Prediction of Deformation Behaviors on Stress-Dependent Unbound Pavement Foundations

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ABSTRACT: There are several major practical consequences of the stress-dependent and deformational properties of unbound pavement foundations. Among those are the modulus and Poisson's ratio's that may change, the compressive stresses that are generated in such materials under load, the stiffening and strengthening effect of repeated loading to progressively increase the unbound pavement materials resistance to permanent deformation. The predictions of deformation on stress-dependent unbound pavement foundations are presented in this paper by finite element analyses and a combination of field observations of accelerated loading tests results.

KEY WORDS: Deformation, Resilient dilatancy, finite element analysis, unbound materials, pavement foundations.

1 INTRODUCTION

Unbound granular materials are made up of a range of odd-shaped particles that tend to lie flat when they are compacted. The stiffness of this collection of particles and its volume change under load changes with stress state and with direction mainly because of the shape and gradation of the particles. This gives rise to stress-dependent stiffness and Poisson's ratios that may rises above 0.5. Making use of the fact that the moduli of unbound granular materials are stress-dependent, and assuming the form of the non-linear and stress-dependent to be exponential (Uzan, 1992; Lytton et al., 1993).

If the moduli are stress-dependent, then both of the Poisson's ratios must be stress-dependent as well. Earlier studies by Allen (1973) show measured Poisson's ratios rising above 1.0. The fact that the horizontal Poisson's ratio routinely rises above 0.5 is a major reason why unbound granular mateials in pavement foundations work as well as they do. If granular materials are prevented from expanding horizontally, it will build up a confining pressure that stiffens the surrounding materials under load. Therefore, these material properties of stress-dependent moduli and resilient dilatancy (Poisson's ratio effect) have a direct effect on the major long-term deformation damages of layers in pavement foundations under repeated traffic loading. These effects and deformation for modeling are presented by finite element analyses and accelerated loading tests results.

2 MODEL FOR RESILIENT AND DEFORMATON BEHAVIORS

It is well known that the behavior of unbound granular materials is non-linear and stress-dependent. The most commonly used non-linear elastic modulus model for characterizing stiffening behaviors of unbound granular materials is the k-θ model proposed by Hicks and Monismith (1971). However, several studies have shown that the model does

not accurately predict the response of granular materials. Because the model assume a constant Poisson's ratio of materials as an initial input data, and the effect of shear stresses on the resilient properties is not considered, nor are the dimensional problems of model itself. The shear stress component is especially responsible for dimensional change and consequently, permanent deformation.

However, the Universal Soil Model (Uzan, 1992) introduced the effect of octahedral shear stress into the K-θ model and added atmospheric pressure as a normalizing factor into Equation 1 to make the stress terms non-dimensional as shown by Equation 1.

$$M_r = K_1 \left(\frac{I_1}{P_a} \right)^{K_2} \left(\frac{t_{oct}}{P_a} \right)^{K_3} \quad (1)$$

where:

Mr = resilient modulus for vertical direction,
Pa = atmospheric pressure,
I_1 = first stress invariant,
τ_{oct} = octahedral shear stress, and
Ki = material constants by regression analysis from repeated load triaxial test.

The octahedral shear stress term is believed to account for dilation effect that takes place when a pavement element is subjected to a large principal stress ratio. Depending on the level of stresses, the first stress invariant or bulk stress term considers the hardening effect associated with higher modulus, while the octahedral shear stress term considers the softening effect (Park and Fernando 1998, Park et al. 2003).

In addition, the Poisson's ratio of unbound pavement materials is also known to be the stress-dependent and should therefore be considered with the stress-dependent modulus simultaneously in a single framework. For this study, a relationship between the Poisson's ratio and the resilient modulus is adopted based on a thermodynamic constraint based on the studies by Lade and Nelson (1987), Liu (1993), and Lytton et al. (1993). This relationship was established between the resilient modulus as expressed in Equation 1 and the thermodynamic constraints to derive an expression that relates the stress state and the rate of change of the Poisson's ratio with a changing stress state as described in Equation 2.

$$\frac{2}{3} \frac{\partial n}{\partial J_2} + \frac{1}{I_1} \frac{\partial n}{\partial I_1} = n \left[\frac{1}{3} \frac{K_3^{'}}{J_2} + \frac{K_2}{I_1^2} \right] + \left[-\frac{1}{6} \frac{K_3^{'}}{J_2} + \frac{K_2}{I_1^2} \right] \quad (2)$$

where:

ν = Poisson's ratio,
$K_3' = K_3/2$,
Ki = material parameters,
I_1 = normalized first stress invariant, and
J_2 = normalized second invariant of the deviatoric stress.

Whereas the understanding and prediction of permanent deformation of unbound pavement materials has been far behind than in the area of resilient responses. Typically, the permanent deformation caused by the number of loadings increases exponentially or asymptotes to a plateau value. Several prediction models have been proposed for

characterizing the permanent deformation behavior of unbound materials under repeated loads.

Among the various models, two models are selected for this study. One of these is the VESYS as a linear model (Kenis, 1978; Kenis and Wang 1997) and the other is the Three Parameter as a non-linear model (Tseng and Lytton 1989). The VESYS model states that the ratio of vertical plastic strain per cycle, de^P/dN, to the resilient strain, e_r, is an

exponent function of the number of load cycles, N.

$$\frac{1}{e_r}\frac{de^P}{dN} = mN^{-a} \quad (3)$$

where:

$\qquad e_p$ = permanent deformation,

$\qquad e_r$ = elastic or/ resilient deformation,

$\qquad$ N = the number of load applications,

$\qquad$ m = parameter representing the constant of proportionality of strains, and

$\qquad$ a = parameter indicating the rate of decrease.

Whereas the non-linear Three Parameter model states that the plastic strain has a limit, e_0^P, and a logarithmic rate of work hardening and also that the plastic strain increases with the number of load applications, N.

$$e^P = e_0^P\, e^{-\left(\frac{r}{N}\right)^b} \qquad (4)$$

where:

$\qquad e_p$ = permanent deformation,

$\qquad$ N = the number of load applications, and

$\qquad e_o$, r, b = threes material parameters

3   METHODOLOGY FOR MODELING DEFORMATIONS

Permanent deformation in flexible pavements is the direct result of the passage of loads over the pavement surface and the strain induced by the load. This induced strain can be simplified as two components, the resilient strain and the plastic (or permanent) strain. It is suggested that the resilient strain remains fairly constant during the major part of the pavement's life, except for at a low number of load repetitions where the material undergoes conditioning and near failure (Uzan et al. 1988). It can also be assumed that the elastic strain is constant throughout the pavement life and the plastic strain per load application is assumed to decrease with the number of load applications.

For the deformation calculation, a 2-D nonlinear elastic finite element program is developed with the use of the layer strain approach. In this program, the nonlinear analysis is made using an incremental loading and an iterative solution technique for each load increment. Stress dependent modulus and Poisson's ratio are used. This algorithm handles both stress-dependency and equilibrium criteria in an incremental scheme. In this procedure,

strains under loads for each season are assessed. In order to predict deformation development of the pavement structure with respect to the specified seasons and years.
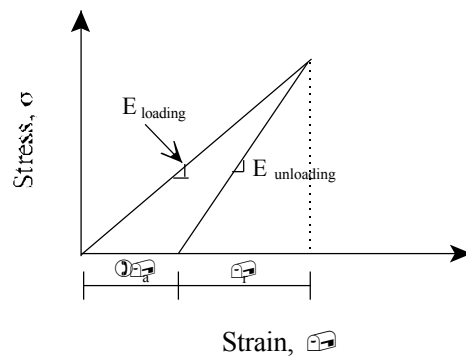

Figure 1: Simplified Relationship between Stress and Incremental Permanent Strain under the Passage of a single Axle Load.

This approach has the assumption that the permanent strain is functionally proportional to the stress state and number of load applications. From the results of structural analysis, deformation at the surface is calculated by summing the products of the permanent strains and the corresponding difference in depths between the layers using the layer strain approach as presented in Figure 2.
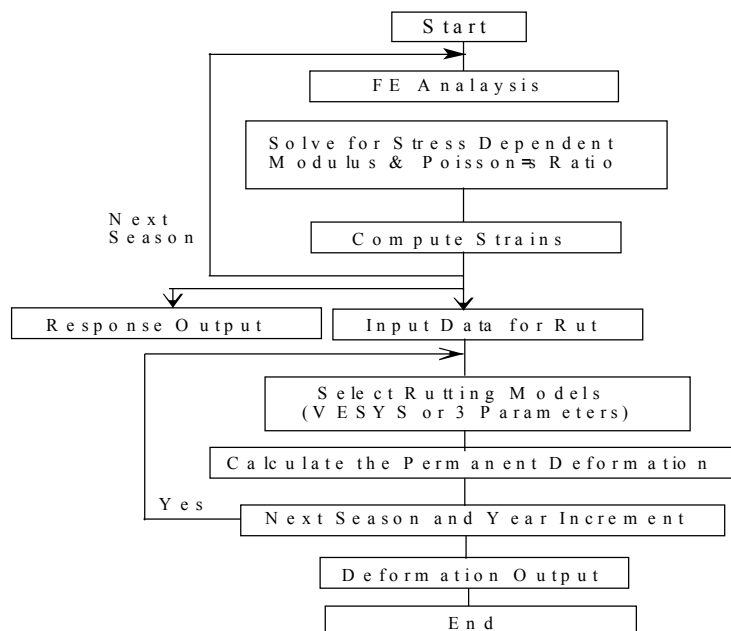

Figure 2: Flow Chart for Predicting Deformation Using Stress-dependent FE Program.

The incremental strain for each load application is calculated by taking the derivative of the permanent strain with respect to load application and is shown in the Equation 3. Two permanent deformation parameters that can define the characteristics of rut in relation to the

materials are the alpha, (α), and the gnu, (μ). These parameters can then be used to substitute for I and S and the incremental strain is calculated by:

$$\Delta e_a^{(N)} = e_r m N^{-a} \qquad (5)$$

when N is equal to 1,

$$m = \frac{e_a^{(1)}}{e_r} S \qquad (6)$$

From Figure 1,

$$E_{lo^{(N)}} = \frac{S}{e_r + \Delta e_a} = \text{modulus during loading} \quad (7)$$

$$E_{un} = \frac{S}{e_r} = \text{modulus during unloading} \quad (8)$$

By rewriting Equations 5 through 8, modulus during loading can be written as:

$$E_{lo^{(N)}} = \frac{S}{\Delta e_a^{(N)} + e_r} = \frac{S}{e_r(mN^{-a}+1)} = \frac{E_{un}}{1+mN^{-a}} \qquad (9)$$

The Equation 9 gives a relation between loading modulus and unloading modulus as a function of permanent deformation characteristics of pavement materials, α and μ, and the number of load applications.

To estimate the incremental permanent strain from a single axle load application, it is assumed that the stress-strain relationship is of the form shown in Figure 1. Using geometry, it can be shown that:

$$\frac{\Delta e_a}{\Delta N} = S\left[\frac{1}{E_l} - \frac{1}{E_u}\right] \qquad (10)$$

The fractional increase of the total strain, F(N), which is the permanent strain with load repetition is given by:

$$F(N) = \frac{\Delta e_a}{e_r + \Delta e_a} \qquad (11)$$

It is assumed that the resilient strain is large in comparison to the increase of the permanent strain with each load repetition. With this assumption, the following approximation can be used:

$$F(N) \cong \frac{\Delta e_a}{e_r} \qquad (12)$$

Since

$$\frac{\Delta e_a}{\Delta N} \approx \frac{\partial e_a}{\partial N} \qquad\qquad (13)$$

the fractional increase of the total strain, F(N), can be expressed as:

$$F(N) \cong \frac{\Delta e_a}{e_r \, \partial N} = \frac{E_u}{E_l} - 1 = mN^{-a} \qquad\qquad (14)$$

The rut depth may be then estimated by:

$$d_a(N) = \int_0^N \int_o^{Z_{max}} e_c(z) \, F(N) \, dz \, dN \qquad\qquad (15)$$

where

Z_{max}	=	depth of the pavement layer, and
$\varepsilon_c(Z)$	=	compressive strain at depth z.

The Equation 15 may be extended to include all layers as follows:

$$d_a(N) = \int_0^N \int_o^{d_1} e_c(z) F_1(N) \, dz \, dN + \int_0^N \int_{d_1}^{d_2} e_c(z) F_2(N) \, dz \, dN + \Lambda$$
$$+ \int_0^N \int_{d_{n-1}}^{d_n} e_c(z) F_n(N) \, dz \, dN \qquad\qquad (16)$$

where $d_1, d_2, ..., d_n$ are the depths of each layer in the pavement, and $F_1, F_2, ..., F_n$ are the fractional increases of the total strain for each layer. So, the rut depth becomes:

$$d_a(N) = \sum_{i=1}^n \left[\int_0^N m_i \, N^{-a_i} \, dN \int_{d_{i-1}}^{d_i} e_c(z) \, dz \right] \qquad\qquad (17)$$

The first integral on the right-hand side of Equation 17 becomes:

$$\int_0^N m_i \, N^{-a_i} \, dN = \frac{m_i \, N^{1-a_i}}{1-a_i} \qquad\qquad (18)$$

Therefore, the total deformation becomes:

$$d_a(N) = \sum_{i=1}^n \left[\frac{m_i \, N^{1-a_i}}{1-a_i} \int_{d_{i-1}}^d e_c(z) \, dz \right] \qquad\qquad (19)$$

Equation 19 can be integrated numerically in each layer beneath the center of loads. Then, the predicted deformation at the surface can be the sum of the vertical permanent strains of all the layers.

Using the Three Parameters model, the fractional increase of the total strain, F(N), can be expressed by replacing the first derivative of Equation 4 with respect to N into Equation 14 as follows.

$$F(N) = \frac{\partial e_a}{e_r \partial N} = \frac{e_0 b r^b}{e_r} e^{-(r/N)^b} N^{-(b+1)}$$

(20)

Therefore, the rut depth becomes:

$$d_a(N) = \sum_{i=1}^{n} \left[\int_0^N \frac{e_{o_i}}{e_{r_i}} e^{-(r_i/N)^{b_i}} dN \int_{d-1}^d e_c(z)dz \right]$$

(2 1)

4 VERIFICATION BY FIELD OBSERVATIONS

A finite element program has been adapted to take the K_1 through K_3 values for vertical direction as input and convert them into stiffness that vary with stress state. Both the moduli and the Poisson's ratios vary within a pavement structure from element to element beneath a load. The stresses are different from what are predicted with linear finite element, linear layered elastic, and non-linear layered elastic methods. The radial strains calculated in the granular base materials using the stress-sensitive approaches are commonly smaller than those calculated by other methods. These results are a consequence of the larger Poisson's ratios and higher continuing pressures that are generated in the stress-dependent finite element program (Park and Lytton 2004). For the permanent deformation prediction, a 2-D non-linear stress-dependent finite element program is used as described earlier.

The permanent deformation data measured in the field by Chen et al. (1999) using the Texas Accelerated Mobile Load Simulator (CTR, 2003) were used to compare with the values calculated with the nonlinear stress-dependent finite element analysis. Figure 3 shows multiple-depth deflectometer installations that were used at each of two field test sites. The layer thickness and materials at each site are shown on that figure as well.

The total deformation and the deformation in pavement foundations at test site 281N1 is shown in Figure 4 compared the values predicted with the developed finite element program. Figure 5 compare the same results for test sites 281S1. The wavy predicted line indicates that

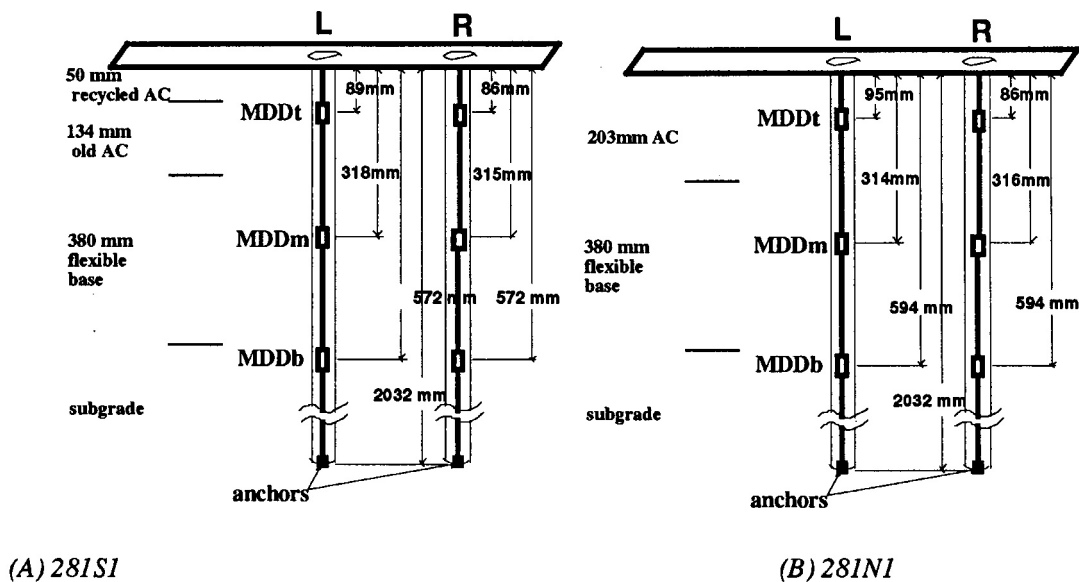(A) 281S1   (B) 281N1

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Figure 3: Structure and MDD Installations for Pavement Foundations (Chen et al. 1999).
the permanent deformation properties of the asphalt surface layer are temperature-dependent.

It was stated that the relative percent of the total deformation in the asphalt surface and in the pavement foundations changed significantly during the course of the loading. The asphalt layer percentage increased and the base and subgrade percentage decreased as the number of load applications increased. These are illustrated in Figures 6 and 7, in which are also shown the predictions by the finite element program. Predicted and observed results from test sites 281N1 and 281S1 are shown in Figures 6 and 7 respectively.
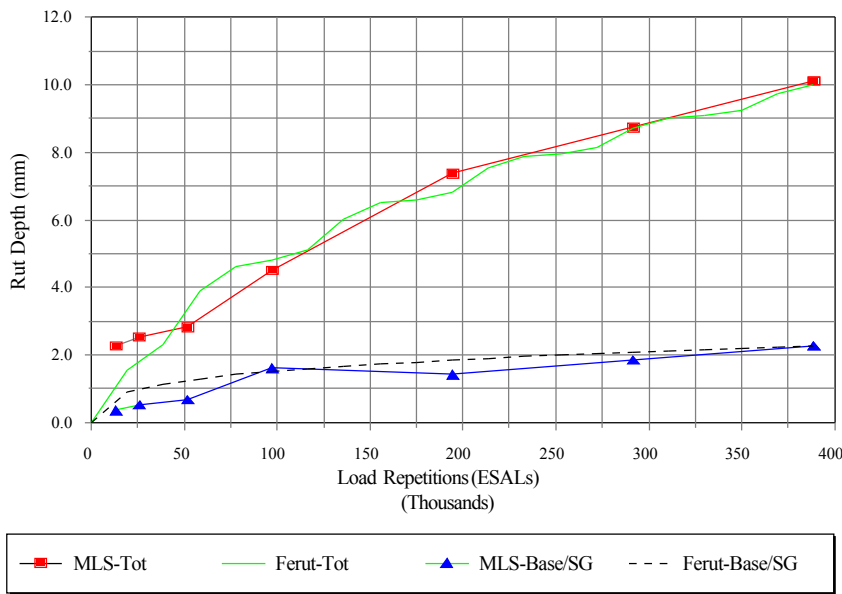

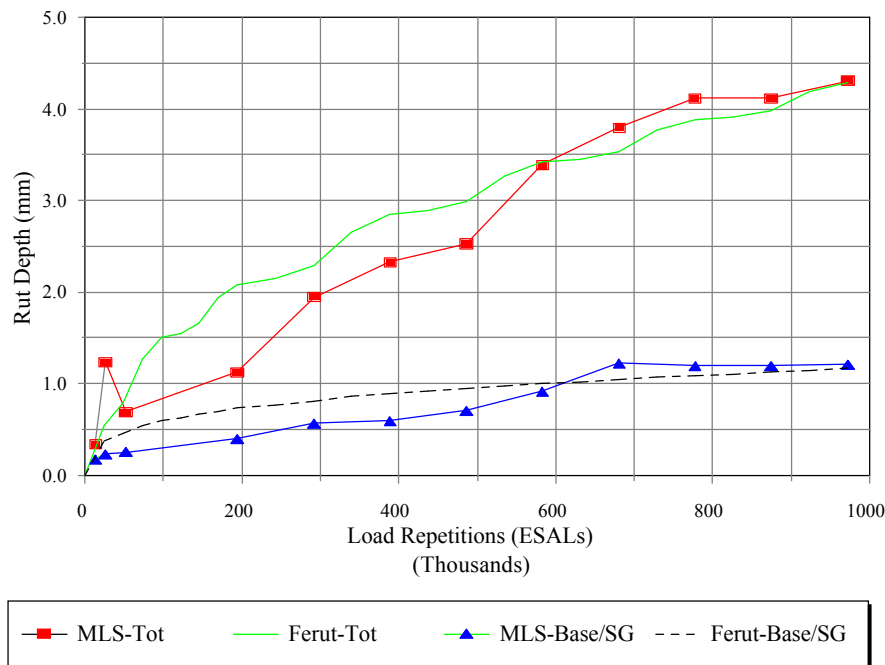Figure 4: Comparison between Measured and Predicted Deformation in 281N1.

Figure 5: Comparison between Measured and Predicted Deformation in 281S1.
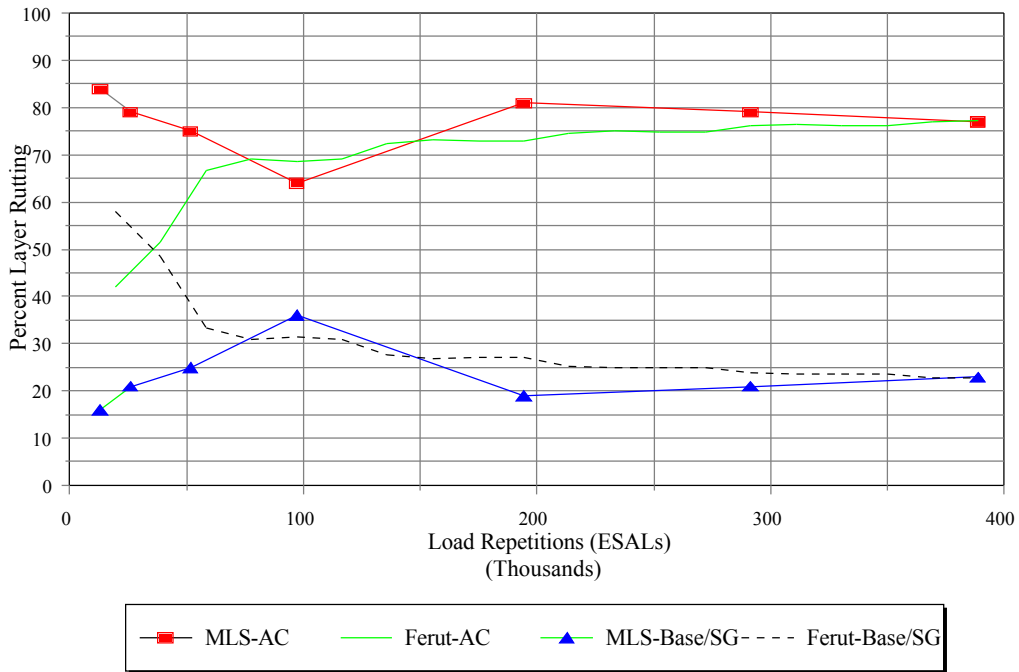
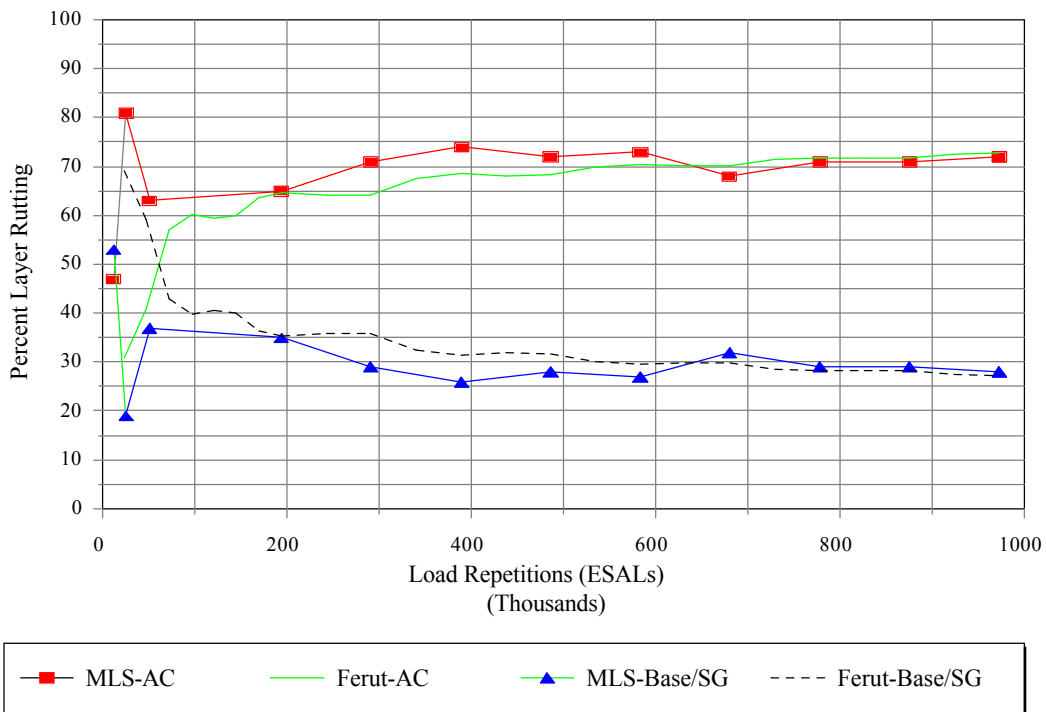Figure 6: MLS Measured and Predicted Percent Layer Rutting in 281N1.

Figure 7: MLS Measured and Predicted Percent Layer Rutting in 281S1.

5   DEFORMATION PREDICTIONS

An analysis was conducted to demonstrate the fact that the stress-dependent behavior has a significant influence in the predicted granular layer deformation in conventional flexible pavements (CFP). Deformations were estimated using the VESYS model. Because of the unavailability of some of the material properties, typical values for the VESYS deformation parameters, a and m, are assumed for each layer in the analyses. Although the VESYS model is only used in this analysis, it is expected that the Three Parameter model has the same trends as those of the VESYS model, because the main efforts were made to identify the layer deformation or rutting trends based on the stress dependency instead of different deformation models. The average values used for an example analysis are noted in Table 1.

Table 1. Resilient and Deformation Parameters for an Example Analysis.

Layer	Thickness (mm)	K_1	K_2	K_3	a	m
AC Surface	100	50,000	0.1	0.0	0.60	0.75
Granular Base	200	700	0.6	-0.3	0.84	0.53
Subgrade	2300	400	0.0	-0.3	0.81	0.08

Note: A single load of 40 kN was applied over a circular area with a radius of 135 mm.

Stress dependency governs the shape of the deformation that forms as well as the amount that occurs in each layer. This is illustrated in Figures 8 and 9 that compare the predicted shapes of the permanent deformation in the asphalt surface, granular base and subgrade layers when stress dependency is considered (Figure 9) and when it is not (Figure 8). This stress-dependent computations show that the subgrade deforms less and the base and asphalt deform more than in the case when stress dependency of unbound granular base was not considered.
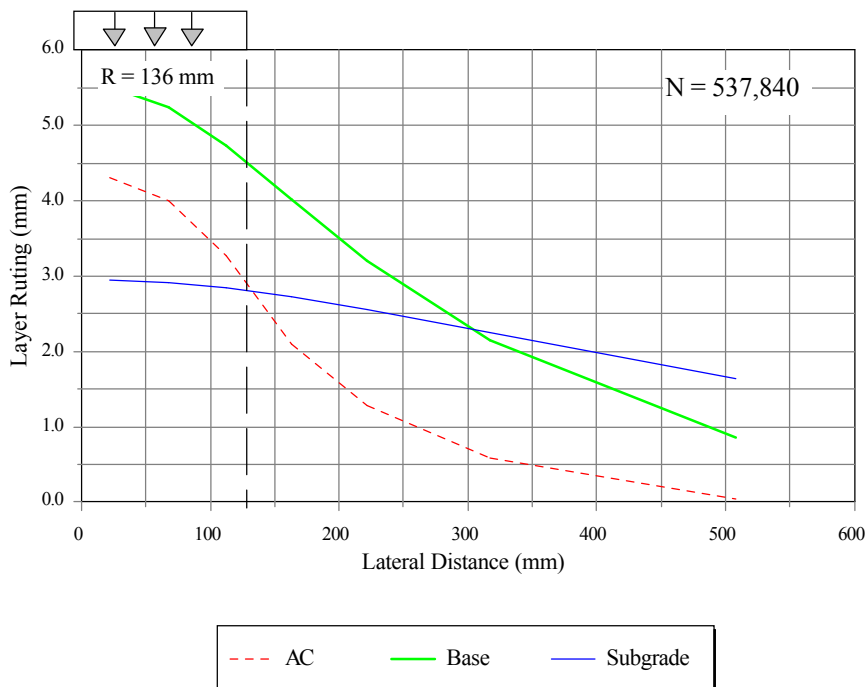

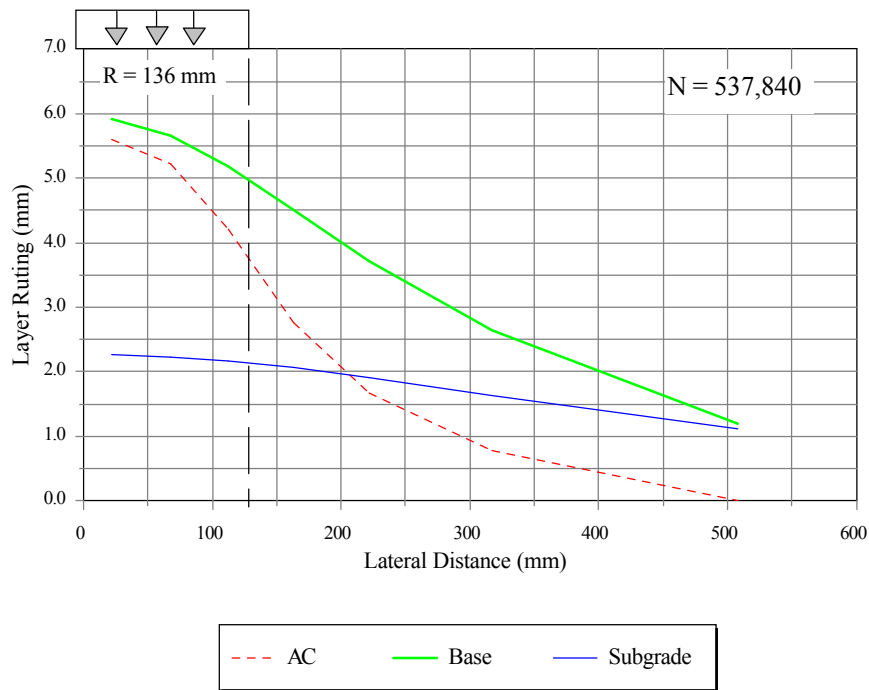Figure 8: Predicted Layer Rut Depth Profile without Stress Dependency.

Figure 9: Predicted Layer Rut Depth Profile with Stress Dependency.

6   CONCLUSIONS

The stress dependent modulus and Poisson's ratios of the unbound pavement materials, when represented properly in a finite element program, are able to closely duplicate the deformational behavior of these materials under accelerated repeated loading such as is provided by the Texas MLS. The overall results presented that the non-linear stress-dependent finite element method was consistently closer to the observed measurements as well. However, the question still arises whether the stress-dependent approach for analyzing the deformational behaviors of unbound pavement materials is more realistic. As answers, the stress-dependent approach is more realistic because it represents actual observed material properties and comparisons of predictions using the stress-dependent program with observed permanent deformation patterns from accelerated load tests demonstrated well. However, many of the important variables and material properties that are still needed to make an accurate evaluation, and a more detailed study have to be made.

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