Interactive modelling and simulation using blending techniques

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Abstract

Blending splines are constructions where local geometry is blended together by a blending function to create global geometry. The different basis functions has different properties, which can be suited for different problems. Different basis functions, properties and the implementation of physical properties with a focus towards utilizing parts of blending splines in an isogeometric analysis (IGA) context constitutes the basis for the current study. This paper gives an introduction to a newly started PhD project regarding the use of blending splines in modeling and simulation environments.

1 Introduction

Through years of research, numerical simulations have become a key technology in both scientific and industry applications. Simulations of real-life large scale problems where models become increasingly more complex, is still a problem, despite the growing capabilities by modern computers. Finite element methods (FEM) are the most used and well known methods for solving partial differential equations (PDEs). FEM is used to find an approximate solution for PDEs and can be applied to a wide variety of problems. There have been, and still are, research topics on both local refinement and adaptivity for finite elements. There are however some bottlenecks which accounts for a lot of the computational cost. One such example is mesh generation. Constructing meshes of good quality in the sense of geometry for complex constructions both for computations as well as visualization can be a difficult and expensive task. IGA was introduced by Hughes et.al in [1] where an integration between Computer Aided Design (CAD) and Finite Element Analysis (FEA) was introduced. In general this integration means using the same functions for constructing the geometry as for performing analysis and there is no need for any mesh generation. B-Splines is the industry standard constructions used for this purpose. NURBS is incorporated into almost all commercial software and industry standards used in CAD. NURBS is one of many different types of

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spline constructions, such as T-Splines [12], PHT-Splines [3, 4], Locally refined B-Splines (LR B-Splines) [5, 7] and Hierarchical B-Splines [13, 6]. Most of the different research environments which conducts research either using or developing spline constructions, often require specific properties and new spline constructions are developed from scratch or extended from existing constructions.

2 Blending splines

Blending splines is a spline construction where local geometry is blended together to create global geometry. Blending splines was introduced through the Expo-Rational B-Splines (ERBS) [8, 9] and the Generalized Expo-Rational B-Splines (GERBS) [2] constructions. The research was first communicated at the international conference "Mathematical Methods for Curves and Surfaces" in Tromsø, Norway in 2004, and are an ongoing research effort at Narvik University College (NUC). With the generalization, non expo-rational basis functions were introduced, such as the Euler Beta Function B-Spline (BFBS) basis. BFBS are polynomial splines and, as such, can be represented as linear combinations of polynomial B-Splines. Other examples of generalized basis functions used with blending splines is Logistic ERBS (LERBS) and as well as the original scalable subset. The different constructions vary for example in the way they are constructed. Some require heavy integration and are more complex in implementation other does not require any integration, but consist of explicitly and exactly computable functions. This can make the different constructions suitable for different aspects and applications of interactive geometric modeling and IGA.

3 Geometric constructions and analysis

The study into blending splines at NUC has created a platform for R&D through projects on PhD, MSc and BSc level as well as our research group. The general methodology for this PhD consist of mathematical modelling and analysis of differential geometry, where experimental programming and implementation of algorithms are used to test both constructions and analysis. Furthermore, though both blending splines, example areas and finite element analysis (FEA) have already been investigated separately, a recent topic, IGA, is a new approach trying to integrate FEA and the conventional mathematical models and this new approach entails an ocean of unexplored territory. The techniques in commercial use today are techniques developed through years of research, and since IGA is recenty established, only by benchmarking every step in both analysis as well as generating geometries it can be improved, changed or modified in order to be reliable and ready for commercial use. This article serves as an introduction to a newly started PhD research project. The study focuses on combining blending splines and its applicability to IGA, where increased editing possibilities and interactivity through blending techniques, hierarchical models and spline methods is the motivation. Using the blending spline construction have some interesting properties which could be applicable in an IGA context. The blending construction is using a blending function to blend together local geometry and the blending function can be adjusted by changing the intrinsic parameters. So far there is no research concerning the shape of the blending function with regards to analysis on the geometry. Other properties which require additional examination are the vanishing derivatives at

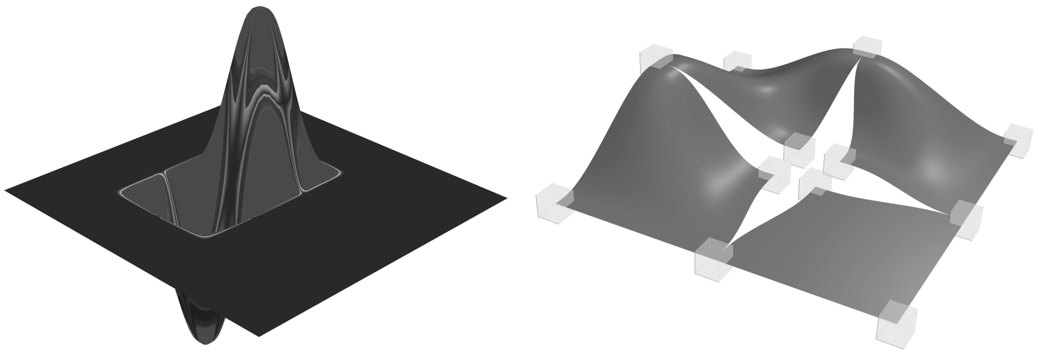

Figure 1: Both figures are blending spline constructions, namely Generalized Expo-Rational B-Splines (GERBS for short). The blending construction on the left consist of 5x5 local patches where two of these are moved in order to show the gaussian curvature. The surface on the right is a 3x3 blending construction which is split in the center and thus consist of more local patches. The latter construction is a surface which is geometrically discontinuous, while still being mathematically C^k-smooth. Both figures are gathered from the master thesis [11].

the knots together with the minimal support and hermite interpolation property. Interesting topics and applications which require further investigation is amongst other, the use of different type of properties, i.e material and geometrical properties such as stiffness, hardness, conductivity and more in combination with the local geometry. There is in general no restrictions for the local geometry regarding the type(scalars, vector, polynomials, trigonometric functions), but the only requirement is that it can be re-parametrized. The vanishing derivatives for the construction; zero derivatives at the knots, is a possible singularity. Singularities of such kind should be explored in combination with fluid-structure interactions, especially when investigating the interface between objects and different phases such as fluids and solids. The focus as of now, before exploring the effects of adjusting properties, studying the zero derivatives, minimal support and hermite interpolation, is with regards to geometrical constructions and initially this has been constructions to represent geometry, and in [10] we presented a construction where the global surface was created by using Coons patches as the local geometry in the blending construction. We investigated the shape of the local patches and splitting the global surface by knot insertion which creates a discontinuous geometry, but still having a C^k-smooth mathematical representation. The main topic for the current study is utilizing the blending construction in IGA and exploring possibilities for this integration with regards to performance compared to common techniques and also applicability with respect to solving partial differential equations, both analysing and geometric constructions.

4 Concluding remarks

Using blending splines as a basis for IGA is a recent topic with a lot of application areas and the main focus ahead for my PhD is deformation and interaction between objects, especially when dealing with terminal ballistics. Deformation by movable and deformable structures with a fluid flow either inside or surrounding these structures are known as fluid-structure interaction (FSI). FSI problems are multidisciplinary and most model equations dealing with FSI have no analytical solution and numerical simulations may be employed. Using the spline basis for both geometry and analysis could constitute a big difference for both the utilization of modern hardware (GPU clusters) and computationally expensive tasks. Using blending splines to solve partial differential equations of hyperbolic, parabolic and mixtype is relevant for problems within multiphysics. Another aspect of utilizing blending splines for both computations and analysis is that the interaction with the geometry during a simulation does not require the simulation to start again and there is no need for mesh generation. This has some restrictions regarding both complex problems as well as models, because of computational costs. Blending splines with multi-layered refinement, where the local geometry for each layer consist of different materials could be used for composite materials or sandwich constructions. Application areas for such constructions could be of use in military perspective as armor protection, projectile, gas and fluid behaviour as well as medicinal use in organ modeling and simulation and many more.

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