Superposition principle to determine properties of bituminous mixtures in the time –temperature domain

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ABSTRACT: In this paper, it will be shown that several properties of asphaltic mixes fall within the domain of applicability of the time-temperature superposition principle. The exchangeability of not only the frequency and temperature (as often reported), but also loading speed and strain rate and temperature, allows application of a unique temperature susceptibility function for any asphaltic mix. This function is applicable, for elastic properties of asphaltic mixes e.g. flexural mix stiffness, as well as for inelastic properties e.g. compressive and tensile strength. An apparent advantage of this approach is the considerable reduction in testing programs.

KEY WORDS: Time temperature superposition, temperature susceptibility function, properties of asphalt mixes, optimization of testing program.

1 INTRODUCTION

Several engineering properties of asphaltic materials are of interest for pavement design and performance e.g. fatigue behaviour, mix stiffness and strength properties etc. Bituminous mixture stiffness needs to be determined in order to estimate both the load-induced and thermal stress/ strain distribution in asphalt pavements. Stiffness has been used as an indicator of mixture quality for pavements and mixture design to evaluate damage and age-hardening trends of bituminous mixtures both in laboratory and the field (Epps, et al, 2000). Monotonic compression and tension tests provide quite useful information e.g.: the relationship between the tensile/compressive strength, temperature and loading speed; post failure behaviour etc. Such information can be used for instance, for material modelling. Furthermore, strength limits may be used for ranking different asphaltic mixes with respect to susceptibility to cracking and permanent deformation.

Computation of parameters of material models normally requires combining different materials properties (e.g. compressive and tensile strength). Several researchers have used different mathematical models to describe different properties of materials, mostly by fitting curves to experimental data. Since the behaviour of asphaltic materials is determined by a combination of properties (e.g. strength, stiffness, dilation, plasticity etc), individual models of these properties need to be combined in numerical tools. The lack of consistency between the models fitted per set of data might generate numerical instability and enhances chances for faulty outputs (Medani and Huurman 2004).

If it can be hypothesized that a unique temperature susceptibility function for all properties of an asphaltic mix exists: a considerable reduction in testing programs, especially when combined with experimental deign techniques, can be achieved. Furthermore, this will result in consistent and logical trends and values of parameters of materials models (Medani and Huurman 2004).

2 TIME TEMPERATURE SUPERPOSITION PRINCIPLE

Asphaltic mixtures are known to behave viscoelastically (i.e. time and frequency dependant) and viscoplastically (i.e. non recoverable strain) at temperatures other than low. When asphaltic mixtures are in their linear viscoelastic phase, the time-temperature superposition principle, or thermorheological simplicity holds. This implies that the same material property values can be obtained either at low temperatures and low frequencies (long loading times) or at high temperatures and high frequencies (short loading times). Chehab et al (2002) have shown based on the work of Schapery (1997) that asphalt concrete in tension is a thermological simple material even with growing damage.

If the time temperature superposition principle is applicable for elastic as well as for inelastic properties, a single temperature susceptibility function should exist for both elastic and inelastic properties of bituminous materials.

3 THE MODEL

It is believed that the sigmoidal model (equation1) suggested by Medani and Huurman (2004) can describe the properties of bituminous mixes appropriately, e.g. their dependency on temperature, frequency and strain rate.

$$\begin{aligned} \Psi &= \Psi_{high} + (\Psi_{low} - \Psi_{high}).S \\ S &= 1 - \exp\left(- \left(\frac{f}{f_0}.\beta \right)^{\gamma} \right) \\ \beta &= \exp\left(- \beta_1 \left(T - \beta_2 \right) \right) \end{aligned} \tag{1}$$

where:

Ψ	: any mix property e.g. flexural mix stiffness, compressive strength etc;
f	: frequency (or loading speed or strain rate);
Ψ_{low}	: value of that mix property when $f \Rightarrow 0$ (same units as Ψ);
Ψ_{high}	: value of that mix property when $f \Rightarrow \infty$ (same units as Ψ);
f_o	: initial frequency (or loading speed or strain/stress rate);
γ	: shape factor related to the slope of the S-function;
β	: temperature susceptibility function (mix property);
T	: temperature in ^{o}C or K;
β_1	: parameter of the temperature susceptibility function [1/^{o}C or 1/K];
β_2	: parameter of the temperature susceptibility function [^{o}C or K]

The parameter γ is related to the curvature of the function. The parameter β controls the horizontal distance of the point of rotation to the origin, which is given by $f = \frac{f_0}{\beta}$ (Figure 1).

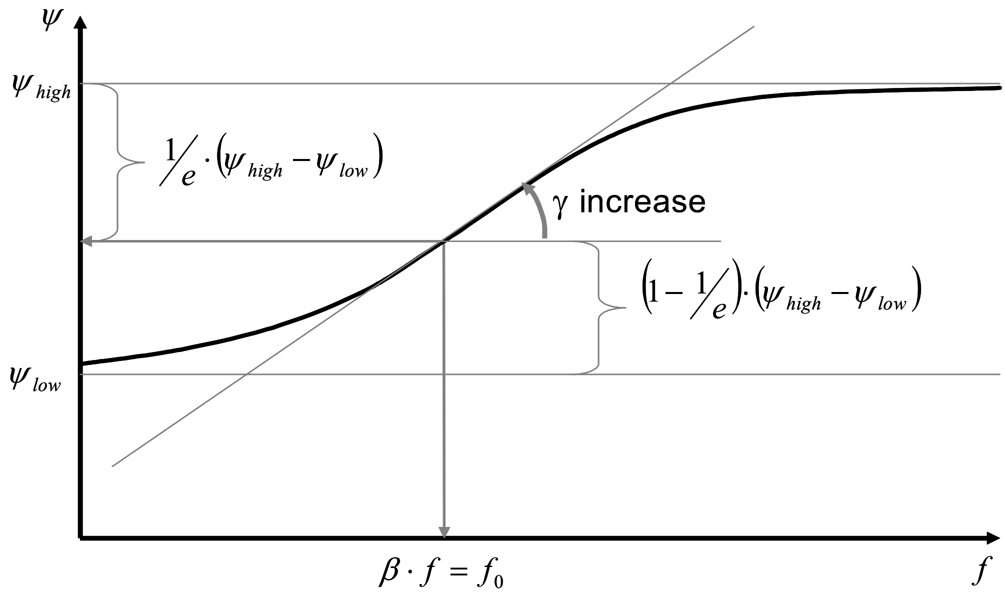

Figure 1: Parameters of the sigmoidal model

3.1 Application of the model

The suggested model has been applied to describe several properties of 10 bituminous mixes with modified and non-modified bitumen. The properties which are described include: flexural mix stiffness, tangential mix stiffness, compressive strength, tensile strength, fracture tensile strength, total energy, compressive stress at which dilation or non-linearity starts in monotonic compressive test. Some of the mixes are standard wearing coarse mixes, others are special mixes which are used or intended to be used for special applications e.g. surfacing of orthotropic steel bridges. The mixes investigated are:

- The ACRe mix (Erkens, 2002).
- Four mixes tested by Scholten (2003), namely: BC (Bestone crushed sandstone aggregate with conventional bitumen), BP: (Bestone crushed sandstone aggregate with polymer modified bitumen), PC (Porphyry aggregate with conventional bitumen) and PP (Porphyry aggregate with polymer modified bitumen).
- Mastic asphalt mix (MA), which was used for resurfacing of the Moerdijk Bridge in the Netherlands in June 2000, Bosch (2001).
- Guss asphalt (GU), which is normally used for surfacing orthotropic steel bridges in Germany.
- Two mixes tested by Airey et al (2002): dense bitumen macadam (DBM) and hot rolled asphalt (HRA).
- Enrobé á Modele Elevé (EME) (Jansen, 2002).

The parameters of the model for the different properties of asphaltic mixes were obtained by using non-linear least square regression techniques, by utilizing the solver function in an Excel spreadsheet. This was done as follows:

- The error was first defined as the square of the difference (or the logarithm of the difference) between the experimental and the model predicted values, for all the properties.

- The errors obtained from individual properties are summed up to obtain the error function of the mix.
- Non-linear least square techniques is then used to determine the temperature susceptibility function β for the mix, and the other model parameters of individual properties namely: $\Psi_{\min}$, $\Psi_{\max}$ and γ.

Examples of the good fit between the experimental and model predicted values are shown in Figures 2 and 3.

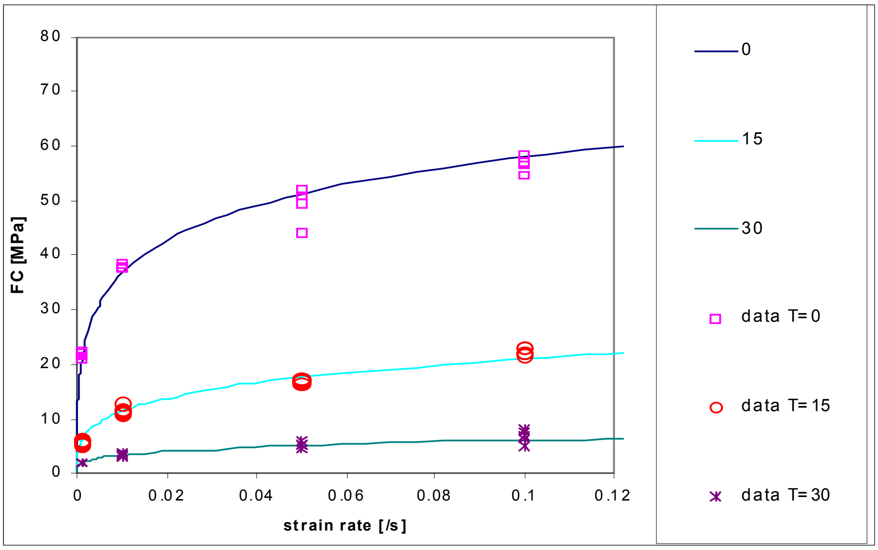

Figure 2: The experimentally and model determined values of the compressive strength of the ACRe mix

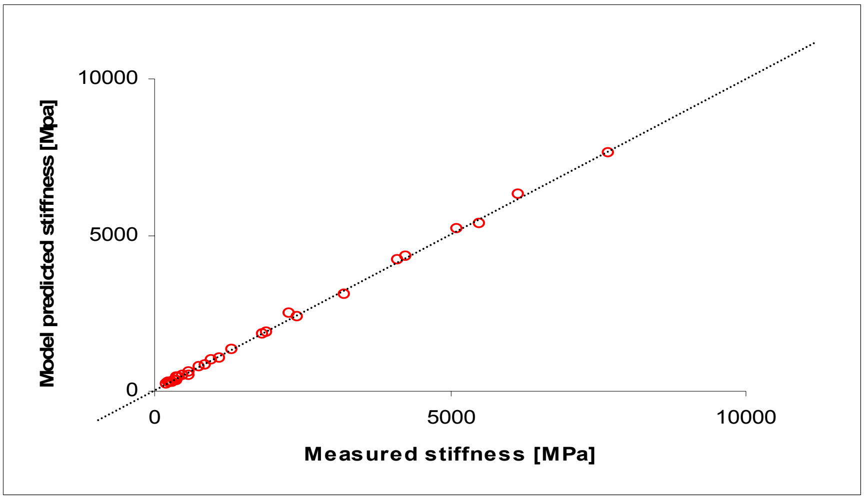

Figure 3: The experimentally and model determined values of the flexural stiffness of the mastic asphalt

The results of application of the model, with a single temperature susceptibility function for each mix, are shown in Table 1.

Table 1: Parameters of the temperature susceptibility function, and the other model parameters for the properties of 10 mixes

Material	ACRe	MA	EME	GU	DBM	HRA	PP	BP	PC	BC
β_1 (1/°C)	0.2880	0.3058	0.2377	0.2514	0.2187	0.2629	0.2696	0.2752	0.1641	0.2140
β_2 (°C)	4.27	3.83	7.84	9.22	17.52	6.83	-2.71	-8.34	2.96	-1.62
Property	Compressive strength [MPa]									
f	Strain rate [1/s]									
Minimum	0.10	0.92	0.33	0.88	0.66	0.11	0.33	0.13	0.26	0.15
Maximum	114.25	73.95	86.00	80.51	40.95	65.25	69.39	66.61	143.65	112.19
γ	0.2911	0.2884	0.2970	0.3278	0.4238	0.2711	0.2917	0.2352	0.4886	0.3546
R^2	0.9957	0.9950	0.9893	0.9963	0.9988	0.9858	0.9866	0.9852	0.9951	0.9976
Property	Tensile strength [MPa]									
f	Strain rate [1/s]									
Minimum	0.0012	0.0700	0.0025	0.0000	0.0717	0.0137				
Maximum	30.98	34.79	17.92	55.96	16.74	13.04				
γ	0.2785	0.3014	0.1706	0.3079	0.4403	0.3971				
R^2	0.9752	0.9576	0.9341	0.8274	0.9652	0.9958				
Property	Compressive stress @ which plasticity starts [MPa]									
f	Strain rate [1/s]									
Minimum	0	0	0	0						
Maximum	69.56	46.51	62.90	63.82						
γ	0.4263	0.4282	0.3081	0.4430						
R^2	0.9098	0.9782	0.9391	0.9934						
Property	Compressive stress @ which dilation starts [MPa]									
f	Strain rate [1/s]									
Minimum	0	0	0	0						
Maximum	110.18	68.03	79.94	74.43						
γ	0.4747	0.4464	0.3734	0.4620						
R^2	0.9911	0.9972	0.9835	0.9783						
Property	Tangential stiffness [MPa]									
f	Strain rate [1/s]									
Minimum	0	0	0	0						
Maximum	18374	21576	23386	21645						
γ	0.4470	0.4597	0.4745	0.3526						
R^2	0.9746	0.9565	0.9461	0.9585						
Property	Flexural stiffness [MPa]									
f	Frequency [Hz]									
Minimum	10	230	215	10			150	120	100	140
Maximum	10430	9560	16300	15720			11200	9860	12010	14200
γ	0.4098	0.4664	0.4601	0.4953			0.4859	0.4714	0.6605	0.4801
R^2	0.9683	0.9981	0.9832	0.9906			0.9901	0.9921	0.9639	0.9721

The high R^2 values obtained for several properties of the different mixes considered suggest the following:

- The sigmoidal model is capable of describing several properties of asphaltic mixes.
- A single temperature susceptibility function can adequately describe elastic and inelastic properties of asphaltic materials.

4 VERIFICATION OF THE HYPOTHESIS

To verify the hypothesis that the time-temperature superposition principle is applicable for a wide range of linear as well as non-linear properties of asphaltic mixes, three examples will be given where the model will be used to predict properties, other than the those considered in developing the model. Furthermore, the model will be used to predict values outside the range of the data (extrapolation), which is in the authors' view the ultimate test for the validity of any model.

4.1 Example 1: ACRe mix

The parameters of the temperature susceptibility function of the ACRe mix determined for the investigated properties reported in section 3.1, were found to be: $\beta_1 = 0.2880\,(1/^{o}\text{C})$ and $\beta_2 = 4.27$ ^{o}C. These parameters will now be used to fit a model for a property, which was not investigated before, namely: shear strength. The results of the four-point shear test carried out by Erkens (2002), which will be used to fit the model, are shown in Table 2.

Table 2: Results of the four-point shear test for the ACRe mix

Temperature (^{o}C)	Strain rate (1/s)	Shear strength $\tau_{\max}$ [MPa]
0	0.001	2.1
0	0.001	1.8
0	0.001	2.0
15	0.01	1.1
15	0.01	1.1
15	0.01	1.0

From the data and the parameters of the temperature susceptibility function, the other three parameters determined using non-linear least square regression are: $\tau_{\min} = 0$, $\tau_{\max} = 4.9576$ MPa and $\gamma = 0.3179$. These parameters and the temperature susceptibility function will now be used to predict values of shear strength by extrapolating both temperature and strain rate. The experimentally determined and model predicted values of the shear strength are shown in Table 3.

Table 3: A comparison between experimentally determined and model predict values

T (^{o}C)	Strain rate (1/s)	Shear strength experiment [MPa]	Shear strength model [MPa]	Difference %
30	0.1	0.5	0.544	
30	0.1	0.6	0.544	2
30	0.1	0.5	0.544	

4.2 Example 2: Guss asphalt mix

The parameters of the temperature susceptibility function of the guss asphalt for the investigated properties reported in section 3.1, were found to be: $\beta_1 = 0.2514\,(1/^{o}C)$ and $\beta_2 = 9.22$ ^{o}C. These parameters will now be used to fit models for properties obtained from indirect tensile test namely: fracture tensile strength, total energy and static stiffness. The results of the indirect static tensile test, which will be used to fit the models, are shown in Table 4.

Table 4: Results of the indirect static tensile test

Temperature (^{o}C)	Speed (mm/s)	Fracture tensile strength [MPa]	Total energy [Nmm]	Static stiffness [MPa]
23.6	0.85	1.31	38825	
23.6	0.85	1.24	47750	
23.6	0.50	1.01		125
23.6	0.50	1.25		135
23.6	0.50	1.11		123
23.6	0.10		23575	102
23.6	0.10		23574	96

From the data and the parameters of the temperature susceptibility function the other three parameters of the model were determined for each property and are shown in Table 5.

Table 5: The model parameters for the data of the indirect tensile test

	Fracture tensile strength [MPa]	Total energy [Nmm]	Static stiffness [MPa]
Ψ_{min}	0	241	0
Ψ_{max}	6.3551	186257	532
γ	0.2933	0.2790	0.2213

The experimentally determined and model predicted values of fracture tensile strength, total energy and static stiffness are shown in Table 6.

Table 6: A comparison between experimentally determined and model predict values

T (^{o}C)	Speed [mm/s]	Fracture tensile strength [MPa]		Total energy [Nmm]		Static stiffness [MPa]		Difference %
		experiment	model	experiment	model	experiment	model	
23.6	0.85					178	150	6.3
23.6	0.85					142	150	
23.6	0.50			34600	34289			0.4
23.6	0.50			32025	34289			
23.6	0.50			35825	34289			
23.6	0.10	0.78	0.72					6.7
23.6	0.10	0.76	0.72					

4.3 Example 3: Mastic asphalt mix

The parameters of the temperature susceptibility function of the mastic asphalt mix for the investigated properties reported in section 3.1, were found to be: $\beta_1 = 0.3058\,(1/^{o}\mathrm{C})$ and $\beta_2 = 3.83$ ^{o}C. These parameters will now be used to fit models for properties obtained from indirect tensile test namely: fracture tensile strength, total energy and static stiffness. The results of the indirect static tensile test (Medani et al. 2002) which will be used to fit the models, are shown in Table 7.

Table 7: Results of the indirect static tensile test

Temperature (^{o}C)	Speed (mm/s)	Fracture tensile strength [MPa]	Total energy [Nmm]	Static stiffness [MPa]
25	0.85	0.6249	40514	
25	0.85	0.5938	38243	
25	0.85	0.6333	42418	
25	0.50	0.5497		38
25	0.50	0.5401		38
25	0.50	0.5294		36
25	0.10		23313	23
25	0.10		22569	20
25	0.10		23146	21

From the data and the parameters of the temperature susceptibility function the other three parameters of the model were determined for each property and are shown in Table 8.

Table 8: The model parameters for the data of the indirect tensile test

	Fracture tensile strength [MPa]	Total energy [Nmm]	Static stiffness [MPa]
Ψ_{min}	0	8343	0
Ψ_{max}	3.7169	380122	403
γ	0.2751	0.3752	0.2703

The experimentally determined and model predicted values of fracture tensile strength, total energy and static stiffness are shown in Table 9.

Table 9: A comparison between experimentally determined and model predict values

T (^{o}C)	Speed (mm/s)	Fracture tensile strength [MPa]		Total energy [Nmm]		Static stiffness [MPa]		Difference %
		experiment	model	experiment	model	experiment	model	
25	0.85					43	44	4.5
25	0.85					41	44	
25	0.85					44	44	
25	0.50			36640	34790			1.4
25	0.50			34235	34790			
25	0.50			35003	34790			
25	0.10	0.3637	0.3561					1.0
25	0.10	0.3607	0.3561					
25	0.10	0.3515	0.3561					

In example 1, the model has been extrapolated in both time and temperature domains, and only in the time domain in examples 2 and 3. The capability of the model in providing a good estimate of the experimentally determined values, demonstrated in the three examples, points towards the existence of a unique temperature susceptibility function for individual materials. This function is applicable for elastic (e.g. static stiffness) and inelastic (e.g. shear strength, fracture strength) properties of several road materials. When coupled with the observed physical response of materials, this has resulted in a model that can successfully be used for estimation of several properties of materials. Furthermore, by successfully predicting values outside the range of the data (extrapolation), it can be concluded that the model has passed its ultimate test.

5 CONCLUSIONS

Based on the material presented in this paper, the following conclusions and remarks can been drawn:

- The exchangeability of not only frequency and temperature (as often reported), but also loading speed and strain rate and temperature allows the application of a single temperature susceptibility function to several properties of mixes.
- There exists a unique temperature susceptibility function for asphaltic mixes. This function explains adequately the time-temperature interaction of asphaltic mixes.
- The domain of applicability of the time temperature superposition principle includes inelastic properties of asphaltic mixes, with conventional and probably elastomer polymer modified binders.
- The parameters of the temperature susceptibility function, together with the parameters of the sigmoidal model, can easily be obtained by using non-linear least square regression techniques, utilising the Solver function in an Excel spreadsheet.
- The applicability of the time-temperature superposition principle to linear and non-linear properties of asphaltic mixes will mean that once the temperature susceptibility function of a certain mix is obtained, from a rather quick (cheap) test e.g. indirect tensile test, it can be used to estimate other properties which are rather difficult or expensive to determine e.g. direct tensile test. This may lead to considerable reduction of test programs, especially when combined with well-designed experimental programs based on statistical techniques.

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