A review and reliability assessment of frost penetration models

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Frost penetration in pavements and subgrade can lead to frost heave and thaw weakening, and hence influences bearing capacity of roads during winter and spring thaw. Several models of frost penetration have been developed based on energy and mass (specifically, water) balances at the surface and within the pavement structure. This describes a review of frost penetration models used in pavement design. It also includes an overview of major field studies and their use to validate frost penetration models. This forms the basis for the reliability assessment of the models and their advantages and disadvantages for predicting frost penetration.

KEY WORDS: Frost penetration, frost penetration models, pavement design, soil freezing

1 INTRODUCTION

Fifty percent of the earth's land area has seasonal or permanent temperatures below freezing. In these areas, the possibility of frost heave and thaw weakening must be considered in the design of infrastructure foundations. Frost penetration is influenced by a number of factors such as solar radiation, wind, air temperature, surface color, material and water content in the pavement and subgrade, ground water level, etc. There are several models available to estimate frost penetration. To evaluate these models, laboratory studies and field observations are necessary.

2 THE THERMAL BALANCE

There are a variety of factors affecting the temperature in a pavement. The heat balance (conservation of energy) is used to predict the temperatures at a given time and the change of temperature over time. According to this principle, the heat flow out of a body must at any time equal the heat flow in plus the change in the storage of heat. Applied to soil freezing:

$$q_- = q_+ + q_f + q_s \qquad (1)$$

where

$q_- = k_f \, \mathrm{grad} T_-$ = Heat flow between the surface and the frost front through the frozen layer (W/m^2)

$q_+ = k_u \, \mathrm{grad} T_+$ = Heat flow between the unfrozen soil and the frost front (W/m^2)

$q_f = L \, dz_0/\mathrm{dt}$ = Heat generated by the phase change of the pore / interstitial water when transformed to ice (W/m^2)

$q_s = L_w \, SP \, \mathrm{grad} T_-$ = heat generated by the additional segregation water during ice lens formation (W/ m^2)

Applying this heat balance to a given volume of a road, for example, the flow of heat into the body from the subgrade during a time increment will be the same as the heat flow to the air from the surface if there is no change in heat storage in the volume. In this situation the temperature of the pavement is constant and the frost front is stationary. If the temperature in the air decreases, or the radiation from the surface increases (for instance caused by clearing of clouds on a winter night), the increased heat loss results in an increased heat flow out of the body. The following mechanisms are then possible:

1) The frost might penetrate and release energy from the unfrozen material by cooling i) the mineral phase or ii) the pore water
2) The frost front might stay at a constant depth, but water migrates to the frozen fringe and freezes
3) A combination of 1) and 2)

Several factors influence whether the frost front moves downward and the velocity at which it will move:

- ✓ The temperature gradient of the frozen ground at the segregation front *(gradT₋)*
- ✓ The temperature gradient of the unfrozen ground at the segregation front *(gradT₊)*
- ✓ Amount of unfrozen water available to freeze
- ✓ Energy release by cooling of the mineral phase
- ✓ Thermal conductivity of the unfrozen ground at the frost front (k_u)
- ✓ Thermal conductivity of the frozen ground at the frost front (k_f)
- ✓ Thickness of the frozen layer
- ✓ Overburden pressure (z_i)

The segregation potential (SP) is used as a material parameter in some frost penetration and frost heave models. This parameter is however an implicit function of suction, hydraulic conductivity, overburden pressure and temperature gradient. The heat balance of a freezing ground is illustrated in figure 1 (Saarelainen 1992)

3 FACTORS INFLUENCING FROST PENETRATION

3.1. The climate regime

The main factors that influence frost penetration are air temperature, solar radiation, wind and precipitation. The time integral of temperature during the frost season, defined as the frost index, is the most important factor. Another factor giving a significant effect on the frost penetration is the solar radiation. This is a function of latitude and is a sinusoidal function over the year. The effect of the solar radiation depends on the surface color, as a dark surface absorbs more heat than a light or white surface. Wind will increase the heat flow between the air and the ground as increased wind speed increases heat loss. Precipitation will have several effects depending on the conditions and temperature; snow cover reduces the heat loss, melting snow or rain releases energy to the frozen surface during the phase change.

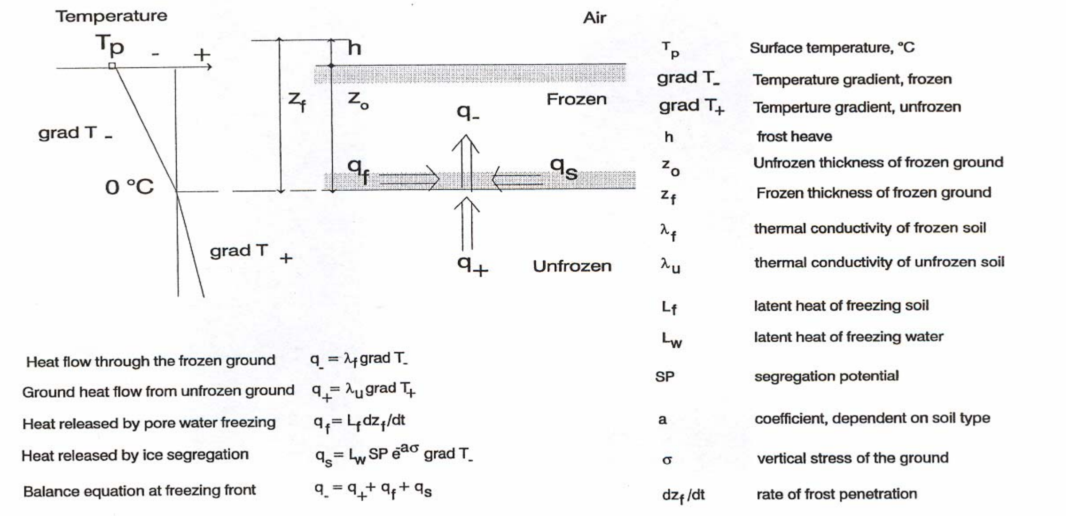


Fig. 1. Heat balance of a freezing ground (Saarelainen 1992).

Figure 1: The heat balance of a frozen ground (Saarelainen, Seppo1992)

3.2. Thermal characteristics of pavement materials and subgrade

The thermal characteristics of soil and pavement materials depend on several factors; the thermal conductivity of the mineral phase (being a function of temperature), the thermal conductivity of water and ice, the water content, the latent heat of fusion for water, and the amount of unfrozen interstitial water when the soil is frozen. Some of these factors are relatively easy to find by laboratory testing, while others must be calculated or estimated based on laboratory testing and field investigations. The thermal properties of soils are very sensitive to quartz content (e.g., Kersten, 1949; Farouki, 1982; Johansen, 1977). Farouki (1982) concluded that regarding the thermal conductivity for coarse soils “Kersten over predicts for those with a low quartz content and under predicts for those with high quartz content…” He further recommends that the Kersten method as applied to coarse soils be limited to soils with intermediate quartz content that he specifies to be around 60% of the soil solids by weight.

4 FIELD MEASUREMENTS OF FROST PENETRATION

4.1 Need for monitoring of frost penetration

Frost penetration models should be calibrated by laboratory experiments or field tests. By using laboratory calibration, it is possible to have accurate control of the material conditions. Laboratory tests might also have some shortcomings in their ability to simulate the more complex in-situ conditions. Therefore field tests are frequently used for evaluation of frost penetration models. For this paper, two field sites were used for the evaluation of four frost penetration models.

4.2 Frost penetration monitoring methods and their strengths and shortcomings

Several methods have been used for monitoring frost penetration. The most frequently used methods are several types of "frost tubes" where an indicator fluid (methylene blue in water) changes color when it freezes, (Henry, 1990). If installed properly, this method is simple and relatively accurate for monitoring the depth of phase change of water. But it is time consuming and only manual readings can be made.

Another method commonly used is monitoring ground temperatures with thermocouples installed in the pavement and subgrade. This method gives enough data to draw isotherm contour maps based on a number of simultaneous temperature readings in several profiles. The monitoring method is relative simple, and the data can easily be monitored by a data logger, (Henry, 1990).

Techniques that use latent heat of fusion to identify phase change have been evaluated to indicate the position of the freezing front (Heydinger, 2003). However, supporting measurements from other sources ("4-point resistivity", "2-point resistance", voltage) are needed to complement this method. In 40% of the cases evaluated by Heydinger (2003), none of the latent heat of fusion methods were successful in identifying freezing conditions. These methods have therefore still to be further developed to be used for practical purposes.

A multisegment time domain reflectometry probe has been developed by the Minnesota Department of Transportation (MnDOT) (Roberson and Siekmeier, 2000, 2004). This instrument combines time domain reflectometry with remote diode switching to provide a profile of dielectric properties for base and subbase aggregate. It works well in locating frost depth, and has been tested under controlled conditions in the laboratory. Field installation and performance evaluations have also been conducted since 1999. This method accurately measures the phase change of water, and the monitoring is easy to record with a datalogger.

4.3 Selected test site in Canada

A pavement test site for the purpose of documenting frost action was constructed in 1988 and 1989 on National Highway RN 155 in St Celestin, Quebec, in the St. Lawrence Valley about 100 km northeast of Montreal, (Boutonnet, et al., 2003).

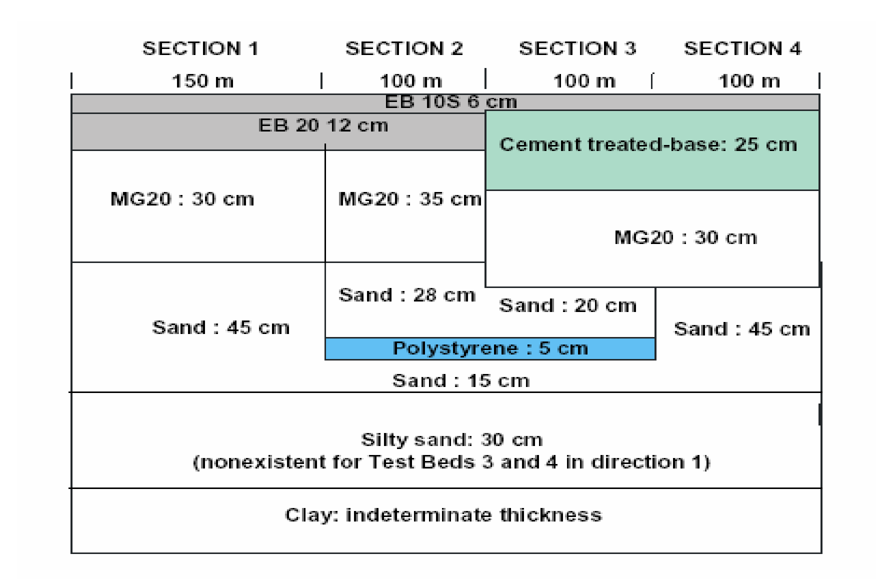


Figure 3: Test bed stratigraphy on National Route RN155 at St-Célestin, Québec, Canada, (Boutonnet, et al., 2003)

The site is subjected to harsh winter climate and heavy traffic. Weather data are taken from the Nicolet weather station located about 20 km west of the test site. The freezing index (FI) calculated from 3 winters ranged from 666 to 1400 ºC-days (average 1150 ºC-days), and the number of freezing days ranged from 94 to 129 days. The mean annual 30-year temperature is 5,1 ºC. The asphalt surface temperature increases according to the solar radiation. The n-factor (FI $_{surface}$ / FI $_{air}$) was calculated from meteorological data and temperatures recorded 5 cm below the surface in the asphalt layer, giving n = 0,9 which was used to estimate n = 0.95 for the surface.

The surface layer of all test sections was an asphalt mixture. Section 1 with an asphalt base course servedas a reference. Section 2 was equally constructed, but with a Polystyrene frost insulation layer 81 cm beneath the surface. This insulation continued in Section 3 having a cement treated base (CTB) substituting for the asphalt base. Section 4 was without frost insulation, but still with CTB. The lower base course was unbound granular material 0/20 (MG 20), and the subbase course consisted of sand resting on silty sand (30 cm) over natural clay. Each test section was instrumented with a set of thermistors to monitor the frost progression, time domain reflectometer sensors (TDR) to measure water content, frost tubes, piezometers and frost heave sensors, (M. Boutonnet, et al, 2003). For our paper, however, only data from Sections 1 and 2 are used.

The thermal conductivity values for granular materials (unbound base, sand and silty clay) used in the analyses were determined by two alternative models; 1): The Kersten formulas (Kersten, 1963) and 2): Mickley's model (Van Rooyen and Winterkorn, 1957). Thermal conductivity values for the asphalt mixtures and the CTB material were taken from benchmark data for road products from Quebec. Segregation Potential values, SP, were evaluated from laboratory tests on the actual materials.

Table 1: Properties of the test bed on National Route RN155 at St-Célestin, Québec, Canada, (M. Boutonnet, et al, 2003)

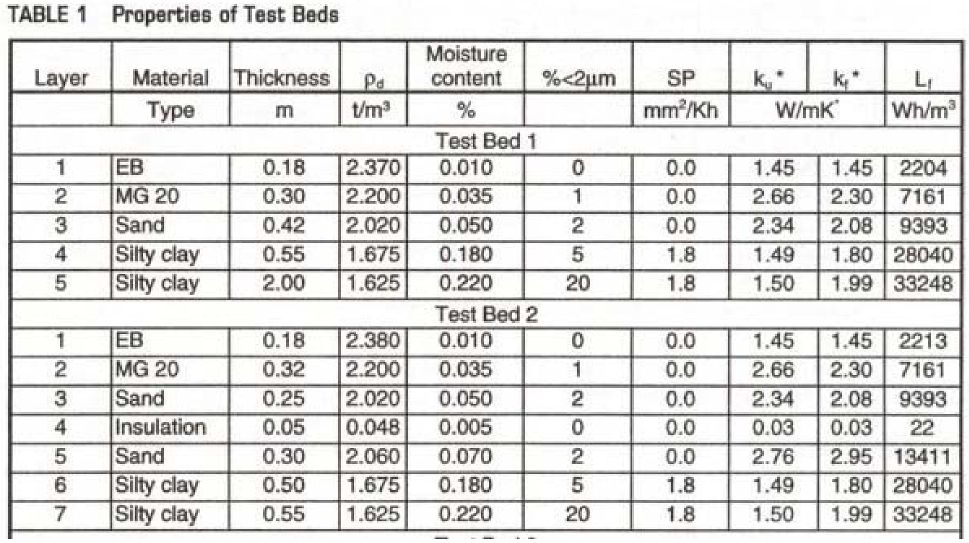


TABLE 1 Properties of Test Beds

Layer	Material	Thickness	ρ_d	Moisture content	%<2µm	SP	k_u *	k_f *	L_f
	Type	m	t/m³	%		mm²/Kh	W/mK˙		Wh/m³
				Test Bed 1					
1	EB	0.18	2.370	0.010	0	0.0	1.45	1.45	2204
2	MG 20	0.30	2.200	0.035	1	0.0	2.66	2.30	7161
3	Sand	0.42	2.020	0.050	2	0.0	2.34	2.08	9393
4	Silty clay	0.55	1.675	0.180	5	1.8	1.49	1.80	28040
5	Silty clay	2.00	1.625	0.220	20	1.8	1.50	1.99	33248
				Test Bed 2					
1	EB	0.18	2.380	0.010	0	0.0	1.45	1.45	2213
2	MG 20	0.32	2.200	0.035	1	0.0	2.66	2.30	7161
3	Sand	0.25	2.020	0.050	2	0.0	2.34	2.08	9393
4	Insulation	0.05	0.048	0.005	0	0.0	0.03	0.03	22
5	Sand	0.30	2.060	0.070	2	0.0	2.76	2.95	13411
6	Silty clay	0.50	1.675	0.180	5	1.8	1.49	1.80	28040
7	Silty clay	0.55	1.625	0.220	20	1.8	1.50	1.99	33248

4.4 Selected test site in USA

Ravalli County Airport in Hamilton, Montana is located approximately 1 mile (1.6 km) east of Hamilton at elevation 3642 ft. (1110.1 m). Ravalli County Airport is located in a relatively flat area of low alluvial fans and flood plains formed by the creeks feeding the Bitterroot

River. The soils at the airport are quite variable; however, they are generally fine-grained silts and lean clay (Henry, 1990).

Table 2: Thermal properties for soils, data from Station 6 Ravalli County Airport, Hamilton, Montana

Layer	**Thick-ness**	**USCS Class**	**Dry density**	**Volum. Water content**	**Heat capacity, frozen** M)	**Heat capacity, unfroz** M)	**Thermal cond., frozen** K)	**Thermal cond., unfroz** K)	**Thermal cond., aver.** US)
	m		**kg m^{-3}**		**kJ m^{-3} °K^{-1}**		**W m^{-2} °K^{-1}**		
1	0.0254	Asph.	2,240^{1}	0.10$^{1)}$	1,877.792	1,877.792	1.49	1.49	(1.49)
2	0.305	(SM)^{2}	2,120	0.127	2,068.237	2,630.691	3.00	2,23	1.7
3	0.61	ML	1,580	0.395	1,320.276	1,518.317	2.02	1.30	1.6
4	2.06	ML	1,960	0.078	1,555.920	1,719.701	1.18	0.80	1.6

1) Default value provided by Modberg

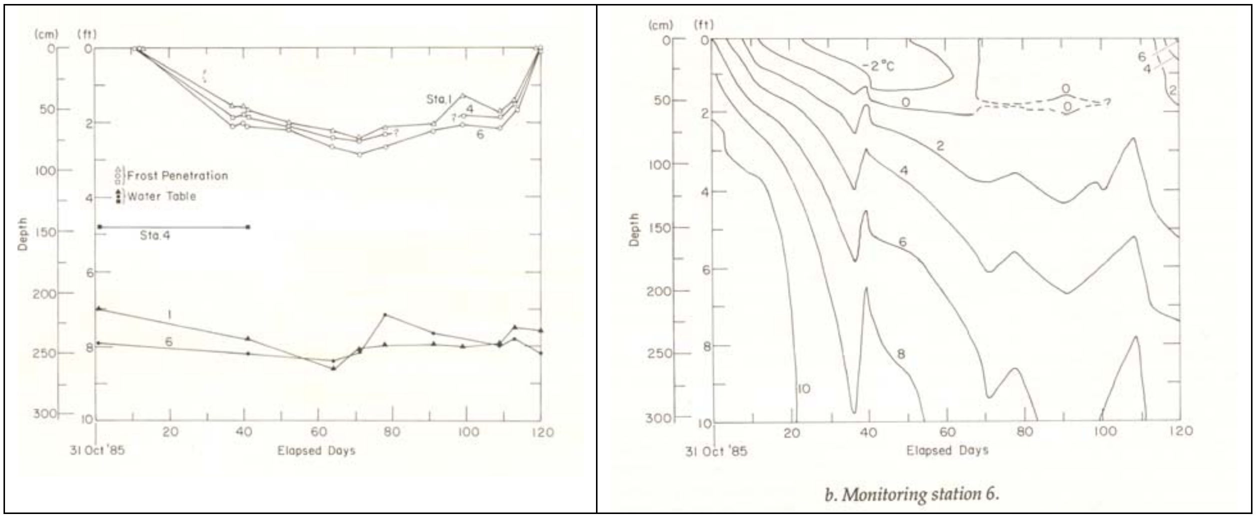
2) "Coarse soil" / sand according to soil description in the boring log

M) Thermal capacity and conductivities as given by Modberg (Chamberlain 1991)

K) Thermal cond. Based on Kersten (1949) and Johansen (1975)

US) Thermal conductivities by empirically based curves presented in TM-5-856-6(US Army 1966), using average thermal conductivity for frozen and unfrozen soil

During the 1985-86 freezing season, a field study was conducted to examine frost heave and its causes on the airport runway (Henry, 1987; 1990). Temperature measurements were made with depth at two locations on the runway—stations 3 and 6. Soil collected from station 6 was very well characterized, including the determination of the soil moisture characteristics at three depths, and this information was further developed for model input. Hence stratigraphic data from station 6 was used for the assessment of the frost penetration models. The water table was 9 ft (2.77 m) below the surface at both survey stations in August of 1985 when the soil collection took place.



b. Monitoring station 6.

Figure 4: Frost penetration measurements a) with frost tubes and b) with thermocouple recordings winter 1985/86, station 6, Ravalli County Airport

Meteorological data (daily surface summaries) were obtained for Hamilton, Montana, for the years 1985 and 1986 from the National Climatic Data Center (NCDC) of the National Oceanographic and Atmospheric Administration (NOAA). The computer program Modberg, downloaded from http://www.pcase.com, was used to estimate the heat capacity (Chamberlain, 1991). Thermal conductivities are calculated according to Kersten (1949) and Johansen (1975) and alternatively by empirically based curves presented in TM-5-856-6(US Army 1966), using average thermal conductivity for frozen and unfrozen soil

Data from the frost tube measurements and isotherms calculated on the basis of thermocouple recordings are shown in Figure 4. The maximum frost penetration measured by the frost tube is a bit deeper than the 0°C-isotherm recorded from the thermocouples (86 cm and 63 cm respectively). The difference measure might be real, with less frost penetration measured by the thermocouples due to the effect of solar radiation beneath the dark asphalt surface on the runway, while frost tubes were installed in the terrain beside the pavement. Another explanation for this disagreement might also be inhomogeneous subgrade and different hydraulic conditions.

5 FROST PENETRATION MODELS AND EVALUATION

5.1 Models evaluated

The thermal models that we evaluated are (Boutonnet, et al., 2003):

- The SSR model used at MTQ, Quebec, Canada
- He GEL1D one-dimensional model used by LCPC used to verify pavement frost in the French Pavement Dimensioning Method
- The CESAR-GELS finite elements model used by the LCPC for specific evaluation requiring a two-dim. or three-dim approach
- TEMP/W model, used in several countries; including Canada, USA and Norway

5.1.1 The SSR model

The SSR model and the segregation potential (SP) concept allow evaluation of frost penetration and frost heave. The frost penetration is based on the equilibrium of the thermal balance at the frost front in homogeneous layers. Frost penetration, Δz_0, is obtained as a function of time increment dt as:

$$\Delta z_0 = (T_f - T_p)dt\,(1 - SP\,L_w/k_f)/(L_f R_f) - S\,gradT_+ k_u dt/L_f \qquad (2)$$

where:

Δz_0 = frost front penetration during a time increment dt
T_f = ground freezing temp (ºC)
T_p = ground surface temp at the time Δt (ºC)
SP = segregation potential (m^2/Kh)
L_w = latent heat of water fusion (Wh/ m^3)
k_u = thermal conductivity of the unfrozen ground at the frost front (W/Km)
k_f = thermal conductivity of the frozen ground at the frost front (W/Km)
L_f = latent heat of frozen ground fusion at the frost front (Wh/ m^3)
R_f = thermal resistance of the frozen layers = $\sum(z_i / k_{fi})$ (m^2K/W)
z_i = thickness of the frozen layer during increment I (m)
S = coefficient of thermal gradient intensity $gradT_+$ (S = 1.0 in November and is assumed to decrease gradually to 0.7 in April)
$gradT_+$ = temperature gradient of the unfrozen ground (K/m)
$gradT_-$ = temperature gradient of the frozen ground at the segregation front (K/m)

The midwinter temperature gradient of the unfrozen ground is estimated from an empirical curve based on mean annual air temperature.

5.1.2 The GEL1D and the CESAR-GELS finite elements models

Both models are software programs developed at LCPC for calculating frost penetration. GEL1D uses the finite differences method. CESAR-GELS uses the finite element method (FEM), and allows 2-dimentional or 3-dimentional calculations.

Determination of the frost front depends on the solution to a coupled problem where the temperature and the rate of frost propagation are unknown parameters. The thermal equilibrium equations are expressed as:

In the unfrozen zone:

$$\rho_u c_u \frac{\partial T}{\partial t} - div(k_u \cdot \overrightarrow{grad}T_+) = 0 \tag{5}$$

Where ρ_u is the density of the unfrozen material and c_u and k_u are the heat capacity and thermal conductivity in the unfrozen zone respectively.

The equilibrium in the frozen zone is expressed as:

$$\rho_f c_f \frac{\partial T}{\partial t} - div(k_f \cdot \overrightarrow{grad}\, T_-) = 0 \tag{6}$$

Where ρ_f is the density of the frozen material and c_f and k_f are the thermal capacity and conductivity in the frozen zone respectively.

The third zone, the frost front, being the intermediate area between the frozen and the unfrozen zone, is the seat of a thermal gradient discontinuity, known as a Stephan number, n. Its law of variation involves the frost front speed, ds/dt, according to the equation:

$$(k_f \cdot \overrightarrow{grad}\, T_- - k_u \cdot \overrightarrow{grad}T_+) \cdot \vec{n} = L_f \cdot ds/dt \tag{7}$$

Each layer is characterized by its thickness (z), dry density (ρ_u, ρ_f) water content (w), heat capacity (c_u, c_f) and thermal conductivity (k_u , k_f). The initial temperature of the structure is given at a certain time, t. The conditions at the surface and several depths can then be given in four different ways:

- ✓ By imposed temperature
- ✓ By imposed heat flow
- ✓ By heat flow proportional to the temp. diff. between the air and the ground surface
- ✓ By heat flow equal to the sum of an imposed flow and a flow proportional to the temperature difference between the air and the ground / structure surface

5.1.3 The TEMP/W model

The governing differential equation used in the formulation of TEMP/W is the general heat flow equilibrium equation (TEMP/W Users manual, 2004):

$$\frac{\partial}{\partial x}\left(kx\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(ky\frac{\partial T}{\partial y}\right) + Q = \lambda\frac{\partial T}{\partial t} \tag{8}$$

where:

T = temperature
kx = thermal conductivity in the x-direction
ky = thermal conductivity in the y-direction
Q = applied boundary flux

λ = capacity for heat storage
t = time

This equation states that the difference between the heat flux entering and leaving an elemental volume of soil at a point in time is equal to the change in the stored heat energy.

The capacity to store heat is composed of two parts. The first part is the volumetric heat capacity of the material (either frozen or unfrozen) and the second part is the latent heat associated with the phase change. In equation form:

$$\lambda = c + L\frac{\partial w_u}{\partial T} \tag{9}$$

where:

c = volumetric heat capacity (material property)
L = latent heat of water
w_u = total unfrozen volumetric water content
T = temperature
λ = capacity for heat storage

TEMP/W uses the unfrozen water content function of a soil to estimate the latent heat absorbed or released by the soil medium due to the phase changes of the soil water. When the unfrozen water content function of a soil is defined, the total unfrozen volumetric water content can be expressed as:

$$w_u = W_u w \tag{10}$$

where:

W_u = unfrozen water content ($0 < W_u < 1$)
w = volumetric water content of the soil

Substituting for w_u in Equation (9) and then substituting for λ in Equation (8) leads to the complete differential equation:

$$\frac{\partial}{\partial x}\left(kx\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(ky\frac{\partial T}{\partial y}\right)+Q=\left(c+Lw\frac{\partial W_u}{\partial T}\right)\frac{\partial T}{\partial t} \tag{11}$$

6 RESULTS AND DISCUSSION

Analysis from Quebec test-beds:
Table 4 gives the calculated and the recorded frost penetration. The frost penetration was measured from the pavement surface, thus is also includes frost heave. The SSR model always gives slightly deeper frost penetration than the CESAR-GEILS models. In some cases the SSR model gave the best estimates, and in other cases the CESAR-GEILS MODEL succeeded best (Boutonnet, et al., 2003). However, both models give estimates of the frost penetration within 10% of the measured values for all parameters used. There is also a very good agreement between the recorded and calculated frost penetrations from the TEMP/W model. Here we observe that the influence of the alternative temperature functions used are negligible, while the influence of the amount of frozen water in the subground is significant for the coldest winter (2000-01) where the freezing front penetrates significantly into the subgrade.

From the test-bed in Ravalli we observe:

Actual frost penetration in station 6 was somewhat less than calculated by the US Army protocol (Berg and Johnson 1983), but not significantely. Calculations with TEMP/W clearly demonstrates the significant influence of uncertainties in the estimated thermal conductivities and the unfrozen water content. Using the thermal conductivities according to the US Army protocol gives in this case very good agreement with the observed frost penetration. However, with the Kersten formula, this model gives too deep frost penetration. This could be explained by the fact that TEMP/W does not take water flow to the frozen fringe into account, as the strucure in this case has frost-suceptible material in the pavement and possible water flow to these layers. The SSR model also gives a bit too deep frost penetration, even when this model does take the water flow to the frost-susceptible layers into account. The explanation might be that the estimated thermal conductivities are too high.

Table 3: Comparison of the measurements in situ and calculated results from the numerical models:

Test bed	Frost depth (m) Recorded (R) & modeled (M)	Winter 1998-99 FI = 666 ˚C day		Winter 1999-00 FI = 796 ˚C day		Winter 2000-01 FI = 1114 ˚C day	
		Kersten	Mikley	Kersten	Mikley	Kersten	Mikley
1	R	1,30		1,45		1,54	
QUEBEC	M:SSR	1,32	1,21	1,46	1,34	1,62	1,51
	M:GEL 1D	1,20	1,09	1,33	1,26	1,48	1,34
	M:CESAR-GELS	1,24	1,10	1,42	1,26	1,55	1,41
	M:TEMP/W a)		1,29		1,36		1,57
	M:TEMP/W b)		1,29		1,36		1,57
	M:TEMP/W c)		1,29		1,37		1,79
2	R	0,79		0,83		0,84	
QUEBEC	M:SSR	0,81	0.83	0.80	0,82	0.80	0,81
	M:GEL 1D	0,78	0,78	0,79	0,79	0,79	0,79
	M:CESAR-GELS	0,78	0,78	0,79	0,79	0,78	0,78
	M:TEMP/W		0,78		0,78		0,78
RAVALLI	Frost depth (m) Recorded (R) And modeled (M)	Winter 1985-86: FI = 459 ˚C day					
		Kersten	US Army 1966) US)	Mod. Berggren	(Frost heave)		
	R	0,86 (0.12)					
	M: SSR	1.06			(0,12)		
	M: TEMP/W a)	1.15	0.84		-		
	M: TEMP/W c)		1,05		-		
	M: Berg&Johnson			0,97	--		

TEMP a): With no unfrozen water in the subgrade
TEMP b): Applying winter temperatures according to part of a “sinus” curve (no unfrozen water in the subgrade)
TEMP c): With some unfrozen water in the subgrade

7 CONCLUSION

All models tested gave relatively good estimates of frost penetration in the case when the frost front did not penetrate significantly into frost-susceptible layers (Quebec test site). The influence of the unfrozen water contents and possible water flow to the freezing front may cause substantial uncertainties in the modeling results for frost-susceptible materials. The main source of uncertainties in the calculations are the estimated thermal conductivities. The water contents and possible water flow to the frozen fringe also influence the frost penetration significantly (Ravalli County Airport). Thus, the thermal and hydraulic characteristics together with the boundary conditions have to be properly modeled in order to succeed in determining frost penetration. The estimation of the n-factor may also give significant errors in the modeling of frost penetration.

Further research should be conducted to improve the estimates of the thermal conductivity of granular materials and subgrade soils, and to improve the modeling of the influence of water. There are also several challenges to be overcome regarding a proper monitoring of the freezing front in-situ for calibration of the models.

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REFERENCES

Ali,H.A., Tayabji,S.D, 1999: Determination of frost penetration in LTPP sections, final repport, FHWA-RD-99-088

Boutonnet, Michel; Savard, Yves; Lerat, Patrick; St-Laurent, Denis; Pouliot, Nadia, 2003: *Thermal Aspects on Frost-Thaw Pavement dimensioning: In situ measurement and numerical Modeling,* TRB record 1821.

Berntsen, Geir,1993: *Reduction of Bearing Capacity under Spring Thaw*, PhD Thesis, Norwegian Institute of Technology, Department of Road and Railway Engineering, IJV-Medd. 25, 1993:64

Chamberlain, E.C.,1991. *Modified Berggren program for determining the depth of freeze or thaw in layered soil systems*, PC version, U.S. Army Corps of Engineers Cold Regions Research and Engineering Laboratory, Hanover, NH.

Farouki, O.T.,1982: *Evaluation of methods for calculating soil thermal conductivity*. CRREL Report 82-8, U.S.Cold Regions Research and Engineering Laboratory, Hanover, NH, 90 p.

Henry, Karen, 1990: *A Case study of Potential Causes of Frost Heave*, CRREL special report 90-9, April 1990

Heydinger, Andrew G., 2003: *Evaluation of Seasonal Effects on Subgrade Soils*, TRB record 1821.

Jean Côté and Jean-Marie Konrad , 2005: Thermal conductivity of base-course materials, Can. Geotech. J. 42: 61–78 (2005)

Kersten, M.S, 1949: *Laboratory research for the determination of the thermal properties of soils*, Final Report., U.S. Army Corps of Engineers, St. Paul District, 227p.

Kersten, M.S., 1963: *Thermal properties of frozen ground,* First International Conference on Permafrost, Lafayette, Indiana.

Van Rooyen, M and Winterkorn, H.F.,1957: *Theoretical and practical aspects of the thermal conductivity of soils and similar granular systems*, Bulletin 168, HRB, National Research Council, Washington D.C., Pg 143-205

Frost action in soils, *Publication no 17, Sikring mot teleskader*, 1976: Public Roads Administration November 1976, Norway (in Norwegian)

Roberson, Ruth. L., Siekmeier, John,2000: Using a multisegment time Domain Reflectometry Probe to Determine Frost Depth in Pavement Systems, TRB Record 1708, paper no 00.1316, Roberson, Ruth. L., Siekmeier, John, 2004: *Determining Frost Depth in Pavement Systems Using a Mult-Segment Time Domain Reflectometry Probe*, TRB Session: Effects of Moisture and Temperature on Pavement Systems

Saarelainen, Seppo,1992: Modeling frost heaving and frost penetration in soils at some observation sites in Finland, the SSR model, Thesis or the degree of Doctor of Technology, VTT publications 95, 119 p

Saarelainen, Seppo,1998: *Pavement design applying allowable frost heave*, ASCE, Cold Regions Conference, Fairbanks

TEMP/W: Users manual, 2004 + website http://www.geo-slope.com/products/geostudio/tempw2004.htm

William, P. J and Smith, M. W.,1989: *The Frozen Earth*. Cambridge University Press, New York