it was not interrupted by solved examples or tasks. If a solved example included sub-tasks (a, b, c, etc.), each sub-task was counted as one unit.

Table 5. *Relative frequencies of units about explanations of solutions of linear equations in the textbooks.*

		Theory text (in body text)	Solved examples	Other
Sweden (*N* = 221)	(%)	17.6	71.9	10.4
Norway (*N* = 111)	(%)	22.5	51.4	26.1

Table 5 shows the proportion of solved examples was larger in Swedish textbooks, while the occurrence of 'other' kinds of presentations was much larger in the Norwegian books. Two examples of this category are summaries and rule boxes.

In Table 6 and Table 7, we demonstrate how the set of codes was utilised in the textbook analysis. The first example is a solution of an equation.

Table 6. *Parts of a solved example (Szabo et al., 2011, p. 70, our translation). Red text in the book is shown by bold text here.*

Solution provided in the textbook		**Assigned code**
$4(2x - 3) = 7 - (5 - 6x)$	Multiply and get rid of the brackets	No code
$8x - 12 = 7 - 5 + 6x$	Simplify the right-hand side	No code
$8x - 12 \mathbf{- 6x} = 2 + 6x \mathbf{- 6x}$	We want to collect the *x*-terms on one side	DSBS particular additive; Procedural objective
$2x - 12 = 2$		No code
$2x - 12 \mathbf{+ 12} = 2 \mathbf{+ 12}$		DSBS particular additive
$2x = 14$		No code
$x = 7$		No code

In this example, there are no comments indicating that DSBS is invoked. Nevertheless, the solution shows when DSBS is applied. This would have been coded as DSBS even if the text had not been highlighted (red). However, in the multiplicative step it is not shown how $2x = 14$ is transformed to $x = 7$, hence no code was assigned.

The second example is a rule box from a Norwegian textbook. Rule boxes like this occurred in several Norwegian textbooks, which we will come back to in the last paragraph of the results section.

Table 7. *Rule box (Heir et al., 2014, p. 69, our translation)*

Rule box in the textbook	Assigned code
1. We can move a term over to the other side of the equals sign, if we at the same time switch the sign.	SSSS general
2. We can multiply or divide with the same number on both sides of the equals sign.	DSBS general

The textbook analysis revealed some interesting connections between the PSTs' explanations and the textbooks in the respective country. Since the grades of lower secondary school are 7–9 in Sweden and 8–10 in Norway, we henceforth call them L1, L2 and L3, and the first course at upper secondary school is called U1. In the Norwegian textbooks, there was a clear change between L2 and L3 in how the operational steps of the solution were presented. Thus, the Norwegian data are also split in two groups in Table 8. Because no corresponding change was identified in the Swedish books, we present these data as one group. Table 8 includes the same codes as Table 4.

Table 8. *Relative frequencies of codes identified in the textbooks' explanations of the operational steps of the solution.*

		SSSS general (merged)	SSSS particular additive	DSBS general	DSBS general additive	DSBS particular additive	DSBS general multiplicative	DSBS particular multiplicative	Unspecified operation on the coefficient
Sweden (*N* = 221)	(%)	0.0	4.5	5.9	3.2	63.3	1.4	67.4	5.9
Norway (*N* = 111)	(%)	14.4	26.1	6.3	12.6	22.5	21.6	55.0	0.0
Nor L1/L2 (*N* = 47)	(%)	0.0	0.0	10.6	17.0	44.7	17.0	46.8	0.0
Nor L3/U1 (*N* = 64)	(%)	25.0	45.3	3.1	9.4	6.3	25.0	60.9	0.0

Table 8 shows the code "SSSS general" appeared only in Norwegian books and that "SSSS particular additive" was clearly more frequent in Norwegian books. Noteworthy is that these codes were not present in Norwegian books for L1 and L2, but appeared in L3 and U1. In fact, only one Norwegian series for lower secondary school treated linear equations thoroughly after L1. This series introduced an SSSS-approach for the additive operations in L3, after only invoking DSBS in L1 and L2. The SSSS approach to the additive operations then continued in all Norwegian book series for U1. Consequently, the relative frequencies of DSBS decreased from L1/L2 to L3/U1.

The "DSBS particular additive" was clearly more frequent in Swedish books, which, with one exception, consistently invoked DSBS for the additive operations. The only textbook series that presented SSSS briefly mentioned that 'it seems like we have moved the number and changed signs'. This approach was also invoked in some solved examples, but it was not expressed as a general rule, and it was clearly stated that this occurred because the same number was added to both sides. In a corresponding way, this series provided some solutions where the multiplicative steps were coded as "unspecified operation on the coefficient". This was introduced as a quicker way to solve e.g., $2x = 8$, with just concluding $x = 8/2$. In the first examples, it was emphasised that "we have divided both sides by 2", but this was omitted later. Apart from these few exceptions, the DSBS aspect was clearly dominating for both additive and multiplicative steps in Swedish textbooks.

Although the relative frequencies for "DSBS particular multiplicative" were rather equal in the two countries, "DSBS general multiplicative" was clearly more frequent in Norwegian textbooks. One explanation is that it is common that Norwegian textbooks for L3 and U1 include rule boxes as in Table 7, where point 2 expresses the general aspect of DSBS multiplicative. The numbers in Table 8 are likely to explain the PSTs' differences in explaining the solution of a linear equation, displayed in Table 4. A fair conclusion is that the appearance of rule boxes influenced the Norwegian PSTs' preferences for SSSS in the additive steps and DSBS in the multiplicative steps.

5 Discussion

The research questions of this paper were 'How do Swedish and Norwegian pre-service teachers explain how to solve a linear equation, and what similarities and differences can be found between the two countries?' and 'What connections can be identified between Swedish and Norwegian pre-service teachers' explanations of how to solve linear equations and how the topic is presented in secondary school textbooks?'. The first research question was studied through PSTs from only one university in each country, which implies some caution must be exercised regarding generalisability of the findings. Nevertheless, our results highlight some important similarities and differences between PSTs' views of linear equations, which justifies the significance of this paper.

The results show that the cultural differences between Swedish and Norwegian PSTs' explanations of how to solve a linear equation are distinct regarding the SSSS- or DSBS-approach for additive steps. That such cultural differences can exist is consistent with a number of studies regarding solution processes of linear equations (Andrews, 2020; Andrews & Xenofontos, 2017; Larson, 2024; Ngu et al., 2015; Xenofontos & Andrews, 2017). Consequently, we argue that not only large-scale cultural differences between areas, such as Western Europe and Southeast Asia, can exist, but also two neighbouring countries with similar school systems can differ regarding specific topics. These differences became even more visible through the lens of cross-cultural comparison, in line with suggestions from Hiebert (2003).

By the textbook analysis, we, in this paper, also provide a possible explanation to the preferences of respective approach. The SSSS-approach for additive steps is literarily stated as an explicit rule in the Norwegian textbooks for the last year of lower secondary school and the first course at upper secondary school (notice the difference between Nor L1/L2 and Nor L3/U1 in Table 8. Furthermore, these rule boxes stated the DSBS approach for multiplicative steps. Since a majority of the Norwegian PSTs preferred to invoke SSSS for the additive steps and an even larger majority invoked DSBS for the multiplicative step Table 4, this implies the textbooks had an important influence on the PSTs' solution process. We did not find such strong connections between Swedish textbooks and the PSTs' preferences for DSBS over SSSS. A possible explanation is that text highlighted in rule boxes might have a stronger influence on students' choices of methods than regular theory text and solved examples.

It is fair to conclude that cultural influences play a role in a country's mathematics education (cf. Andrews & Larson, 2017). School textbook authors often come from the country in question and have been formed by its traditions. Hence, the textbooks often provide solution methods similar to what the authors learnt, and the impact of textbooks in schools (Johansson, 2006; Kongelf, 2015; Larson, 2014) makes new students adopt the same approach. In turn, these students are the future's authors. These traditions are also likely to continue through the teachers' understanding of the concept, perceived from their own education. Thus, it is plausible that stressing DSBS might be more frequent among Swedish upper secondary teachers than among their Norwegian colleagues. As Andrews and Öhman (2017) described, their interviewees claimed their teacher did not allow them to use SSSS. One upper secondary student's statement, that when applying SSSS you will not understand what you are doing, is possibly connected to the Swedish teaching culture. This does not mean providing SSSS is wrong, but to strengthen algebraic reasoning, SSSS should be backed up by DSBS (cf. de Lima & Tall, 2008; Otten et al., 2020; Star & Seifert, 2006; Wasserman, 2014).

Choosing one method for the additive step does not imply you do not understand the other. It might be just that you prefer one of the methods, possibly by cultural reasons (cf. Andrews & Larson, 2017). In this study, the participants gave a written reply, with no possibilities for follow-up questions. It is not unlikely that several PSTs would have given an alternative method, or a justification of the SSSS additive method, if prompted. A few participants included such a justification anyway, by explaining that the consequence of DSBS is that the term appears on the other side with the opposite sign. Although just a few PSTs included this justification, it is plausible that more PSTs are aware of it. Correspondingly, the Swedish PSTs who provided explanations coded as "Unspecified operation on the coefficient" might give DSBS as an alternative, if prompted. However, from our data, it is not possible to draw further conclusions about that.

Considering the approaches of SSSS and DSBS themselves, arguments for SSSS is mostly consistent with the product of equation solving. For example, Ngu et al. (2015) suggest that it is less error-prone and more efficient. However, today there are several technical tools aiding solutions of equations, that might decrease the need of efficient solution processes, which would weaken arguments for SSSS. Arguments for DSBS, on the other hand, rather focus on development of algebraic reasoning, including a relational understanding of the equals sign (see Blanton et al., 2015; Otten et al., 2020; Wasserman, 2014). Since the PSTs are going to teach algebra, including linear equations, to their students in primary and lower secondary school, developing an DSBS-approach is essential for their transition from students to teachers. Hence, teacher educators need to design the education towards a sustainable understanding of algebra suitable for PSTs' future work. This includes discussing the different properties of equations that are made more or less discernible by different approaches. The balance model for a relational understanding of the equals sign is one such property that has been suggested as a fundamental representation to promote algebraic reasoning (Otten et al., 2020). The balance model is compatible to DSBS, both concerning additive and multiplicative steps among positive numbers. To design teacher education courses that support PSTs' development of algebraic understanding, we claim that knowledge of their earlier experiences and preferences is critical. Therefore, we suggest that both Swedish and Norwegian teacher educators are supported by the results of this paper, for example that DSBS needs more attention to multiplicative steps in Sweden and for additive steps in Norway.

This paper highlights some important differences in Swedish and Norwegian PSTs' ways of explaining the solution steps of a linear equation. We also identified a clear connection between the PSTs' explanations and how the solution was presented in the respective country's textbooks. Considering the PSTs' future work as compulsory school teachers and the pivotal role of algebra as 'gatekeeper' for future studies (Blanton et al., 2015; Cai et al., 2010; Ladson-Billings, 1997; Moses & Cobb Jr, 2001), it is essential that PSTs develop a broad understanding of linear equations. However, since cultural influences risk restricting this broad understanding of linear equations, it is important that PSTs become aware of this issue. Thus, it is important that teacher education emphasises multiple views of linear equations, as well as highlights the importance of cultural influence.

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6 Appendix

The secondary school mathematics textbook series analysed:
Faktor (Cappelen Damm). Hjardar, Pedersen.
Matemagisk (Aschehoug). Lerø Kongsnes, Wallace.
Maximum (Gyldendal). Normann Tofteberg, Tangen, Stedøy-Johansen, Alseth.
Matematikk (Aschehoug). Toft Norderhaug, Melander Vie, Heir, Engeseth, Moe, Haug.
Sigma (Gyldendal). Øgrinn, Bakken, Pettersen, Skrindo, Thorstensen, Thorstensen.
Sinus (Cappelen Damm). Oldervoll, Orskaug, Vaaje, Svorstøl, Hals.
Matematik XYZ (Liber). Undvall, Johnson, Welén.
Matte Direkt (Sanoma Utbildning). Carlsson, Hake, Lundkvist.
Prio (Sanoma Utbildning). Cederqvist, Larsson, Gustafsson, Szabo.
Vektor (Natur & Kultur). Domert, Bjermo, Lundin Jakobsson, Madej, Öberg, Amberntsson, Ristamäki, Söderberg.
Exponent (Gleerups). Gennow, Gustafson, Silborn.
Matematik Origo (Sanoma Utbildning). Szabo, Larson, Viklund, Dufåker, Marklund.
Matematik 5000 (Natur & Kultur). Alfredsson, Bråting, Heikne, Erixon.
M-serien (Liber). Holmström, Smedhamre, Sjunnesson.