Optimal Design of Wind Farm Collector System using a Novel Steiner Spanning Tree

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Abstract. This poster paper addresses the issue of optimal cable layout design of a wind farm collector system. The objective is to minimize the total cable length which is the sum of length of all the branches of the collector system tree. We propose a graph theoretic solution and propose improvements to cater to the constraints of wind farm collector system. The solution is based on minimum spanning tree algorithm to connect the wind turbines with the minimal trenching length. Thereafter, to further minimize the trenching length, we propose a novel Steiner tree algorithm which finds a minimal path to connect the wind turbines. Our algorithm solves the problem in polynomial time with time complexity O(V^2), since finding a minimum Steiner spanning tree is a NP-hard problem.

Keywords: Wind farm collector system design · Cost optimization · Minimum spanning trees · Steiner spanning trees.

1 Introduction

A wind farm is a collection of wind turbines whose power should be collected and distributed through the existing electrical network. The wind turbines generate the power from the wind energy. The power produced by the wind turbines should be transmitted to a specific substation. The substation then further transmits the collected power to the Smart Grid. The substation transfers the power generated from the turbines to the Smart Grid. The electrical collector system, which consists of cables, transformers, junction boxes, switch-gear, and other electrical equipment's collects the power generated by the turbine units(distributed across the geographical area of the wind farm) to the substation. The topology of the layout of the collector system is decided based on the factors such as climatic conditions, wind conditions, the position of wind turbines, geography, landowner requirements, aviation restrictions and, construction restrictions[1]. We assume that the cable costs are known. There are several works on the optimal design of the collector system [4, 5] to minimize the cable length, but our work solves it in the minimum time as compared to the others.

2 Mathematical Considerations

The location of wind turbines is considered as the vertices while, the costs of cables is regarded as the weights of the edges connecting the wind turbines, forming a graph. The problem is modelled as, *Given an undirected graph* $G = (V, E)$*, in which each edge* $(u, v) \in E$*, has a weight* $w(u, v)$ *specifying the cost to connect* u *and* v*. The problem is finding an acyclic subset* $T \subseteq E$ *that connects all of the vertices so that the total weight of the edges is minimal.* $w(T) = \Sigma w(u, v), \forall w(u, v) \in T, w(T)$ *is minimal.* A Steiner spanning tree (SST) is the most optimized spanning tree. We introduce additional vertices called Steiner vertices and, with their help we connect the vertices in the minimal edge weight. Fig. 1 illustrates the Steiner spanning tree for a graph with 5 vertices. Yellow colored vertices are the vertices which should be connected with total minimum weight. When we introduce an additional vertex represented by red color then, these 5 terminal vertices can be connected in the minimum possible weight. We have wind turbine vertices which should be connected mandatorily and, are called *Terminal* vertices. For a large scale wind farm collector system optimization, the euclidean Steiner tree problem is NP-hard, and hence an optimal solution can not be found using a polynomial time algorithm. Therefore, we define the minimum Steiner tree problem as, *Given a connected undirected graph* $G = (V, E)$*, the weight of each edge* $w(E)$ *is an integer, and a set of vertices* N *is known as the terminals,* $N \subseteq V$*, then the problem is to find a subtree* $T \subseteq E$ *spanning all the terminal vertices and the total weight of* T *is minimal.*

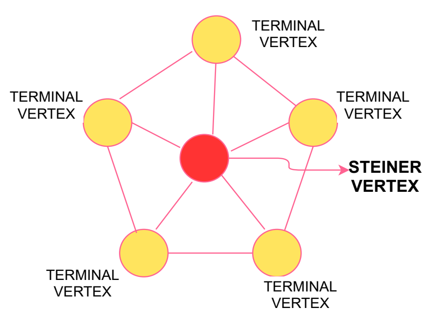

Fig. 1. Five vertices connected in the minimum possible weight using Steiner vertex.

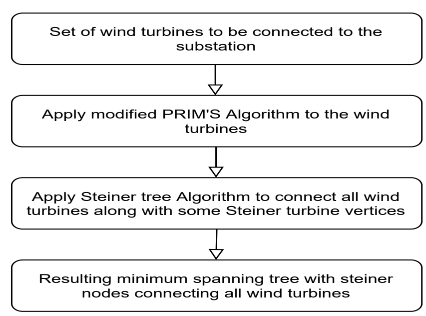

Fig. 2. Flow diagram of the proposed approach.

Our proposed algorithm achieves this goal in polynomial time as explained in section 3.

3 Methodology

We propose a two step approach to find the minimum cable length together with steiner vertices. First, Prim's minimum spanning tree algorithm is modified to

```
PRIM_Modified (G.V, G.E, T)
Input: G.V – set of vertices
       G.E – set of edges
       T – set of terminal vertices
Output: MST - Minimum Spanning Tree
--------------------------------------------
% initialization
sort E in nondecreasing order by weight w
extract e1 = (u1, v1) from E with minimal w
A = {u1, v1}
eliminate e1 from E
  V = G.V – A
% the main loop
while V ≠ Ø or (V U T) ≠ Ø
  | extract e = (ui, vi) ϵ E with minimal w
  |   such that ui ϵ A and vi ϵ (V – A)
  | A = A U { vi }
  | V = V – vi
  | eliminate e from E
```

Fig. 3. Modified Prim's Algorithm.

```
Steiner_Spanning_Tree (G.V, G.E, T)
Input: G.V – set of vertices
       G.E – set of edges
       T – set of terminal vertices
Output: SST – Steiner Spanning Tree
--------------------------------------------
% step-1: set T(1) as the source terminal vertex
set the source terminal vertex T(1) = tS
% step-2: run Prim's modified algorithm
MST = Prim_modified(G.V, G.E, T)
% step-3: the main loop
while NOT(exist(vi a Steiner_vertex with 1-degree))
  | Eliminate vi from MST
% the result: assign the remaining MST as the
% Steiner spanning tree
  SST = MST
```

Fig. 4. Steiner Spanning tree algorithm.

generate the least total length of the wind farm. It traverses all the possible terminal vertices and the Steiner vertices and finds a minimum length spanning tree. Second, Steiner spanning tree algorithm is proposed where all the Steiner vertices having one degree of connection with the terminal vertices are deleted from the tree iteratively, until, there are no more Steiner vertices with one degree of connection. The modified Prim's algorithm is show in Figure 3, and the Minimum steiner spanning tree algorithm is shown in figure 4. The modified Prims's algorithm take run time, $\mathcal{O}(|V|^2 + |V|log|V|)$ [2] for dense graphs. For the Steiner spanning tree algorithm, there will be a maximum of $|V| - |N|$ number of non-terminal vertices. Thus, this step takes $O(|V| - |N|)$ time. Hence, the run time of the algorithm is $O(1) + O(|V|^2) + O(|V| - |N|) = O(V^2)$.

4 Experimental Results

The algorithm is implemented in the MATLAB platform as a function of the tool known as the General-Purpose Petri Net Simulator (GPenSIM). GPenSIM is developed by the second author of the paper, and is freely available [3]. As shown in the Fig. 5, the wind farm collector system optimization problem is defined as a graph problem. The initial connection and cable length is as shown in the figure 5. The wind turbines which should be connected are shown in green colored vertices and, are also called Terminal vertices. Blue color vertex is the substation. The cable lengths are shown as edge length. The next step is to apply the modified Prim's algorithm that connects all the turbines with minimum possible cable length. After applying the modified Prim's algorithm, we get the minimum spanning tree with vertices D-A, A-B, B-E, B-I, B-C, C-F, I-H and, H-G. The total weight comes out to be 49. Thereafter, we apply the Steiner tree algorithm. It identifies two Steiner vertices with one-degree of connection G

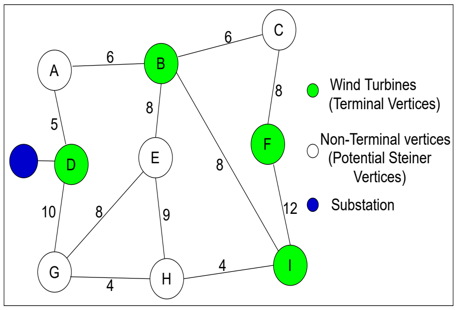

Fig. 5. Wind turbine collector system optimization problem.

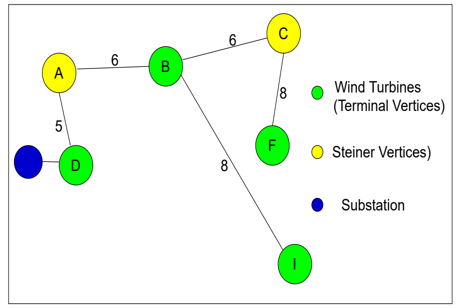

Fig. 6. Resulting Steiner Spanning tree after running the proposed algorithm with minimum cable length.

and E, so, these are deleted. In the next iteration, it identifies a vertex H with one degree of connection. And so, vertex H is deleted. The algorithm stops as there are no more Steiner vertices with one degree of connection. The resulting minimum spanning Steiner tree has vertices D-A, A-B, B-I, B-C and, C-F. The minimum weight calculated comes out to be 33. The resulting Steiner tree is shown in Fig. 6.

5 Conclusion

In this paper, we proposed a novel Steiner tree algorithm for generating a design of wind farm collector system with minimum cable length. Our algorithm reduces the construction cost and the maintenance cost. Also, our algorithm solves a NP-hard minimum Steiner tree problem, in polynomial time.

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