# Geosynthetic Material Properties for Use in 2-D Finite Element Pavement Response Models

S.W. Perkins
*Department of Civil Engineering, Montana State University, Bozeman, Montana, USA*

G. Eiksund
*SINTEF Civil and Environmental Engineering, Trondheim, Norway*

ABSTRACT: Modern finite element response models for flexible pavement analysis and design are traditionally two-dimensional axisymmetric models. The inclusion of a geosynthetic reinforcement layer in such a model is generally accomplished by the insertion of a horizontal layer of membrane elements. These elements are particularly well-suited for describing geosynthetics in that they carry loads in tension while having zero bending stiffness. The use of a 2-D axisymmetric response model requires that the reinforcement be described by an isotropic material model. Geosynthetics commonly have direction dependent properties, the most notable being an elastic modulus that differs between the machine and cross-machine directions of the material, which are described best by an orthotropic constitutive model. This paper presents an approach that allows a geosynthetic's orthotropic linear elastic properties to be converted to equivalent isotropic linear elastic properties for use in 2-D finite element response models. This is accomplished through a work-energy equivalency equation developed from a general stress application to a geosynthetic material modeled by an orthotropic and an isotropic linear elastic sheet. Parameters contained within the equation are calibrated by the comparison of pavement response of a completely 3-D finite element model containing a geosynthetic having an orthotropic material model to a 2-D finite element model having an isotropic material model for the geosynthetic. The study results in a simple equation to convert orthotropic properties to equivalent isotropic properties.

KEY WORDS: Pavements, geosynthetic, reinforcement, finite elements, response model.

## 1 INTRODUCTION

Modern finite element response models for flexible pavement analysis and design are traditionally two-dimensional axisymmetric models. The use of three-dimensional models is generally not currently practical given the excessively long computational time needed for these analyses. The inclusion of a geosynthetic reinforcement layer in a finite element response model is generally accomplished by the insertion of a horizontal layer of membrane elements. These elements are particularly well-suited for describing geosynthetics in that they carry loads in tension while having zero bending stiffness.

The use of a two-dimensional axisymmetric response model requires that the reinforcement be described by an isotropic material model, which is specified by two elastic constants, namely an elastic modulus and a Poisson's ratio. It is well-known that reinforcement materials exhibit direction dependent properties. Most notably, the elastic modulus differs between the machine and cross-machine directions of the material. An orthotropic material model best describes the direction dependent properties of reinforcement materials but cannot be used directly in a 2-D axisymmetric finite element model.

Given the need to use two-dimensional models in routine practice, a method is needed to convert the orthotropic elastic constants for the geosynthetic to equivalent isotropic constants. It is tacitly assumed that geosynthetic materials, which are most often discontinuous, can be modeled as a continuum within the context of a finite element response model. While this assumption is inherently incorrect, it is necessary in order to develop an efficient mechanistic response model within the context of a finite element program.

## 2 ORTHOTROPIC AND ISOTROPIC ELASTIC CONSTANTS

Geosynthetic reinforcement materials generally exhibit direction dependent material properties that within the framework of linear elastic theory are best described by an orthotropic linear elastic model. This material model contains 9 independent elastic constants, of which the four describing behavior within the plane of the material ($E_{xm}$, $E_m$, $\nu_{xm\text{-}m}$, $G_{xm\text{-}m}$) are pertinent to a reinforcement sheet modeled by membrane elements in a pavement response model. These parameters are defined as:

$E_{xm}$: Elastic modulus in the cross-machine direction
$E_m$: Elastic modulus in the machine direction
$\nu_{xm\text{-}m}$: Poisson's ratio in the cross-machine/machine plane
$G_{xm\text{-}m}$: Shear modulus in the cross-machine/machine plane

Various testing methods have been proposed for identification of these elastic constants for conditions pertinent to the small strains seen in these materials in reinforced pavement applications and are discussed in Cuelho et al. (2005), Cuelho and Perkins (2005) and Perkins et al. 2004.

The constitutive equation for an orthotropic linear-elastic material containing the above constants is given by Equation 1 where the subscripts *xm* and *m* denote the in-plane cross-machine and machine directions, and *n* denotes the direction normal to the plane of the geosynthetic.

$$\begin{Bmatrix} \varepsilon_{xm} \\ \varepsilon_m \\ \varepsilon_n \\ \gamma_{xm-m} \\ \gamma_{xm-n} \\ \gamma_{m-n} \end{Bmatrix} = \begin{bmatrix} 1/E_{xm} & -\nu_{m-xm}/E_m & -\nu_{n-xm}/E_n & 0 & 0 & 0 \\ -\nu_{xm-m}/E_{xm} & 1/E_m & -\nu_{n-m}/E_n & 0 & 0 & 0 \\ -\nu_{xm-n}/E_{xm} & -\nu_{m-n}/E_m & 1/E_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{xm-m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{xm-n} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{m-n} \end{bmatrix} \begin{Bmatrix} \sigma_{xm} \\ \sigma_m \\ \sigma_n \\ \tau_{xm-m} \\ \tau_{xm-n} \\ \tau_{m-n} \end{Bmatrix} \quad (1)$$

Poisson's ratio, $\nu_{m\text{-}xm}$, is related to $\nu_{xm\text{-}m}$ through Equation 2.

$$\nu_{m-xm} = \nu_{xm-m} \frac{E_m}{E_{xm}} \quad (2)$$

When using membrane elements, values for the remaining elastic constants can be set to any values that ensure stability of the elastic matrix.

The constitutive matrix for an isotropic linear-elastic constitutive matrix is given by Equation 3 and contains two ($E$, $\nu$) independent elastic constants. The third elastic constant in Equation 3 ($G$) is expressed in terms of $E$ and $\nu$ by Equation 4.

$$\begin{Bmatrix} \varepsilon_{xm} \\ \varepsilon_m \\ \varepsilon_n \\ \gamma_{xm-m} \\ \gamma_{xm-n} \\ \gamma_{m-n} \end{Bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{Bmatrix} \sigma_{xm} \\ \sigma_m \\ \sigma_n \\ \tau_{xm-m} \\ \tau_{xm-n} \\ \tau_{m-n} \end{Bmatrix} \tag{3}$$

$$G = \frac{E}{2(1+\nu)} \tag{4}$$

## 3 ELASTIC CONSTANTS EQUIVALENCY FORMULATION

Equivalency of measured orthotropic elastic constants ($E_{xm}$, $E_m$, $\nu_{xm-m}$, $G_{xm-m}$) to isotropic constants ($E$, $\nu$) is established using a work-energy equivalency formulation. It is assumed that two materials, one containing orthotropic properties and the second containing isotropic properties, experience an identical in-plane general state of stress given in Figure 1.

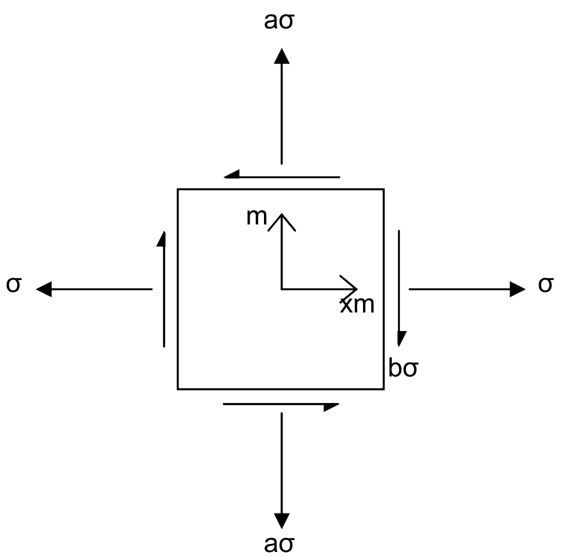


Figure 1:General state of stress experienced by a reinforcement element.

According to Equations 1 and 3, the three in-plane strains produced by this stress state in the orthotropic material are given by Equations 5 – 7 and by Equations 8 – 10 for the isotropic material.

$$\varepsilon_{xm} = \sigma\left(\frac{1}{E_{xm}} - a\frac{\nu_{m-xm}}{E_m}\right) \tag{5}$$

$$\varepsilon_m = \sigma\left(\frac{a}{E_m} - \frac{\nu_{m-xm}}{E_m}\right) \tag{6}$$

$$\gamma_{xm-m} = \frac{b\sigma}{G_{xm-m}} \tag{7}$$

$$\varepsilon_{xm} = \frac{\sigma}{E}(1 - a\nu) \tag{8}$$

$$\varepsilon_{m} = \frac{\sigma}{E}(a - \nu) \tag{9}$$

$$\gamma_{xm-m} = 2b\sigma\frac{(1+\nu)}{E} \tag{10}$$

The work energy produced by the application of the stress state shown in Figure 1 is given in general by Equation 11. Substitution of Equations 5 – 7 and 8 – 10 into Equation 11 results in the work energy for the orthotropic and isotropic materials given by Equations 12 and 13, respectively.

$$W = \frac{1}{2}\left(\sigma\varepsilon_{xm} + a\sigma\varepsilon_{m} + b\sigma\gamma_{xm-m}\right) \tag{11}$$

$$W = \frac{\sigma^2}{2}\left(\frac{1}{E_{xm}} + \frac{a^2}{E_m} - \frac{2a\nu_{m-xm}}{E_m} + \frac{b^2}{G_{xm-m}}\right) \tag{12}$$

$$W = \frac{\sigma^2}{2E}\left(1 - 2a\nu + a^2 + 2b^2(1+\nu)\right) \tag{13}$$

Equivalent isotropic elastic constants are chosen to produce equivalent work energy by the orthotropic and isotropic materials given by Equations 12 and 13. Since an infinite number of combinations of the two isotropic elastic constants ($E$, $\nu$) to establish equivalency between Equations 12 and 13 are possible, a value for the isotropic Poisson's ratio, $\nu$, is assumed such that the value of isotropic elastic modulus, $E$, can be calculated. Setting Equations 12 and 13 equal to each other and solving for $E$ results in Equation 14. Assuming a value of $\nu = 0.25$ and substitution of Equation 2 into Equation 14 results in Equation 15.

$$E = \frac{1 - 2a\nu + a^2 + 2b^2(1+\nu)}{\frac{1}{E_{xm}} + \frac{a^2}{E_m} - 2a\frac{\nu_{m-xm}}{E_m} + \frac{b^2}{G_{xm-m}}} \tag{14}$$

$$E = \frac{1 - 0.5a + a^2 + 2.5b^2}{\frac{1}{E_{xm}} + \frac{a^2}{E_m} - 2a\frac{\nu_{xm-m}}{E_{xm}} + \frac{b^2}{G_{xm-m}}} \tag{15}$$

Equation 15 provides a means of establishing an equivalent isotropic elastic modulus ($E$) for an assumed value of isotropic Poisson's ratio ($\nu$) for a single state of stress given in Figure 1. In Figure 1, the stress factors $a$ and $b$ describe the magnitude of normal stress in the machine direction and the magnitude of shear stress acting on the reinforcement element. The stress state in a reinforcement layer in a pavement system varies from point to point, meaning that values of $a$ and $b$ vary from point to point. This situation creates the need to assess values of $a$ and $b$ in an average sense for the entire reinforcement layer. Since pavement response models are ultimately used for the prediction of pavement performance, the true test of equivalency lies in the comparison of performance predictions between response models using orthotropic and isotropic elastic constants. This comparison is done below by examining common pavement response variables from a 3-D pavement response model using orthotropic elastic constants and comparing those to the same response variables using the same 3-D model but with isotropic elastic constants.

## 4 DETERMINATION OF STRESS FACTORS *a* AND *b*

The stress factors $a$ and $b$ are estimated using the following procedure:

1. A 3-dimensional model of a reinforced pavement system was created. The pavement cross-section consisted of three layers (75 mm of asphalt concrete, 300 mm of base aggregate, 3.435 m of subgrade) with the reinforcement placed between the base and subgrade layers. The distance from the pavement load centerline to the edge of the square model was 2.4384 m.

A linear elastic model was used for the asphalt concrete layer. A non-linear elastic model with tension cutoff with resilient modulus given by Equation 16 was used for the base and subgrade layers. Three sets of material properties for three model analyses were used and are given in Tables 1 – 3. These properties were chosen to represent common materials encountered in pavements.

$$M_R = p_a k_1 \left( \frac{\theta}{p_a} \right)^{k_2} \left( \frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} \qquad (16)$$

Table 1: Material property set 1 for the 3D model.

| Layer | Unit Weight (kN/m$^3$) | Poisson's Ratio, ν | Elastic Modulus (kPa) | | | |
|---|---|---|---|---|---|---|
| Asphalt Concrete | 23 | 0.2762 | 3,337,169 | | | |
| | | | $p_a$ (kPa) | $k_1$ | $k_2$ | $k_3$ |
| Base (finite) | 20 | 0.25 | 101.3 | 957 | 0.906 | -0.614 |
| Subgrade(finite) | 18 | 0.25 | 101.3 | 139 | 0.187 | -3.281 |

Table 2: Material property set 2 for the 3D model.

| Layer | Unit Weight (kN/m$^3$) | Poisson's Ratio, ν | Elastic Modulus (kPa) | | | |
|---|---|---|---|---|---|---|
| Asphalt Concrete | 23 | 0.49 | 3,337,169 | | | |
| | | | $p_a$ (kPa) | $k_1$ | $k_2$ | $k_3$ |
| Base (finite) | 20 | 0.25 | 101.3 | 957 | 0.906 | -0.614 |
| Subgrade(finite) | 18 | 0.25 | 101.3 | 139 | 0.187 | -3.281 |

Table 3: Material property set 3 for the 3D model

| Layer | Unit Weight ($kN/m^3$) | Poisson's Ratio, ν | Elastic Modulus (kPa) | | | |
|---|---|---|---|---|---|---|
| Asphalt Concrete | 23 | 0.49 | 3,337,169 | | | |
| | | | $p_a$ (kPa) | $k_1$ | $k_2$ | $k_3$ |
| Base (finite) | 20 | 0.25 | 101.3 | 957 | 0.906 | -6.14 |
| Subgrade(finite) | 18 | 0.25 | 101.3 | 139 | 0.187 | -3.281 |

Rough contact was used between the reinforcement and the base, and between the reinforcement and the subgrade. The model with each set of material properties listed in Tables 1 – 3 was analyzed using two sets of orthotropic elastic properties for the reinforcement layer (Table 4). Each set of reinforcement properties approximates the behavior of two common geogrid types.

Table 4: Orthotropic linear-elastic properties for the reinforcement layer.

| Case | $E_{xm}$ (kPa) | $E_m$ (kPa) | $E_n$ (kPa) | $\nu_{xm-m}$ | $\nu_{xm-n}$ | $\nu_{m-n}$ | $G_{xm-m}$ (kPa) | $G_{xm-n}$ (kPa) | $G_{m-n}$ (kPa) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 595,000 | 365,925 | 595,000 | 0.813 | 0.25 | 0.25 | 2919 | 2919 | 2919 |
| 2 | 325,000 | 220,000 | 325,000 | 0 | 0 | 0 | 987 | 987 | 987 |

2. The 3-D model described above with each set of material properties listed in Tables 1 – 3 was analyzed using an isotropic linear-elastic model for the reinforcement with Poisson's ratio set equal to 0.25 for all analyses and elastic modulus varied between 50,000 and 1,000,000 kPa. While a 2-D model could have been used for this step, a 3-D model was used to avoid any problems with comparison of models with different boundary conditions.
3. Two sets of response parameters were extracted from each of the analyses described in steps 1 and 2. One set of parameters consisted of the maximum horizontal tensile strain in the asphalt concrete layer, which was then used to determine the number of cycles to fatigue failure according to Equation 17 (NCHRP, 2004).

$$N_f = k_1\beta_1\left(\frac{1}{\varepsilon_t}\right)^{k_2\beta_2}\left(\frac{1}{E}\right)^{k_3\beta_3} \qquad (17)$$

where:
$N_f$ = traffic repetitions to AC fatigue
$k_1$, $k_2$, $k_3$ = laboratory material properties, taken as:
$k_1$ = 1.0
$k_2$ = 3.9492
$k_3$ = 1.281
$\beta_1$, $\beta_2$, $\beta_3$, = field calibration coefficients, taken as:
$\beta_1$ = 1.0
$\beta_2$ = 1.2
$\beta_3$ = 1.5
$\varepsilon_t$ = resilient horizontal tensile strain from the response model taken as the maximum tensile value with the AC layer
$E$ = AC complex modulus used in response model (psi)

The second set of parameters consisted of the vertical strain extrapolated to each node along the model centerline, which was then used to determine the number of cycles needed to reach 25 mm of permanent surface deformation. Permanent strain in the AC layer was determined according to Equation 18 (NCHRP, 2004) while permanent deformation in the base and subgrade layers was determined according to Equation 19 (NCHRP, 2004).

$$\log\left(\frac{\varepsilon_p}{\varepsilon_r}\right) = k_1\beta_1 + k_2\beta_2 \log T + k_3\beta_3 \log N \quad (18)$$

where:
$\varepsilon_p$ = permanent vertical strain as a function of $N$
$\varepsilon_r$ = resilient vertical strain from the response model taken along the model centerline
$k_1$, $k_2$, $k_3$ = laboratory material properties, taken as:
$k_1$ = -3.3426
$k_2$ = 1.734
$k_3$ = 0.4392
$\beta_1$, $\beta_2$, $\beta_3$, = field calibration coefficients, taken as:
$\beta_1$ = 0.8369
$\beta_2$ = 0.5
$\beta_3$ = 2.2
$T$ = temperature of AC (°F)
$N$ = traffic repetitions

$$\delta_a = \xi_1 \left(\frac{\varepsilon_o}{\varepsilon_r}\right) e^{-\left(\frac{\rho}{N}\right)^{\xi_2\beta}} \varepsilon_v\, h \quad (19)$$

$\delta_a$ = permanent deformation for the layer/sublayer
$N$ = traffic repetitions
$\varepsilon_o$, $\beta$, $\rho$ = material properties (Table 5)
$\varepsilon_r$ = resilient strain imposed in laboratory test to obtain material properties $\varepsilon_o$, $\beta$, and $\rho$
$\varepsilon_v$ = average vertical resilient strain in the layer/sublayer as obtained from the response model
$h$ = thickness of the layer/sublayer
$\xi_1$, $\xi_2$ = field calibration coefficients (Table 5)

Table 5: Permanent deformation properties for base and subgrade materials

| Material | $(\varepsilon_o/\varepsilon_r)$ | $\rho$ | $\beta$ | $\xi_1$ | $\xi_2$ |
|---|---|---|---|---|---|
| Base | 88.58 | 7342 | 0.1271 | 0.4318 | 1.336 |
| Subgrade | 4683 | $4.13\times10^{26}$ | 0.03614 | 2.500 | 1.089 |

4. For each of the models analyzed using isotropic linear-elastic properties, the values of elastic modulus used in these analyses for the reinforcement were plotted against the number of cycles to fatigue and the number of cycles to 25 mm surface deformation. Figures 2 and 3 provide these plots for the model material parameters listed in Table 1.
5. For the 3D analyses with the two sets of orthotropic properties listed in Table 4, the number of cycles to fatigue and the number of cycles to 25 mm surface deformation were calculated using Equations 17 – 19 for each of the 3 sets of model material properties, with values listed in Table 6.
6. With the values listed in Table 6, figures similar to Figures 2 and 3 were used for each material property set to determine the equivalent value of isotropic elastic modulus, with values given in Table 7.

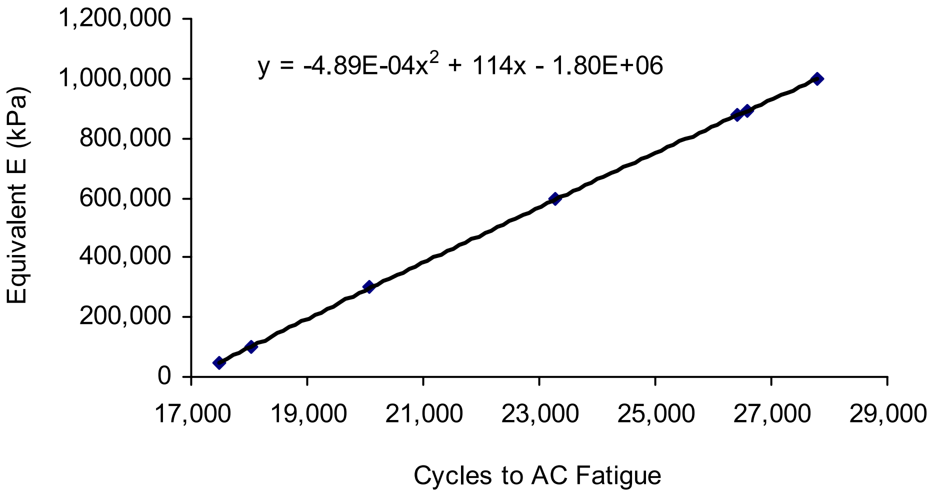


Figure 2: Cycles to AC fatigue versus isotropic reinforcement elastic modulus for model parameters listed in Table 1.

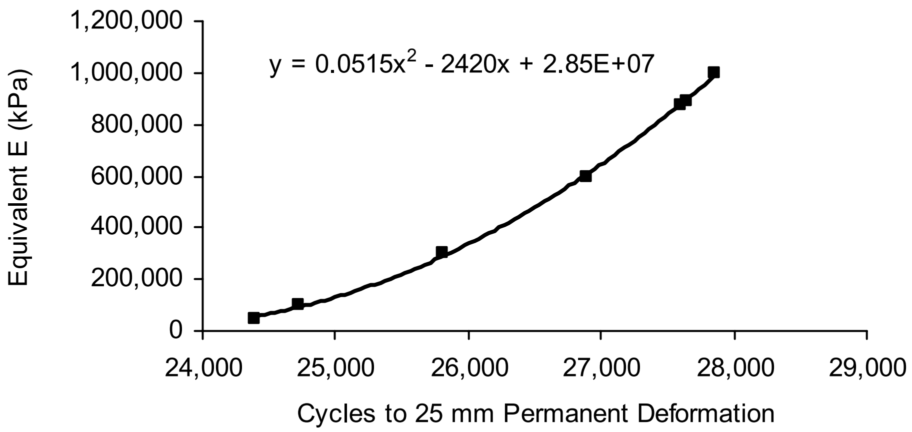


Figure 3: Cycles to 25 mm permanent surface deformation versus isotropic reinforcement elastic modulus for model parameters listed in Table 1.

Table 6: Cycles to AC fatigue and 25 mm permanent surface deformation for orthotropic reinforcement cases 1 and 2.

| Model Material Property Set | Reinforcement Case | Cycles to AC Fatigue | Cycles to 25 mm permanent surface deformation |
|---|---|---|---|
| 1 | 1 | 23,318 | 27,131 |
| 1 | 2 | 19,200 | 25,252 |
| 2 | 1 | 31,393 | 46,417 |
| 2 | 2 | 26,137 | 42,372 |
| 3 | 1 | 9314 | 12,116 |
| 3 | 2 | 9128 | 11,745 |

Table 7: Equivalent isotropic elastic modulus for reinforcement cases 1 and 2.

| | | Equivalent isotropic elastic modulus (kPa) | |
|---|---|---|---|
| Model Material Property Set | Reinforcement Case | AC Fatigue | 25 mm permanent surface deformation |
| 1 | 1 | 598,464 | 692,917 |
| 1 | 2 | 213,850 | 173,694 |
| 2 | 1 | 657,909 | 663,647 |
| 2 | 2 | 198,903 | 237,810 |
| 3 | 1 | 569,260 | 678,139 |
| 3 | 2 | 158,197 | 186,954 |

Equation 15 was solved for parameters *a* and *b* to minimize the difference between equivalent *E* values predicted by Equation 15 and those listed in Table 7. This resulted in values of $a = 0.35$ and $b = 0.035$. With these values of *a* and *b*, Figure 4 shows equivalent *E* values predicted by Equation 15 versus those given in Table 7. The differences between the predictions from Equation 15 and those based on the two damage criterion are relatively minor.

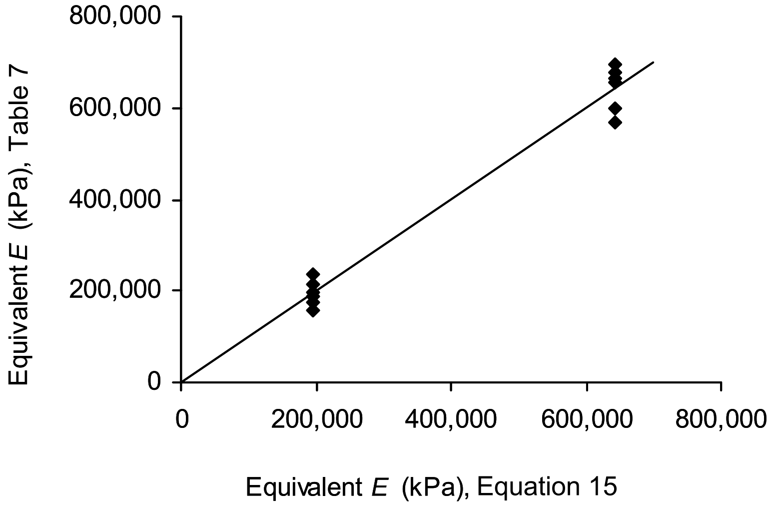


Figure 4: Comparison of predicted and analyzed equivalent *E* values.

## 5 INFLUENCE OF ORTHOTROPIC PARAMETERS ON EQUIVALENT ISOTROPIC ELASTIC MODULUS

The importance of variations of the four principal orthotropic elastic constants ($E_{xm}$, $E_m$, $\nu_{xm\text{-}m}$, $G_{xm\text{-}m}$) on values of equivalent isotropic elastic modulus was examined by varying the ratio of $E_{xm}$ to $E_m$ from 1 to 4, varying $\nu_{xm\text{-}m}$ between 0 and 0.75, and varying the value of $G_{xm\text{-}m}$ by a factor of 200 %. These variations were made while holding all other elastic constants equal and with equivalent elastic modulus calculated from Equation 15. Figure 5 shows the influence of these variations. Variations in the ratio of $E_{xm}$ to $E_m$ shows up to a 23 % decrease in equivalent isotropic elastic modulus as the ratio goes from 1 to 4. An increase in $\nu_{xm\text{-}m}$ has a substantial impact on equivalent modulus. An increase in $G_{xm\text{-}m}$ of 200 % has an impact on equivalent modulus that is comparable to that for variations in the ratio of $E_{xm}$ to $E_m$. Variations in the values of $E_{xm}$ to $E_m$ themselves has an impact on equivalent modulus that is comparable to the magnitude of the variation.

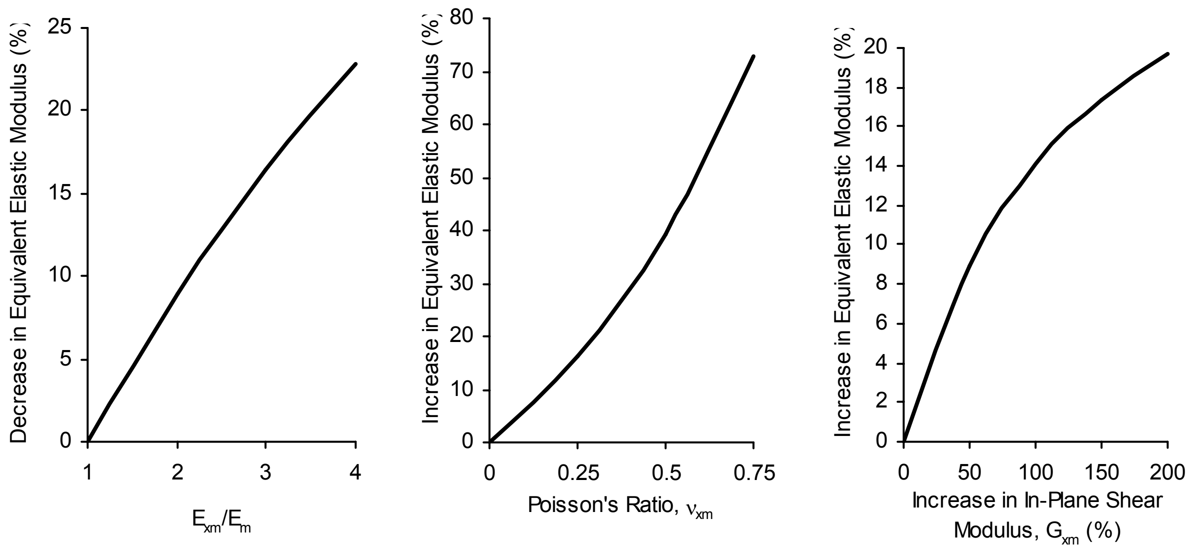


Figure 5: Impact of orthotropic elastic constants on equivalent isotropic elastic modulus.

## 6 SUMMARY AND CONCLUSIONS

Commonly used two-dimensional finite element models used for pavement analysis and design must, by necessity, use isotropic linear elastic models for the geosynthetic reinforcement layer. The direction dependent elastic properties of an actual reinforcement material must be converted to equivalent isotropic values when using a two-dimensional model. This paper has presented a method for converting direction dependent orthotropic elastic properties to equivalent isotropic properties. The method is based on a work-energy approach and results in a relatively simple equation containing two stress constants. These constants were calibrated by comparison of two common pavement damage features (asphalt fatigue and permanent surface deformation) determined from pavement response models using isotropic and orthotropic elastic constants.

Variation of orthotropic elastic constants showed that the in-plane Poisson's ratio had the greatest impact on the equivalent isotropic elastic modulus while variations in the ratio of elastic modulus in the cross-machine to machine directions and the in-plane shear modulus had appreciable but less important effects.

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