Evaluation of Three Frost Heave Models

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ABSTRACT: Simulating the water and ice content of the ground, with an emphasis on the capacity to bear traffic, is important for planning the use of existing roads. Of most interest is how weak the ground becomes during thaw and the length of time during which the ground is weakened. If a model accurately predicts frost heave and the locations of the ice lenses that form based on ground surface temperatures, it then is a good starting point for understanding the location of the moisture during thaw. For this reason, we evaluated and compared three frost heave models based on the results of laboratory soil freezing tests. Of the three frost heave models evaluated, the PC-Heave model and the Porosity Rate model accurately simulated the frost heave. FROST was less accurate than the other two models.

KEY WORDS: Frost heave, soil freezing, frost heave models.

1 INTRODUCTION

Predicting the state of the ground, including the water content and the phase of the water, with an emphasis on its capacity to bear traffic, is important for designing roads and planning logistics. Frozen ground carries significantly higher traffic loads than thawing ground. Thaw weakening occurs after soils experience frost heave, and of most interest is how weak the ground becomes during thaw and the length of time during which the ground is weakened. If frost heave is accurately modeled, the location of water upon commencement of thawing is known. Over the past few decades, several numerical models have been developed that predict frost depth and heave in one dimension based on empirical observations, constitutive relations, thermodynamic principles, or some combination thereof (e.g., Kujala 1997). This paper provides an overview and evaluation of three frost heave models. The models were first calibrated and then evaluated based on laboratory freezing tests.

2 FROST HEAVE

For soil that freezes from the surface downward, ice lenses usually form some distance above the freezing front (where the water first freezes) at a lower temperature than the temperature at which pore water first freezes. The zone between the freezing front and the bottom of the warmest ice lens is called the frozen fringe, and it is saturated, or nearly so, with ice and water. The thickness of the frozen fringe varies, depending on the temperature gradient, the overburden pressure, and the soil. For a saturated, frozen soil, the ice is a continuous body from the frozen fringe up through the lens, and it moves by regelation (continuous ice–water phase change) accompanied by local liquid flow (Miller 1978). There is a pressure difference between the ice and the water in a soil pore, as indicated by the curved interface between the phases (Loch 1978). The total pore pressure is the weighted sum of the pressures of ice and water. When the pore pressure equals the overburden pressure, the effective stress becomes zero, and the soil particles separate and ice lenses form (in a process similar to liquefaction).

Frost heave models usually assume local (microscopic) equilibrium between the phases of water in the soil pores. The generalized Clapeyron equation describes the equilibrium thermal dependency of soil ice and pore water in the pores (e.g., Loch 1978):

$$\frac{P_i}{\rho_i} - \frac{P_w}{\rho_w} = -L_f\left(\frac{\Delta T}{T_o}\right) \tag{1}$$

where ρ_i and ρ_w are the densities of ice and water (Mg m^{-3}), P_i is the ice pressure (Pa), P_w is the total soil water potential (Pa), L_f is the latent heat of fusion of water per unit mass (333.519×10^6 J Mg^{-1}), T_o is the freezing point of pure water (273.15 K), and ΔT is the freezing-point depression (K).

3 PREVIOUS EVALUATIONS OF FROST HEAVE MODELS

Kujala (1997) classified the physical basis of frost heave models into empirical, semi-empirical, hydrodynamic, rigid ice, and thermomechanical, which generally follows the progression of model development—i.e., empirical models are the earliest and thermomechanical models are the most recent. All models he discussed are one dimensional in terms of heat transfer and mass balance.

Empirical and semi-empirical: Empirical models are based on field and laboratory observations of heaving soil. Semi-empirical refers to empirical models that have some basis in the physics of frost heave. The “Segregation Potential,” or *SP*, is semi-empirical (Konrad and Morgenstern 1980).

Hydrodynamic: Hydrodynamic models indicate that frost heave occurs when the pore ice content exceeds a percentage of the total pore content. FROST (Guymon et al. 1993) is the best-known hydrodynamic model.

Rigid ice: Rigid ice models reflect the frost heave process described above. These numerical models predict the location and thickness of ice lenses (e.g., O’Neill and Miller 1982).

Thermomechanical: Recent frost heave models consider the global response of soil to temperatures and pressures, in which constitutive functions describe behavior (e.g., Hartikainen and Mikkola 1997, Michalowski 1993). This constitutive modeling is called thermomechanical because the soil mechanical behavior is related to heat and water transfer.

4 MODELS EVALUATED

Table 1 summarizes models we evaluated. Two models that are used today for pavement design include the SSR, as used in Finland, utilizing the *SP* (e.g. Saarelainen 1992, Kujala, personal communication, 2002), and FROST. However, PC-Heave by Sheng (1994) is also well developed and could easily be transitioned into use as a design tool. In addition, Michalowski and Zhu (2004) are developing a thermomechanical model. We could not adapt the SSR model to predict laboratory tests; therefore, we only evaluated three models. Because it is currently used, a description of the SSR model, with *SP*, is included, but the model is not evaluated.

Most models treat the energy balance of freezing soil similarly but differ in how they model the flow of water that causes heave. The energy balance is $q_{out} - q_{in} = q_{net}$, where q_{out} and q_{in} are the energy flows out of and into the freezing zone, assumed to be conductive heat transfer only, i.e., $q_{out} = -\lambda_f \frac{\partial T_f}{\partial z}$ and $q_{in} = -\lambda_u \frac{\partial T_u}{\partial z}$, where λ_f and λ_u are the thermal conductivities of frozen and unfrozen soil. The term q_{net} represents change in heat storage in the freezing zone caused by heat loss due to the phase change of water. The former is typically represented as $q_{net} = L_f \left(\frac{v_I}{1.1} + \theta_w \frac{\partial s}{\partial t} \right)$, where v_I is the velocity of ice lens growth, θ_w is volumetric water content, and $\frac{\partial s}{\partial t}$ is the rate of frost penetration. Different expressions for q_{net} have been used in various models.

Table 1: Frost heave models evaluated.

Model/ Type	**Output**	**Material properties**	**Boundary conditions**	**Initial conditions**
FROST/ Hydrodynamic	Frost heave, penetration; ice content; thaw penetration; thaw consolidation	A_w, A_k, α, β, λ_{dry}, C_s.	Pore water pressures, temperatures, element lengths	Surcharge pressure, temperature distribution, pore water pressure distribution, ice content distribution
PC-Heave/ Rigid Ice	Frost heave and penetration, ice lens location, segregation temperature, pore water suction in the frozen fringe	Per soil layer: ρ_d, w, λ, K_s, S and one w_u at –1°C	Temperatures at the top and bottom of soil column	Depth to water table, initial temperatures
Porosity Rate/ Thermomechanical	Frost heave and penetration, ice content, porosity	Porosity rate function parameters: $\dot{\eta}_m$, T_m, α, ζ	Boundary temperatures, surcharge	Initial temperatures, surcharge, initial porosity

A_w, α = coefficients required for Gardner's equation used to describe the soil moisture characteristic function, as given by Guymon et al. (1993), A_k, β = coefficients required for Gardner's equation used to describe unsaturated hydraulic conductivity, C_s = volumetric heat capacity of dry soil, λ_{dry} = thermal conductivity of the dry soil, ρ_d = dry density of soil, w = gravimetric water content, K_s = saturated hydraulic conductivity, S = percentage saturation, w_u = liquid (unfrozen) water content, $\dot{\eta}_m$ = maximum porosity rate, T_m = temperature at maximum porosity rate, and α and ζ are parameters that account for temperature gradient and stress state.

4.1 Segregation Potential

Konrad and Morgenstern (1980) developed a theory of ice lens formation in freezing of fine-grained soils that became known as the Segregation Potential (or *SP*) Concept. There has been considerable effort dedicated to using *SP* for engineering (e.g., Saarelainen 1992), and now

Finland uses it in pavement design (Saarelainen 2000). The key idea is that the water intake for formation of the final ice lens in a one-dimensionally freezing soil, subjected to constant temperatures, is proportional to the temperature gradient in the frozen fringe:

$$v_w = SPgradT \tag{2}$$

where v_w (m s^{-1}) refers to the water flux during formation of the final ice lens (and thus the frost heave rate can be estimated as 1.09 times v_w) and $gradT$ is the temperature gradient in the frozen fringe when a "final" ice lens is forming in a freezing soil. This is a simplification of the field situation. The *SP* is essentially a "constitutive relation" that describes water flow as a function of temperature gradient and then is used to predict frost heave based on estimates of the temperature gradient at the freezing front. Frost heave is influenced by overburden pressure and rate of heat loss; thus, *SP* for a given soil is also a function of these variables (e.g., Konrad and Morgenstern 1982). Finnish pavement design for frost heave is based on past field observations and estimates of *SP* for specific locations based on accurate measurements of frost heave, frost penetration, and temperatures (e.g., Saarelainen 1992). *SP* can also be determined by laboratory freezing of undisturbed soil or by correlation of the soil type to the *SP* of soils that have been tested.

4.2 FROST

FROST was developed for non-cohesive, frost-susceptible (FS) soils subjected to seasonal freezing where the depth of frost penetration does not reach the water table and there is up to 60 kPa of overburden pressure at the freezing front. FROST models a "freezing zone." The freezing zone descends through the soil, importing unfrozen soil and exporting frozen soil. It gains water and heat through the lower boundary and loses only heat through the upper boundary. Heaving occurs when the volumetric ice content exceeds the initial soil porosity minus some minimum unfrozen water content. FROST uses the following equation to determine the hydraulic conductivity in the freezing zone, K_f (Guymon et al. 1993):

$$K_f = K(h) \cdot 10^{-\mathrm{E}\vartheta_i} \tag{3}$$

where K (cm hr^{-1}) is the hydraulic conductivity of the soil as a function of the pressure head h (m), ϑ_i is the volumetric ice content, and E is an empirical parameter. Thus, water flow is related to the ice content and water pressure and not to the temperature or ice pressure. The soil heaves only when volumetric ice exceeds the porosity of the soil; therefore K_f controls frost heave.

Parameter E in Equation 2 greatly influences heave predicted. Guymon et al. (1993) forced FROST to fit experimental and observational data for nine different soils to determine an empirical expression for E, which they give as:

$$E = \frac{5}{4}\left(k_s - 3\right)^2 + 6 \tag{4}$$

where k_s is the saturated hydraulic conductivity given in cm hr^{-1}. Equation 4 is based on nine frost heave tests, in which E ranged from 4.5 to 20 (Bigl and Shoop 1994). For four soils observed over two winters, the E values for accurate simulation of frost heave were different for each year, changing from 6 to 80% (Guymon et al. 1993). Although rate of heave is influenced by overburden pressure, rate of heat loss from the frozen fringe, and temperature distribution (e.g., Loch 1979), the E value calculated by Eq. 3 is not directly affected by any of these conditions.

FROST is used in a mechanistic pavement design model known as the Integrated Climatic Model (ICM) (Lytton et al. 1993). The ICM is a one-dimensional coupled heat and moisture flow model used to analyze climatic effects on pavement soil systems. The ICM does not use E to determine hydraulic conductivity in the freezing zone (Lytton et al. 1993). Rather, it uses a thermodynamic relationship to determine suction as a function of temperature. Some boundary conditions for soil water suction were changed in the ICM version of FROST to improve performance. However, even in the improved form, FROST occasionally predicts tens of feet of frost heave, and predicted heave has been capped at 20% of the original element length. In the future, FROST will likely be replaced with a more accurate program for predicting frost heave in the ICM.

4.3 PC-Heave

Sheng (1994) developed a numerical model of frost heave based on Rigid Ice, called PC-Heave (e.g., O'Neill and Miller 1985). PC-Heave models stratified, saturated, and unsaturated soils. Soil layers are initially classified as frost-susceptible (FSL) or non-frost-susceptible (NFSL). In all NFSLs, which are typically insulation, gravel, or dry soil, only heat flow is modeled. Mass and heat flow are modeled in the FSLs. The modeling equations are the mass and heat balances at the base of the warmest ice lens and for the frozen fringe, Darcy's Law, and an expression for the pore water pressure in the frozen fringe that incorporates the generalized Clapeyron equation (Sheng 1994). The hydraulic conductivity of the frozen fringe is given by one of the following:

$$K = K_s e^{c(T-T_f)} \tag{5}$$

or

$$K = K_s e^{-b(z-z_f)} \tag{6}$$

where the units of K and K_s are m s^{-1}, z and z_f are depths to the frozen fringe and the freezing front (m), respectively, and c and b are parameters input by the user or b can be obtained by substituting Equation 7 with $\gamma = 9$ into Equation 6, or

$$K = K_s \left(\frac{\vartheta_w}{\eta} \right)^{\gamma} \tag{7}$$

where η is the soil porosity and γ is a coefficient that was experimentally determined to be between 7 and 9. The unfrozen water content in the frozen fringe (volumetric fraction) is given by

$$\vartheta_w = \eta e^{\left(\frac{T}{\chi} \right)^{\xi}} \tag{8}$$

where χ and ξ are experimentally determined coefficients and T and χ are expressed in °C (e.g., Kujala 1989). The PC-Heave model can be calibrated for the specific FSL by backcalculating the percentage of unfrozen water content at a temperature of –1°C based on heave measurements. The mean air temperature at the surface for the time simulated is usually input as a constant surface temperature. This may underestimate frost penetration early in the season and overestimate heave late in the season (Sheng 1994).

4.4 Porosity Rate Model

Michalowski (1993) and Michalowski and Zhu (2004) are developing the Porosity Rate Model. The porosity rate function gives the increase in porosity due to ice lens growth; however, individual ice lens growth is not modeled (Blanchard and Frémond 1985):

$$\dot{\eta} = \dot{\eta}_m \left(\frac{T - T_0}{T_m} \right)^2 e^{1-\left(\frac{T-T_0}{T_m}\right)^2} (1 - e^{\alpha \frac{\partial T}{\partial x_l}}) e^{-\frac{\bar{\sigma}_{kk}}{(1-\eta)\varsigma}} \tag{9}$$

where $\dot{\eta}$ and $\dot{\eta}_m$ are the porosity rate and maximum porosity rate, respectively, and T_m is the temperature where the maximum porosity rate occurs. Parameters α and ζ take account of the temperature gradient and the stress state, and $\bar{\sigma}_{kk}$ is the first invariant of the average Cauchy stress tensor. This function applies only to soil freezing. The porosity increase is then related to the expansion of the soil due to heave through a function analogous to a strain rate tensor. The gravimetric unfrozen water content is given by

$$w = w^* + (\bar{w} - w^*)e^{a(T-T_0)} \tag{10}$$

where water content as a function of temperature is discontinuous at the freezing point, $\bar{w}$ is the minimum unfrozen water content at the freeing point, w^* is the residual liquid water content at some low reference temperature, and a helps describe the decaying curve.

5 MODEL EVALUATIONS

The models were used to predict heave based on experiments described in Henry (1998). Results from tests in which 150-mm-high specimens of “Soil A,” a silt collected at the Anchorage International Airport, frozen with a 2.44-kPa overburden pressure (with water freely available at the base) were used to calibrate and evaluate them. We used one set of thermal conditions for calibration and a second set to test how well the models predicted heave and frost penetration. Prior to freezing there was a three-day period in which the top applied temperature was 0.5°C while the bottom was 0.7°C. The calibration conditions are –1.4°C applied at the top and 0.7°C applied at the bottom, and the models were tested at –3.0 °C applied at the top and 0.7°C applied at the bottom.

5.1 FROST

The version of FROST used is part of a larger pavement design package of software, SLED (Seasonal Layered Elastic Design). The relevant physical characteristics and Gardner’s coefficients of the soil that were input to FROST were based on unsaturated flow tests of the soils (Henry 1998). The value of 0.2 as the liquid water content after freezing was selected because the equivalent values for the two soils that most closely resembled Soil A by all descriptors were 0.263 and 0.152, respectively. There was a good match of the soil moisture characteristic curves (measured and predicted by the Gardner’s coefficients) in the range of measured suctions, and the shape of the characteristic in the extrapolated portion was typical of silt. The hydraulic conductivity for Soil A used in FROST is half an order of magnitude greater than the average value measured. This is because FROST predicted negligible heave when the average value (5×10^{-6} cm s^{-1}) was used and because 1×10^{-5} cm s^{-1} is still within the range of hydraulic conductivities measured. The values of the other physical parameters input to FROST include a porosity of 0.297, dry density of 1.97 Mg m^{-3}, A_w of 0.14, α of 0.628, A_K of 0.003053, and β of 2.665. FROST was difficult to calibrate; it computed a value for E of 17 for Soil A. Values of E ranging from 0 to 20 were also manually input, and a value of 0 produced the most frost heave and most closely modeled the observed test results.

5.2 PC-Heave

A copy of PC-Heave was provided by Dr. Daichao Sheng. The properties for Soil A and the calibration temperatures were input into PC-Heave to calibrate for the unfrozen water content at –1°C. The thermal conductivity is that of the soil solids only, and a value of 3.0 W m^{-1} K^{-1} was used, the K_{s} was the average value measured (0.5×10^{-5}), and 100% saturation was assumed.

5.3 Porosity Rate model

The Porosity Rate model was implemented using the finite element system ABAQUS. A column of 30 elements was used to simulate the one-dimensional heat flow and deformation process. As the fundamental constitutive relation is the porosity rate function, the model does not require specifying the hydraulic conductivity. The parameters used in computations were a maximum porosity rate ($\dot{\eta}_m$) of 1.08×10^{-4}, T_m of –1.1°C, and α and ζ values of 0.001 m °C^{-1} and 1.5 MPa, respectively. The unfrozen water content was described by the relation in Eq. 10, with these parameters: a = 1.5 (°C^{-1}), w^* = 1.0%, and $\bar{w}$ in the range of 10.11% to 11.47%. The initial moisture content was taken equal to $\bar{w}$ (no discontinuity at T_0). The initial porosity was in the range 0.20 to 0.23. The heat capacity for soil skeleton, water, and ice was assumed to be 0.9×10^3, 4.18×10^3, and 2.1×10^3 J kg^{-1}K^{-1}, respectively. The thermal conductivity is the function of the composition of the mixture, and it is dependent on the temperature. Thermal conductivity was assumed to be 3.0, 0.60, and 2.22 W m^{-1}K^{-1} for soil skeleton, water, and ice, respectively, and the latent heat of fusion of water is 333×10^3 (J kg^{-1}).

5.4 Model evaluation

Figures 1 and 2 show calibration curves for each model compared to the experimental data obtained, and Figures 3 and 4 show the results of the subsequent simulation. PC-Heave and Porosity Rate both predict frost heave quite accurately. The shapes of both the calibration and the simulation frost heave curves are very similar to the experimental curve; they indicate relatively high heave rates initially, with a gradually decreasing rate as frost penetrates. There is a one-day delay in heave and a nearly linear frost heave curve for both the calibration and prediction curves of FROST. FROST predicted a linear rate of frost heave, when we would expect a gradual decrease in the rate of heave with constant temperatures applied on the ends of the specimen.

All three models are in close agreement with respect to frost penetration prediction, both for calibration and for simulation. However, they predict greater frost penetration than what was actually observed. It is possible that there was excessive water available to freeze in the laboratory specimen, resulting in relatively shallow frost penetration.

The first author obtained and worked with both FROST and PC-Heave. Of these two models, PC-Heave was the easiest to work with. The Porosity Rate model is currently being developed and is not yet available for general use. FROST and PC-Heave are both one-dimensional freezing models, and the laboratory data simulated were from one-dimensional freezing tests in well-controlled conditions.

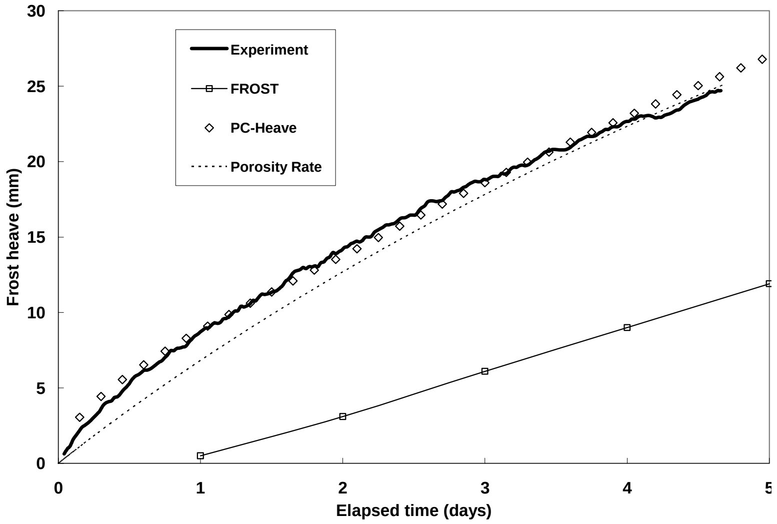

Figure 1: Calibration frost heave curves for FROST, PC-Heave, and Porosity Rate frost heave models, with experimental data for Soil A.

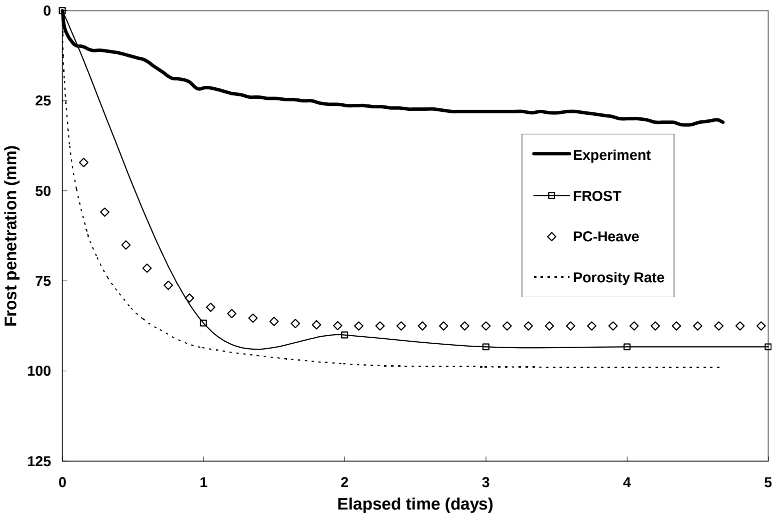

Figure 2: Calibration penetration curves for FROST, PC-Heave, and Porosity Rate frost heave models, with experimental data for Soil A.

6 CONCLUSIONS

Of the three frost heave models evaluated by simulating laboratory freezing of saturated, frost-susceptible soil, the PC-Heave model and the Porosity Rate model accurately simulated the frost heave of soil for one set of thermal conditions after being calibrated with a different set of freezing conditions. The numerical model FROST was less accurate than the other two models, under-predicting the frost heave rate as well as predicting a delayed frost heave. Perhaps most notably, FROST predicted a linear increase of frost heave, when a gradual decrease in the rate of heave with constant temperatures applied on the ends of the specimen is observed. All three models closely agree with each other with respect to frost penetration prediction. However, they predict greater initial frost penetration than what was actually observed. It is possible that there was excessive water available to freeze in the laboratory specimen, resulting in relatively shallow frost penetration.

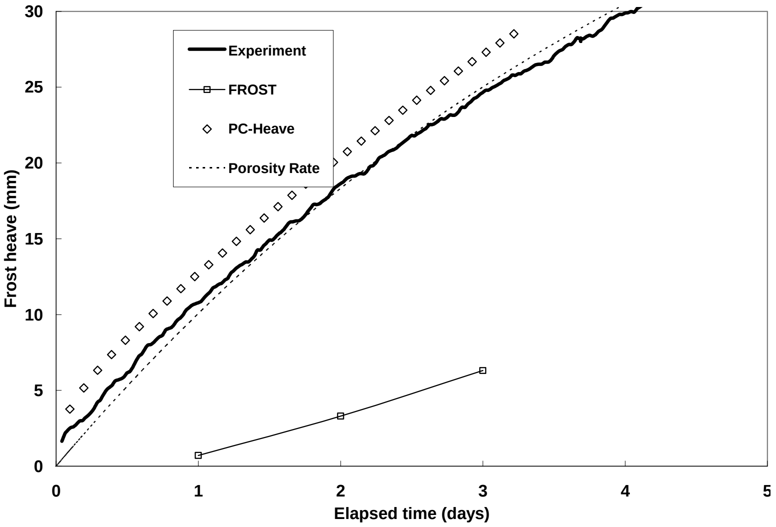

Figure 3: Simulation heave curves for FROST, PC-Heave, and Porosity Rate frost heave models, with experimental data for Soil A.

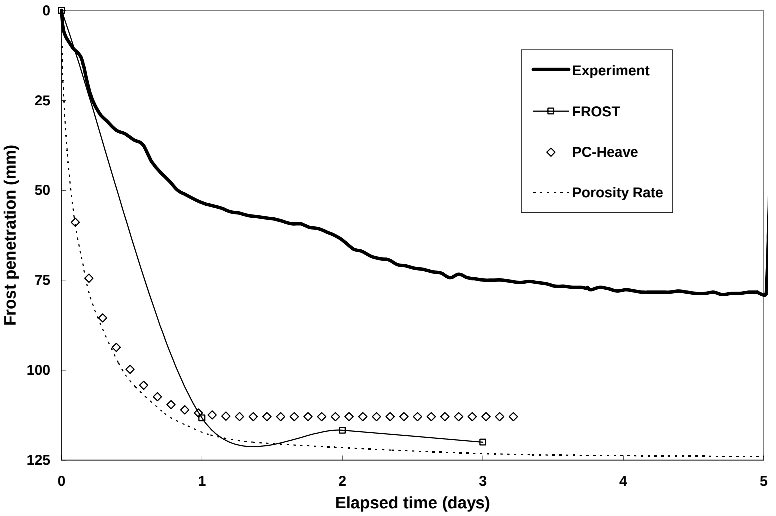

Figure 4: Simulation penetration curves for FROST, PC-Heave and Porosity Rate frost heave models, with experimental data for Soil A.

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