A non-linear elastic material model and its application in pavement design

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ABSTRACT: A non-linear elastic material model with a stress level control defined by the Mohr-Coulomb yield criterion has been developed, implemented, and tested in practice. The model is developed for unbound granular base materials in conjunction with relevant test methods. In the design process the material model is used to predict pavement response and theoretically confirm a stabilized behavior in the base material during cyclic loading. Input parameters for the model are directly linked to practical field tests where design parameters can be validated in-situ during the construction of the pavement. Critical parameters are void ratio, isotropic mean stress, and matric suction. The in-situ mean stress is here the accumulated effect of both compaction induced residual stresses and the contribution due to matric suction. The in-situ mean isotropic stress is estimated indirectly from seismic measurements. The matric suction in the unbound granular material is measured with gypsum blocks. The practical usefulness of the proposed model is demonstrated with a case study where the pavement construction is economically optimized based on available materials. Results show that the model can accurately predict the non-linear pavement response based on the material properties measured during construction.

KEY WORDS: Unbound granular material, non-linear elastic material model, SwePave.

1 INTRODUCTION

Geotechnical aspects in design and construction of infrastructures have a major influence on the performance of road pavements and on their maintenance costs. The state of the art of pavement design has been empirical, with trial and error as basis for the decisions.

The new pavement construction concept, SwePave (Ekdahl et al 2004), is an attempt towards a more scientific approach to the practice of pavement geotechnology. To get an entirety in the concept for analytical pavement design it was necessary to develop a few missing important pieces.

- A new non-linear elastic material model with a Mohr – Coulomb yield criteria which represent a serviceability limit line stress boundary. Cyclic stresses below this boundary will result in stable behavior.
- A new measurement and evaluation technique to evaluate the parameters defining the threshold line for plastic deformations directly in the field.
- A new unified approach to use test methods both in the laboratory and in the field. Essential common parameters which must be considered for unbound granular material (UGM) are the void ratio, isotropic mean stress and the matric suction.

- A new seismic measurement and evaluation technique (Rydén 2004). The master curve of the asphalt layer can now be determined directly in the field from non-destructive testing. The compaction induced residual horizontal stress in the UGM can now be derived by seismic measurement performed directly in the field. In the case of stabilized soil it is now also possible to evaluate both the stiffness and indirectly the unconfined compressive strength by seismic measurements.
- Implementation of the new non-linear elastic material model has been incorporated in a 3D FEM program called ZSOIL.

In this paper the new non-linear elastic material model will be described and the application in practice will be demonstrated.

2 ANALYTICAL PAVEMENT DESIGN

The design process proposed is simply a check if stabilized behaviour will occur in the UGM pavement layer during cyclic loading. However the number of cycles corresponding to fatigue failure is also checked for the asphalt cover. Should stabilised behaviour not occur which mean that we get plastic deformation in the UGM then the pavement must be redesigned. For example the pavement design can then be altered by increasing thickness of the asphalt cover. The horizontal stress assumption in the UGM are however critical in this calculations and must be determined indirectly in the field from preferable seismic wave speed measurements.

2.1 New non-linear elastic model for the UGM layer

The stiffness of the soil is not a material constant it is a state parameter that is dependent on the void ratio of the soil, the stress level and the strain level.

The understanding of the importance of the non-linear relationship between stress and strain and the development of new in-situ methods to determine this relationship have increased the possibilities to perform a prediction of deflections with good precision. The author has developed a revised method to determine the non-linear stress-strain relation for soil in-situ with a modified procedure to perform a pressuremeter test (Ekdahl 1997). The pressuremeter test is here performed with unloading and reloading loops and also a load step with longer duration to determine the time-dependent behaviour. Now a new material model which describes the non-linear elastic behaviour for preferably UGM has been developed. The same fundamental methodology as used for the pressuremeter test has been used but now the basis for the development is the triaxial test. The base for the small strain behaviour are the work done by Biarez and Hicher (1994) and the base for the large strain behaviour are the extensive triaxial tests done by the Swedish National Road Administration for the test road project Sunninge in Sweden (Ekdahl et al 2004).

In the new model the secant modulus E_s is defined as:

$$E_s = K_1 \left(\frac{\max(p, p_L)}{p_{ref}} \right)^{K_2} \left(1 - K_3 \min(\eta, \eta_{max})\right) \qquad (1)$$

where

$$\text{h} = \left(\frac{q - q_0}{p_{ref}}\right)^{K_4} \left(\frac{\max(p, p_L)}{p_{ref}}\right)^{-K_2} \quad \textit{(2)} \qquad \text{and} \qquad \text{h}_{\max} = \frac{1 - \text{x}}{K_3} \quad \textit{(3)}$$

Max means that the biggest value of p or p_L should be used.
Min means that the smallest value of η or η_{max} should be used.

p = Mean stress which is the sum of the in-situ mean stress and the mean isotropic stress change caused by traffic load.
p_L = Limit mean stress. When calculating E_s, p is not allowed to be smaller than this value.
p_{ref} = Reference pressure.
q = Deviatoric stress.
q_o = In-situ deviatoric stress.
ξ = Preserves that $E_s > 0$. ξ can be very small for instance 0,001 but must be verified.

The in-situ mean stress is the accumulated effect of both compaction induced residual stresses and the contribution due to matric suction in the UGM. The in-situ mean isotropic stress is estimated indirectly in the field from seismic measurements. The matric suction in the UGM is measured with gypsum blocks.

If the change in deviatoric stress is negative which means that we have an unloading situation the η parameter is set equal to zero. The secant modulus E_s is therefore for unloading situation calculated as:

$$E_s^{\,unload} = K_1 \left(\frac{\max(p, p_L)}{p_{ref}}\right)^{K_2} \qquad \textit{(4)}$$

The new non-linear elastic model has been incorporated in the FEM code ZSOIL.

The constants K_1 to K_4 can be estimated by:

K_1 = 500/e (Biarez et al 1999)
If E, p and q is in MPa. The expression is valid if the UGM is newly compacted and not affected by ageing.
e = void ratio which is derived from the water content and dry density measured directly in the field by the Troxler method.

K_2 = 0.5 (Biarez et al 1994)
A unique value for all kind of UGM

K_3 = 0.65 – 0.74e
This expression ought to be valid for all kind of UGM. (see Figure 1)

K_4 = 0.32 - 0.17e
This expression ought to be valid for all kind of UGM. (see Figure 1)

The correlation between K_3 and K_4 against the void ratio e is based on triaxial tests done on base material of crushed rock (0-32 mm). The different rock material consisted mainly of quarts, feldspar, amphiboles and biotite. There were no carbonates or harmful fines in these materials. The differences were mainly in mica content which varied between 6 and 34 %. In total 96 determinations of E-modulus was done with the triaxial test apparatus for different isotropic and deviatoric stresses in the load cycles. The result of the correlation is shown in Figure 1. The secant modulus predicted by the new non-linear model is compared by measured secant modulus in triaxial tests. The result of a comparison for one of the tested materials is shown in Figure 2.

For small strains, the increase in deviatoric stress is equal to zero, the model gives therefore Eq. (4). With the values suggested for the constants K_1 and K_2 Eq. (5) is attained.

This modulus is in the same order as the modulus determined with seismic measurement technique.

$$E_{\max} = \frac{500}{e} * \sqrt{p} \qquad (E_{max} \text{ and } p \text{ in MPa}) \qquad (5)$$

This is in accordance to recommendations presented by Biarez et al. (1999) and Santamarina et al. (2001). The expression has been developed by using high quality triaxial tests and applies to road aggregates not affected by ageing.

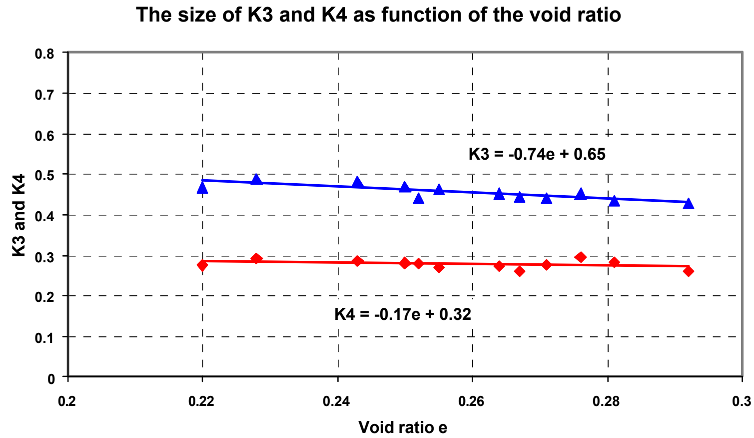

Figure 1: Correlation of the constants K_3 and K_4 against void ratio.

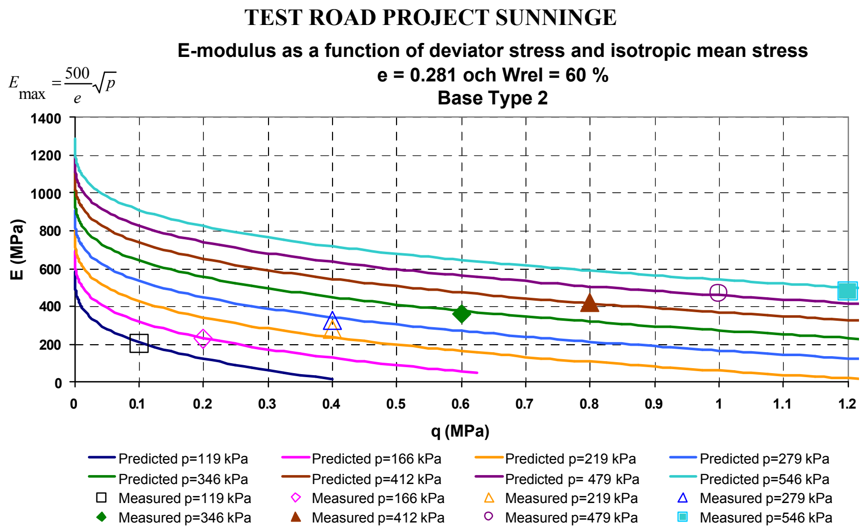

Figure 2: Measured E-modulus compared with predicted modulus using the non-linear elastic material model for different level of isotropic mean stress.

2.2 Mohr-Coulomb yield criteria

The size of the strength parameters c´ and ϕ´ are chosen so they represent a serviceability limit state yield boundary. If we have stresses below this yield criterion we will get a stabilized behavior in the UGM pavement layer with only resilient deformations during cyclic loading.

It is shown for cyclic triaxial loading tests that the plastic strains increases when the dilation phase starts (Tam 1986). The transition stress state from contraction to dilation may be identified by static triaxial tests as the stress state when the slope of the volumetric strain curve is zero which means that no dilation work has been started. Therefore the friction angle is chosen as the critical state value. The critical friction angle for UGM is dependent on its mineral composition (Larsson 2001). This means that if the mineral composition in the UGM is known the inclination of the yield plane for the serviceability limit state yield boundary could be estimated.

It is possible to evaluate the size of the c´ value by use of the so called creep load determined with a carefully done static plate load test (SPLT) performed directly in the field. The principle for this methodology is shown in Figure 3.

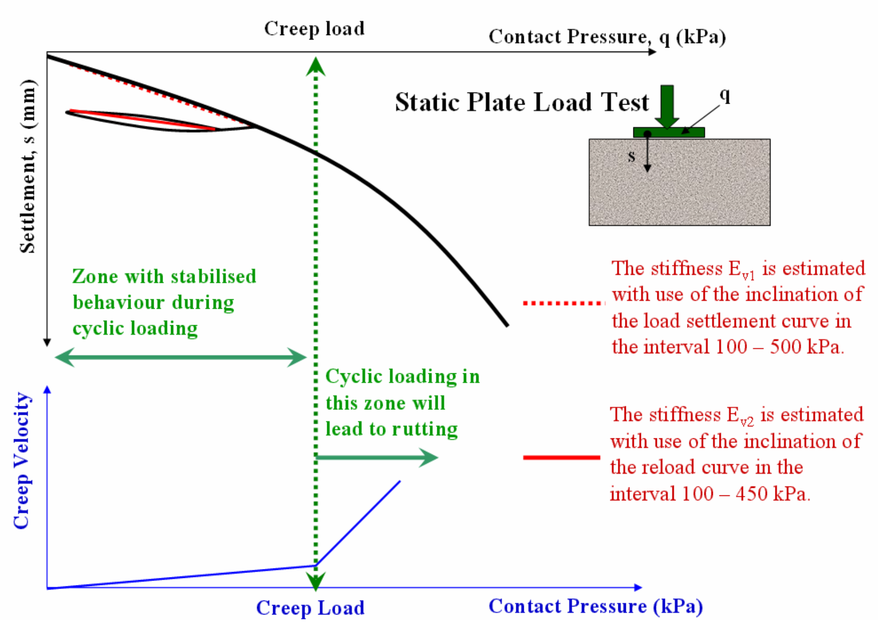

Figure 3: Determination of the creep load with a SPLT.

The load settlement curves for different c´ values are calculated with use of an axial symmetrical model in ZSOIL. In Figure 4 the idea to find out this strength parameter representing the yield criteria for the serviceability limit state is shown. Typical data for a pavement UGM consisting mainly of crushed gneiss are used in this example. The critical friction angle, valid just before dilation starts, is set to 30 degrees according to recommendations by Larsson (2001). The compaction induced stresses is taken into consideration through the horizontal stress which is estimated indirectly in the field from seismic measurements to 25 kPa. The so called creep load is determined when we start to have plastic deformation in the continuum and the load settlement curve starts to bend. With the normal loading procedure when SPLT test is performed it is possible to verify that we have a c´ up to about 30 kPa for the serviceability limit state yield boundary condition. It is also possible to determine the c´ value for the yield criteria representing a stabilized behavior during cyclic loading with use of the heavy falling weight deflectometer FWD. Cyclic loading tests are performed on the UGM for successive increasing stress levels on the same spot. An example with results from such a test is shown in Figure 5. The tests are performed on a conventional UGM base and the compaction induced residual stress in this UGM has been evaluated to 25 kPa. If the modulus increases with the number of blows the behavior is contractible in the UGM. On the other hand if the modulus decreases with the number of blows we have a dilatable behavior in the UGM. It is seen in Figure 5 that the transition state between contraction and dilatation corresponds to a contact pressure between 500-600 kPa. The c´ value is then evaluated with the same approach as described for the static plate load test SPLT in Figure 4. Therefore according to Figure 4 we have a c´ about 35 kPa which represent a serviceability limit state yield boundary condition.

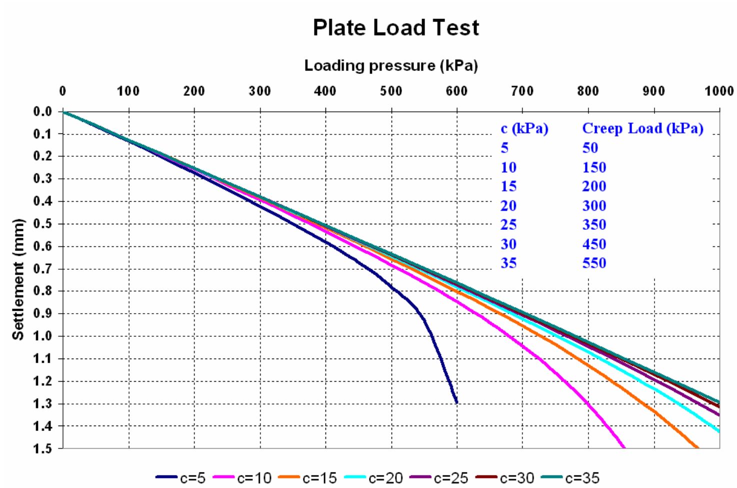

Figure 4: Determination of the effective cohesion c′ as function of the creep load.

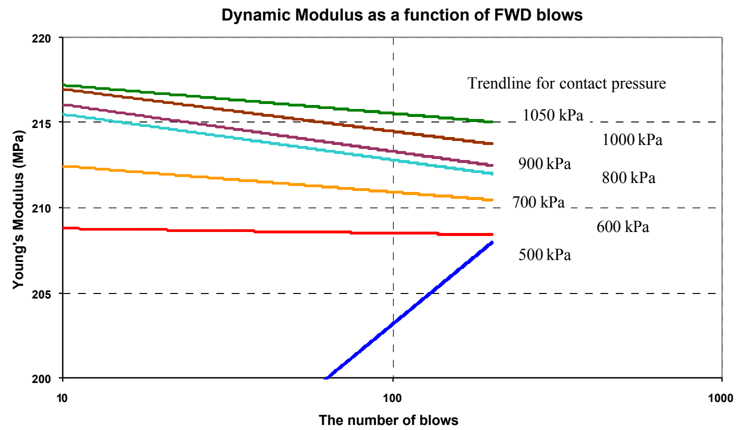

Figure 5: Measured dynamic modulus with FWD tests as function of the number of blows and the contact pressure.

2.3 The compaction induced residual in-situ stress

In UGM plastic deformation will occur due to compaction and traffic load. Because the material within the layer is confined, the stresses in the unloaded state will also progressively change under the effects of repeated load. These residual stresses have an important influence on the granular layer. An increased residual stress level due to compaction has the effect of increasing the strength of the UGM. The residual horizontal stress is the key factor limiting permanent deformation. The accuracy of permanent deformation prediction for layered systems thus will depend on the ability to predict the horizontal in-situ state of stress.

The stiffness, the resilient modulus, is also very dependent of the size of the residual stresses, see Eq. (1) and (5). The horizontal stress in the UGM must therefore be measured indirectly by preferably seismic measurements performed directly on the compacted UGM in the field. The in-situ mean isotropic stress p_o in the UGM could be evaluated from the measured horizontal compression wave speed, v_p, with use of the formulas given in Eq. (6) and (7). Eq. (7) is derived from Eq. (5).

$$E = \frac{v_p^2 \rho (1+\nu)(1-2\nu)}{(1-\nu)} \quad (6) \qquad \text{and} \qquad p_o = \left(\frac{E*e}{500}\right)^2 \quad (7)$$

where
(E and p_o in MPa)

ρ = Bulk density
E = Young's modulus
e = Void ratio determined with the Troxler method
ν = Poisson's ratio

2.4 The hydrological cycle has a significant effect on the UGM loading response

A change in isotropic stress will result in both a change in strength and stiffness of the UGM pavement material. A change of matric suction in the UGM will cause a change in the isotropic stress and therefore it is straightforward to take this influence into account when doing the pavement design. The size of the matric suction is dependent of the saturation degree in the UGM. The matric suction could be measured with gypsum blocks installed in the UGM and the saturation degree is calculated with help of the water content and the dry density determined with the Troxler method.

The isotropic stress due to suction in the UGM can be calculated as the saturation degree multiplied with the matric suction. With ZSOIL and the new developed material model we have a possibility to take the effect of the hydrological cycle into account due to the fact that in ZSOIL it is possible to model flow both in partial and fully saturated soil. In ZSOIL for unsaturated conditions a Van Genuchten (1980) relationship is used. The saturation degree S is calculated as given in Eq. (8).

$$S = S_r + \frac{1 - S_r}{\sqrt{1 + (\alpha * \frac{s}{\gamma_F})^2}} \qquad (8)$$

where

S_r = Residual saturation degree.
α = A parameter which is chosen so the water retention curve agree with the measured saturation degree.
s = Matric suction.
γ_F = Unit weight of water.

For positive pore pressures the saturation degree is equal to 1.

3 CASE RECORD

In Malmö a new road, Lorensborgsgatan, has recently been designed and constructed according to the SwePave concept. The asphalt thickness is 150 mm and is placed on a 300 mm crushed base layer (0–90 mm) where its surface is sealed and leveled by crushed material (0-40 mm). The upper part of the subgrade which consists of clay till is stabilized with 2 % hydrated lime. The thickness of the stabilized layer is 300 mm.

3.1 Measured properties in the UGM pavement layer

Compaction controls performed with the MSOR method (Rydén 2004) on the UGM base layer resulted in P-wave velocities around 520 m/s. With a Poisson's ratio equal to 0.12 the seismic modulus becomes 600 MPa (Eq. 6). MSOR measurements on the asphalt surface resulted in E_{max}-modulus equal to 639 MPa. The in-situ mean stress which is the accumulated effect of both compactions induced residual stresses and the contribution due to matric suction in the UGM can then be calculated with Eq. (7). If the void ratio is set to 0.15, evaluated using dry density and water content determined with the TROXLER method, and the E_{max}-modulus is set equal to 600 MPa then the in-situ mean stress p_o becomes 32 kPa.

The suction in the UGM has been measured with embedded gypsum blocks. In Figure 6 measured matric suction on two gypsum blocks are shown.

The saturation degree is calculated with Eq. (8) using the measured matric suction. The parameter α is chosen so the water retention curve agrees with the calculated saturation degree using dry density and water content determined with the TROXLER method.

The suction dependent effective cohesion can be calculated as:

$$c'(suction) = S * s * \tan(\phi'_{cr}) \qquad (9)$$

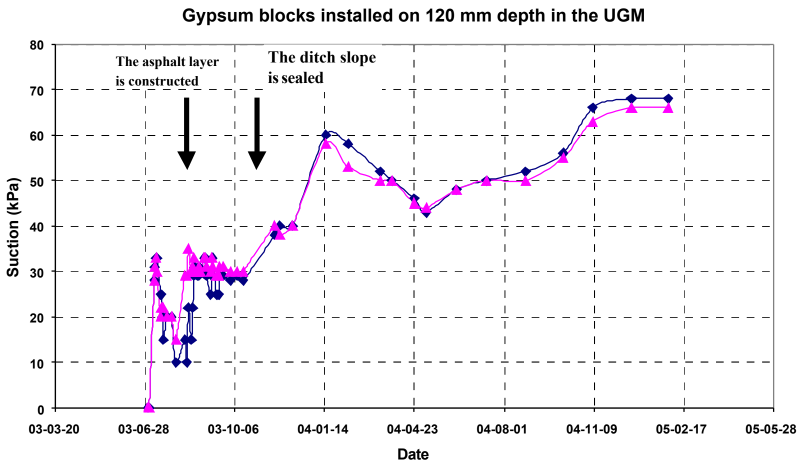

Figure 6: Measured suction in the UGM from two embedded gypsum blocks.

If the in-situ saturation degree is set to 50 %, the suction to 35 kPa and the critical friction angle to 30 degrees then the suction dependent effective cohesion becomes 10 kPa according to Eq. (9). This means that if this UGM becomes saturated the total effective cohesion will decrease from 35 kPa to 25 kPa. At the same time the seismic modulus E_{max} will decrease from approximately 600 MPa to about 400 MPa. Consequently both the seismic modulus and the effective cohesion will decrease with about 30 % if this UGM becomes saturated.

3.2 Validation of the new non-linear elastic model for the UGM

Heavy FWD tests with contact pressures 700, 800 and 900 kPa have been compared with FEM calculated deflection curves. It should be noted that it is only the UGM layer and the subgrade that is modeled with the new non-linear elastic material model. All other layers are modeled as linear elastic. The properties of the pavement materials have been chosen with the aid of performed investigations:

Asphalt	Temperature 7 degrees Celsius E = 9800000 kPa $\nu = 0.26$ $\gamma = 23$ kN/m^3
UGM, base layer	Void ratio e = 0.15 $\nu = 0.12$ $p_{ref} = 1000$ kPa $K_1 = 3333330$ $K_2 = 0.5$ $K_3 = 0.54$ $K_4 = 0.29$ $\gamma = 23$ kN/m^3 c' = 35 kPa ϕ' = 30 degrees Compaction induced horizontal stress including the effect of suction = 45 kPa, the corresponding total mean isotropic stress is 32 kPa
Lime stabilized subgrade layer	E = 600000 kPa $\nu = 0.12$

	$\gamma = 22$ kN/m^3	c' = 115 kPa	ϕ' = 30 degrees
<u>Clay till</u>	$K_1 = 1653000$	$K_2 = 0.5$ $K_3 = 0.45$	$K_4 = 0.27$
	$\nu = 0.35$	$p_{ref} = 1000$ kPa	$K_o = 1.0 - 0.5$
	$\gamma = 21$ kN/m^3	c' = 10 kPa	ϕ' = 30 degrees

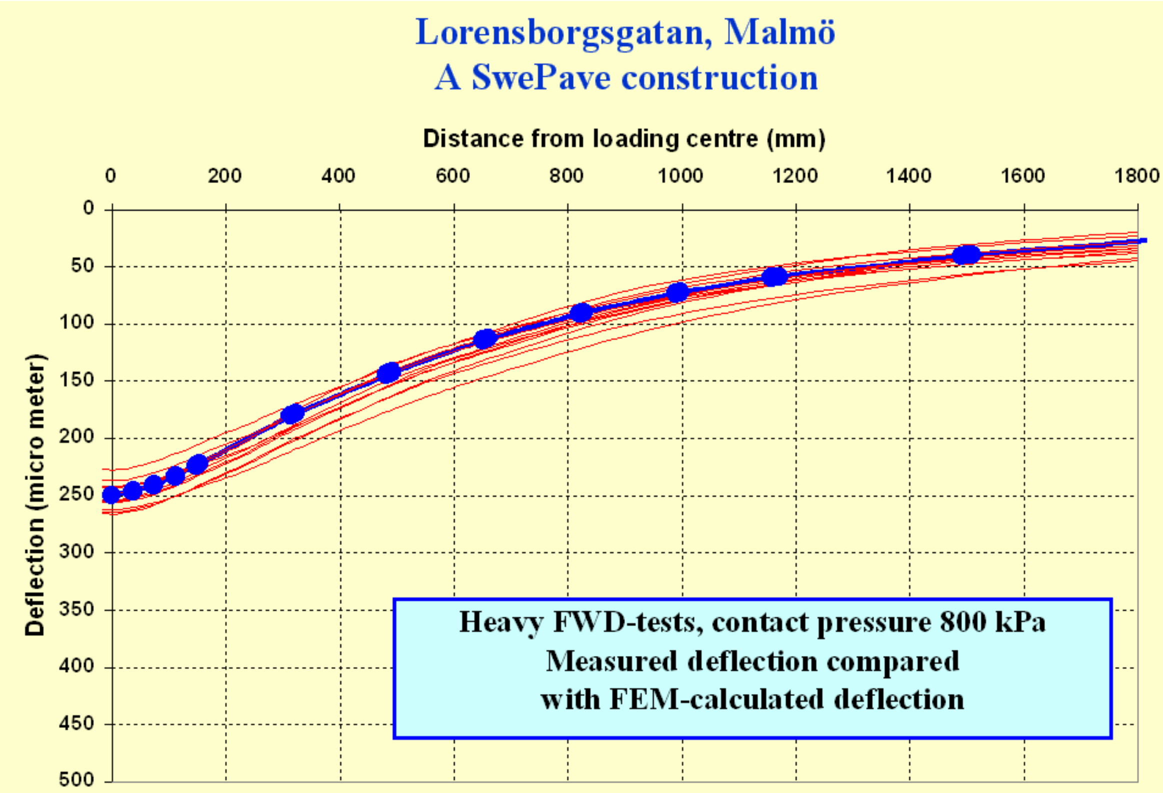

Figure 7: Heavy FWD test deflections for contact pressure 800 kPa compared with FEM calculated deflection.

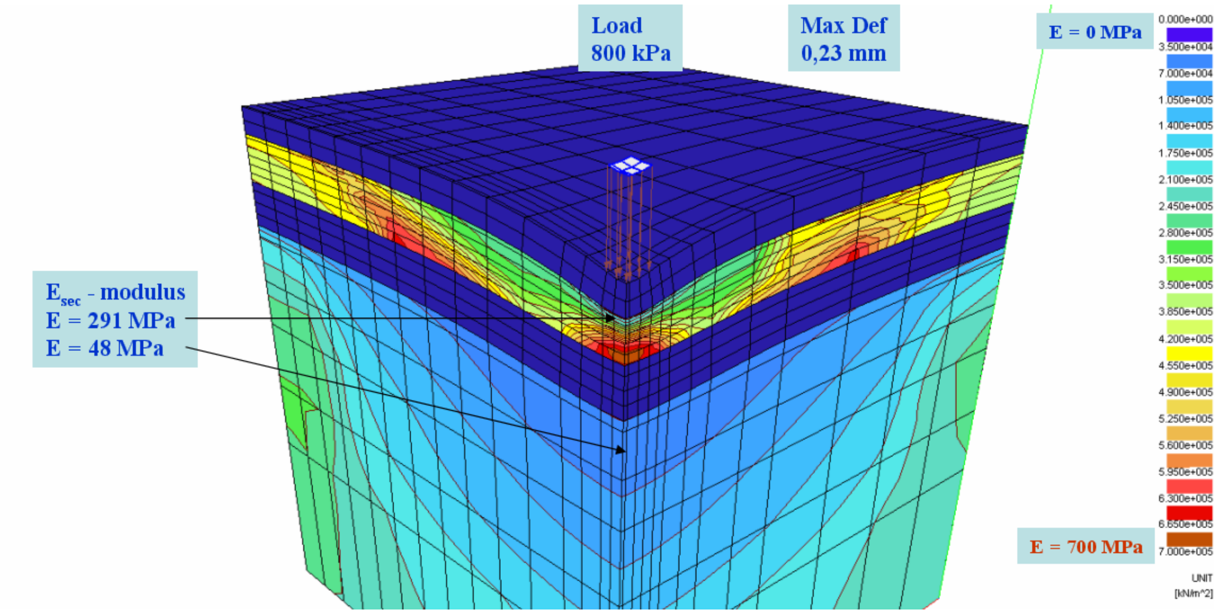

Figure 8: Calculated distribution of the secant elastic modulus when the contact pressure is 800 kPa in the FWD test.

The sizes of the strength parameters c´ and ϕ´ have been chosen so they represent a serviceability limit state yield boundary. The FEM calculated stresses is below this yield criterion which means that we will get a stabilized behavior in the pavement layers with only resilient deformations during cyclic loading. FEM calculated and measured deflections for the contact pressure 800 kPa are shown in Figure 7. Thin red lines represent all the different sections from the FWD measurements. In Figure 8 the calculated distribution of the secant elastic modulus is shown when the contact pressure is 800 kPa in the FWD test.

4 CONCLUDING REMARKS

The new framework for analytical pavement design, SwePave, constitutes efficient methods of pavement design aimed at producing solutions which are less disruptive to the environment and to the road user. An important part in this concept is the new developed non-linear elastic material model with a serviceability limit state yield boundary. In this paper it is demonstrated that the model can accurately predict the non-linear pavement response based on the material properties measured during construction.

The success in the approach depend mainly by the fact that the input parameters for the model are directly linked to practical field tests where design parameters can be validated in-situ during the construction of the pavement. Critical parameters are void ratio, isotropic mean stress, and matric suction.

5 ACKNOWLEDGMENTS

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