Evaluation of the effect of particle shape on the bearing capacity of railroad ballast with discontinuous analysis

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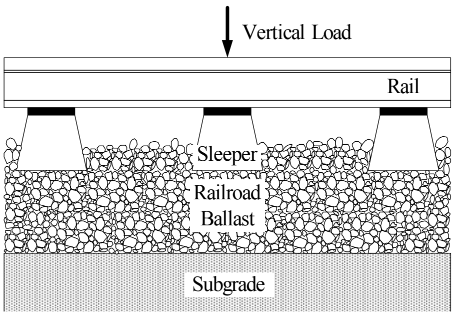
ABSTRACT: This paper examines the effects of particle properties such as particle shape and grain size distribution of particle assemblage on the bearing capacity of railroad ballast in numerical simulations with discontinuous analysis. To evaluate the effect quantitatively, a series of numerical simulations that regard a ballast particle as a regular or an irregular polygon were performed. By comparing model tests of ballasted track with the numerical simulations, the validity of modeling methods of railroad ballast in discontinuous analysis was examined in terms of the shape of ballast particles and the grain size distribution of railroad ballast. As the result, it can be observed that the particle shape of numerical models, especially the angularity has a greater effect on the bearing capacity of railroad ballast than the grain size distribution, and that the imitation of the shape of real ballast particles makes analytical results similar to experimental results.

KEY WORDS: Railroad ballast, Discontinuous analysis, Particle shape, Bearing capacity.

1 INTRODUCTION

The study of “Track deterioration” is one of the principal assignments in railway engineering because track deterioration has serious consequences on the safety of train operation. Track deterioration observed mainly at ballasted tracks is a phenomenon such that the rail level at train passages is irregular toward the longitudinal direction of railway track with repeated train passages. Incidentally, the ballasted track structures are composed of rails, sleepers, railroad ballast and subgrade as shown in Figure 1. The railroad ballast component in which sleepers are embedded is a pile of well-compacted crushed stones. The mechanism of track deterioration is being researched in many countries. In general, a dominant factor of track deterioration is supposed to be uneven subsidence of railroad ballast caused by cyclic wheel loading. Nevertheless, there are a number of points uncertain in the mechanism of track deterioration at the present stage. The reason for this is that the properties of the substructure components consisting of railroad ballast and subgrade are much more variable and complicated than those of the superstructure components consisting of rails, fasteners and sleepers.



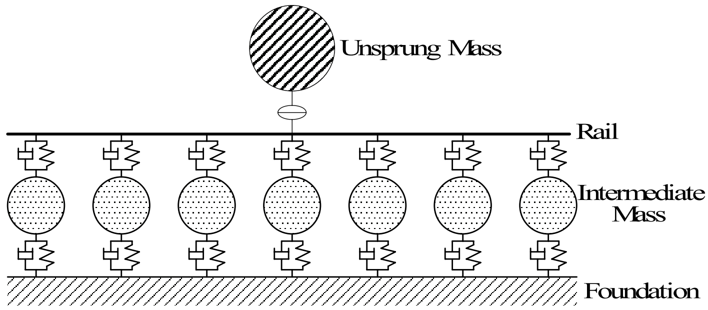


Figure1: Ballasted track structure. Figure2: Mass-spring model in conventional theory of track deterioration.

Therefore, it is essential to examine the cumulative irreversible (plastic) deformation characteristics of railroad ballast in detail.

The railroad ballast is usually composed of single-grained crushed andesite stone, namely "ballast." Traditionally, angular, crushed, hard stones and rocks, uniformly graded and free of dust and dirt, have been considered good ballast materials because such ballast can bear high contact pressures from a sleeper at train passages over a long period of time. Besides, the mechanical properties of ballast such as the strength and deformation characteristics can be characterized by a combination of the particle properties of individual ballast particles and their in-situ laminated state such as density and thickness of ballast layer, namely "ballast depth." To sum up, the mobility of individual particles seems to be concerned with the mechanical properties of ballast. However, the conventional analytical model composed by the mass - elastic spring - dashpot system like Figure 2 cannot evaluate the non-uniform fabric transformation of particle alignment. Accordingly, the mechanical properties of ballast, even though they have a major influence on the mechanism of track deterioration, have been given much less consideration so far.

In general, the mechanics of granular materials is commonly employed to analyze the complex behavior of coarse granular materials such as ballast. From the viewpoint of the mechanics of granular materials, non-linear behaviors of railroad ballast are caused by the movement of ballast particles. Accordingly, discontinuous analysis seems to be effective in simulating the mechanical behavior of railroad ballast because it regards a ballast particle as an element of discontinuous analysis. However, the precision of numerical simulations with discontinuous analyses seriously depends on how to model imhomogeneous internal structure of granular materials. This paper describes a fundamental study to improve the reliability of discontinuous analysis more than the conventional mass-spring model by evaluating the mobility of individual particles in terms of the mechanics of granular materials.

2 2 OBJECTIVES OF RESEARCH

The objectives of this paper are:

- To examine the validity of modeling methods of railroad ballast with discontinuous analysis in terms of the particle properties such as particle shape and grain size distribution of ballast.
- To evaluate the influence of the particle properties of an element of discontinuous analysis on the mechanical behavior of railroad ballast in bearing capacity test of ballasted track.

- To examine the difference in the bearing capacity of railroad ballast which results from the difference in the ballast depth in terms of the mechanics of granular materials.

This paper introduces the numerical simulations of bearing capacity tests for ballasted track performed with discontinuous analysis regarding a ballast particle as a regular or an irregular polygon block (the term "block" is used here in the same way as the term "element"). The above assignments are discussed by comparing model tests of ballasted track with the numerical simulations.

3 ANALYTICAL METHODS

3.1 Modeling

The simulations of bearing capacity tests for model ballasted track (Ishikawa and Sekine, 2002) were performed with two-dimensional DDA models. The Discontinuous Deformation Analysis (DDA, proposed by Shi and Goodman, 1985) is a kind of discontinuous analysis. In DDA, each block is separated by its boundaries and moves individually. DDA is based on the principle of minimum potential energy like FEM (Finite Element Method), however DDA is different from FEM in that DDA is based on discontinuum mechanics like DEM (Distinct Element Method). Figure 3 shows the size, dimension and boundary condition of analytical model tracks, which simulates a one-fifth scale model of a full-scale track. However, for bearing capacity tests, both the analytical model and the real model tracks have a single aluminum sleeper and no rail. The analytical model, which simulates a longitudinal section of the ballasted track, is in the plane strain state with the longitudinal section assumed to infinitely continue like the real model track.

Figure 4 shows the element meshes of DDA models before loading. The DDA model is composed of some polygon blocks, named "ballast blocks," which represent andesite ballast particles and rectangular blocks, named "a sleeper block" which represents a aluminum sleeper, named "a roadbed block" which represents steel subgrade and named "side blocks" which represent a rigid soil container. Furthermore, three types of the element shape, regular icosagon, regular hexagon and expanded hexagon, are employed for a ballast block in view of the angularity of actual ballast particles because the block shape has a strong influence on the stress-strain behavior of granular materials (Kohata et al., 1999). Here, an expanded hexagon block is made by extending a hexagon block in consideration for the shape of real ballast particles as shown in Figure 5. The aspect ratio of an expanded hexagon block was selected at random from the distribution of aspect ratios for the sieve mesh close to diameter of the original hexagon block.

Besides, two types of ballast which had different mean grain size from each other were employed in this paper. The gradation curves for railroad ballast of DDA models are shown in Figure 6, together with their mean grain sizes D_{50} and uniformity coefficients U_c. They are equivalent to the gradation curves of the one-fifth scale model for real bearing capacity tests. The proper grading of railroad ballast provided by the Japanese railway specification has a grain size distribution between approximately 60 mm and 10 mm. Both types of ballast have one-fifth mean grain size distribution of real railroad ballast and the grain size distribution similar to the proper grading of real railroad ballast. The term "A ballast" is used to refer to the model ballast which uniformity coefficient is smaller, and the term "B ballast" is used to refer to the other.

Railroad ballast of DDA models were made with the above-mentioned three types of ballast blocks as follows. First, ballast blocks were thrown one by one into a frame of track profiles to model the random particle alignment in real railroad ballast. After filling the frame with ballast blocks, the frame was removed. Second, a stability analysis was done by gravity force of 1.0 G. In this paper, the state of DDA models after stability analysis is called “the initial loading state”. Figure 4 shows the initial loading state of each model. Here, three type of ballast depth, 20mm, 50mm and 80 mm, are employed. Accordingly, in the simulations, eighteen types of analytical model which differ in particle shape of a ballast block, grain size distribution of ballast blocks and ballast depth were employed as shown in Figure 4.

Table 1 shows the feature of DDA models in comparison with experimental conditions.

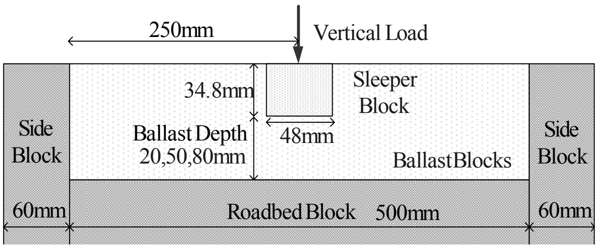


Figure3: Schematic section of DDA model.

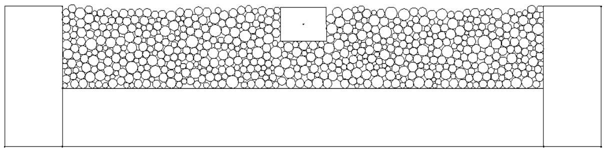


Figure4a: Element mesh (ballast depth 50mm, A ballast, regular icosagon).

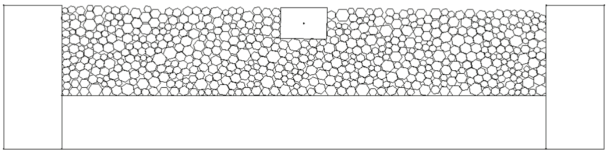


Figure4b: Element mesh (ballast depth 50mm, A ballast, regular hexagon).

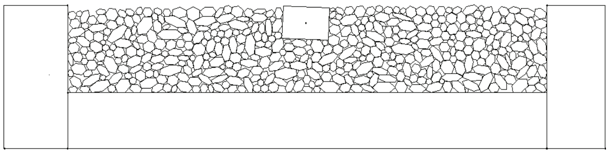


Figure4c: Element mesh (ballast depth 50mm, A ballast, expanded hexagon).

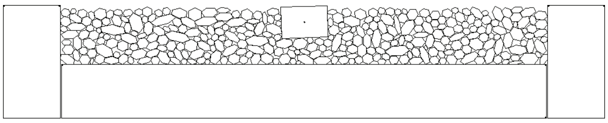


Figure4d: Element mesh (ballast depth 20mm, A ballast, expanded hexagon).

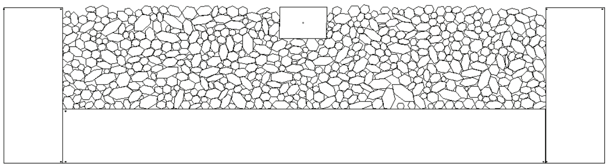


Figure4e: Element mesh (ballast depth 80mm, A ballast, expanded hexagon).

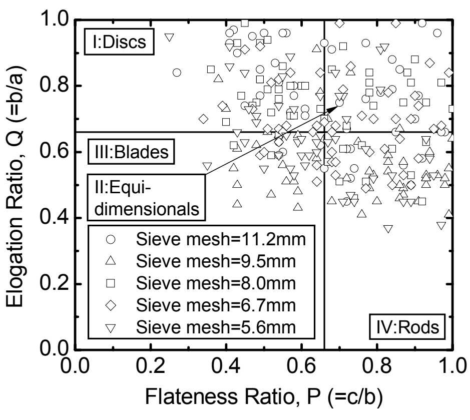


Figure5: Classification of particle shape for ballast particles with the method proposed by Zigg, 1935.

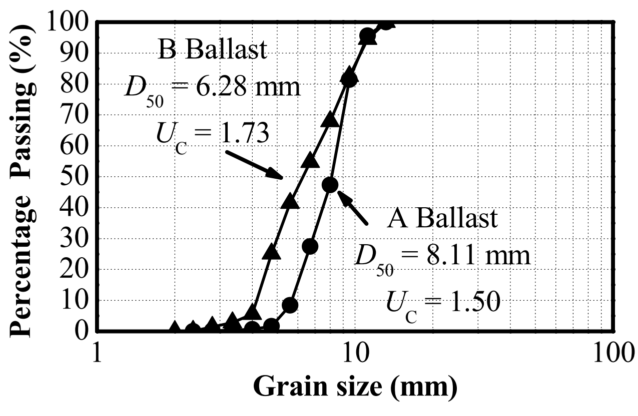


Figure6: Grain size distribution of ballast blocks.

Seeing Table 1, it is recognized that for the same ballast depth and particle shape the total number of ballast blocks for DDA models of A ballast is smaller than that for DDA models of B ballast, and that for the same ballast depth and grain size distribution the total number of ballast blocks increases in order of regular icosagon, regular hexagon and expanded hexagon. In the meantime, the porosity of DDA models is much smaller than that of experiments as shown in Table 1, though there is little difference in the porosity among all analytical models. This phenomenon seems to be mainly caused by the reason that the DDA models used in this study are two-dimensional while the laminated state of experimental specimens is three-dimensional.

Table1: Features of DDA models.

Model		A ballast			B ballast		
		Number of Ballast Blocks	Porosity n(%)	Density (tm^{-3})	Number of Ballast Blocks	Porosity n(%)	Density (tm^{-3})
ballast depth 20mm	regular icosagon	465	20.1	2.16	610	18.4	2.20
	regular hexagon	445	18.7	2.20	552	18.3	2.21
	expanded hexagon	318	21.8	2.11	389	17.5	2.23
	experiment	-	48.2	1.40	-	-	-
ballast depth 50mm	regular icosagon	610	19.0	2.19	913	17.7	2.22
	regular hexagon	598	20.1	2.16	853	18.3	2.21
	expanded hexagon	451	21.4	2.12	552	16.9	2.24
	experiment	-	46.0	1.46	-	-	-
ballast depth 80mm	regular icosagon	803	18.5	2.20	1153	17.5	2.23
	regular hexagon	751	18.5	2.20	1061	16.7	2.25
	expanded hexagon	548	21.0	2.13	718	16.3	2.26
	experiment	-	46.3	1.45	–	–	–

3.2 Analytical Conditions

DDA analyses were performed with two-dimensional models using linear elastic blocks with plane strain. Table 2 shows the material properties of blocks and the interface properties of block edges. In this study, when two DDA blocks come in contact, springs and a slider are created at contact points as shown in Figure 7. Accordingly, the analytical input parameters of DDA blocks are characterized by the material properties of a block, namely unit mass (γ), Young's modulus (E) and Poisson's ratio (ν), and the interface properties of block edges, namely block friction angle (ϕ_μ) and cohesion of surface (C_μ). As for the material properties, the parameters were set by referring to the design values and past experimental results. On the other hand, the interface properties were set as follows. First, the block friction angle (ϕ_μ)between ballast blocks was set equal to 55° because the internal friction angle derived from the analytical results in the case of ϕ_μ=55° indicated the best fit to experimental results in simulating triaxial tests with DDA (Ishikawa et al, 1997). As for ϕ_μ between a ballast block and other kinds of blocks, the value was set equal to 37° by referring to conventional studies. Besides, as for C_μ between all materials, the value was set equal to zero by considering that ballast is a coarse granular material.

Vertical load was applied to DDA models in the initial loading state. The loading point was the center point of the sleeper block as shown in Figure 3. Vertical load slowly increased from 0 kN to 5 kN at the constant loading speed which was regarded as static loading like the real experimental condition. Assuming P=5kN a load to act on a single rail, it corresponds to a simulated wheel load of 194 kN (fn. the track modulus C = 100 MN/m^2, dynamic load factor

S = 1.5, rail type: 50N, sleeper type: PC3 and tie spacing 600 mm) when calculated by applying the theory of elasticity in consideration of track rigidity. Furthermore, the gravity force was applied to DDA models throughout the simulation.

Table 2. Material Properties of Analytical Model.

Property	Sleeper	Ballast	Roadbed	Side
Unit Mass γ	$2.7tm^{-3}$	$2.7tm^{-3}$	$7.8tm^{-3}$	$7.8tm^{-3}$
Young's Modulus E	70GPa	20GPa	210.0GPa	210.0GPa
Poisson's Ratio ν	0.3	0.1	0.3	0.3
Cohesion C_{μ}	0	0	0	0
Friction Angle ϕ_{μ}	37.0°	55.0°	37.0°	37.0

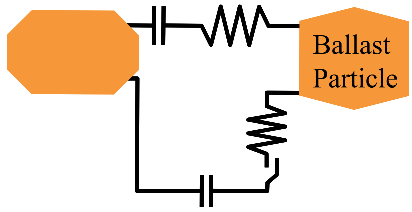
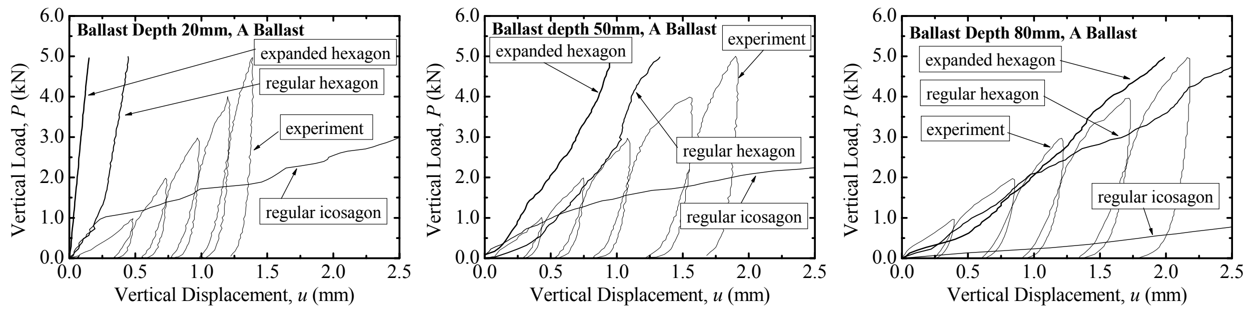


Figure7: Contact mechanism between DDA blocks.

4 ANALYTICAL RESULTS

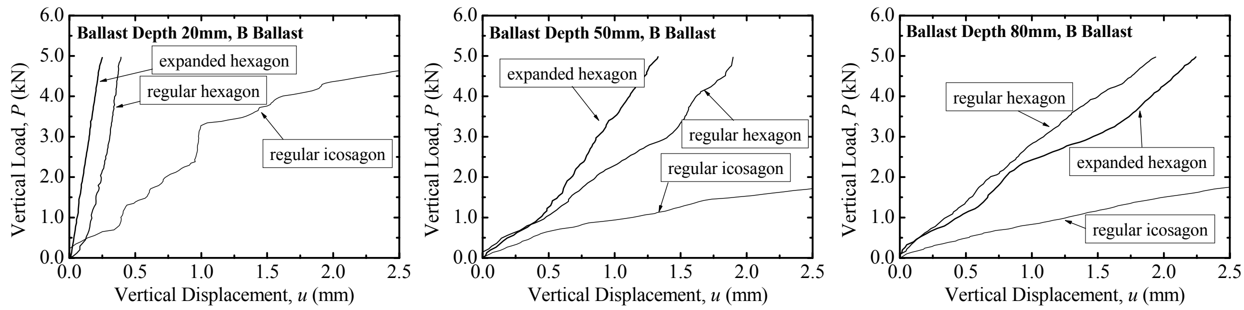
4.1 Load–Displacement Relations

Figure 8 and Figure 9 show the relations of the sleeper block between the vertical load P and the vertical displacement u at the center of gravity of the sleeper block derived from the bearing capacity test simulations. Figure 8 shows the load-displacement relations for DDA models using A ballast at the respective ballast depths of 20mm, 50mm and 80mm comparing three types of the element shape, and the relations for DDA models using B ballast are shown in



(a) ballast depth 20mm (b) ballast depth 50mm (c) ballast depth 80mm

Figure8: Load – displacement relations(A ballast).



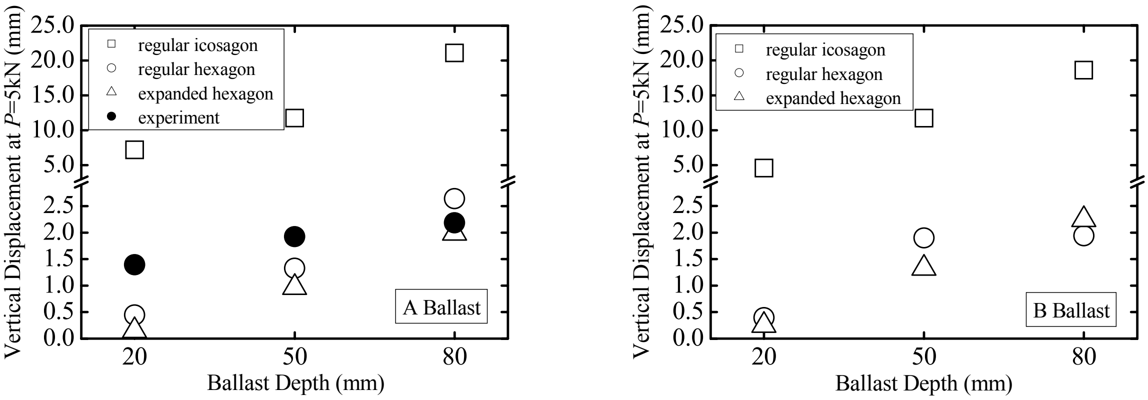
(a) ballast depth 20mm (b) ballast depth 50mm (c) ballast depth 80mm

Figure9: Load – displacement relations(B ballast).

Figure 9. These figures also compare analytical results with experimental results. In this paper, the bearing capacity of DDA models can be defined as the vertical displacement at *P*=5kN, it is considered that the DDA model has stronger bearing capacity in case of obtaining smaller vertical displacement at *P*=5kN.

First, the influence of the particle properties of ballast blocks on the bearing capacity of ballasted track is examined. At the beginning, discussed is the difference in the load-displacement relations due to the difference in the particle shape of a ballast block. Comparing analytical results of each model at the same ballast depth in Figure 8, the bearing capacities of DDA models for A ballast decrease in order of expanded hexagon, regular hexagon, and regular icosagon. Also, with increasing the total number of ballast blocks, the bearing capacity decreases as can be seen when comparing Table 1 and Figure 8. The same is true for analytical results of B ballast as shown in Figure 9. These results indicate that the difference in the particle shape of a ballast block has a considerable influence on the bearing capacity of DDA models.

Furthermore, Figure 10 compares the vertical displacement at *P*=5kN derived from analytical results of each DDA models. In Figure 10, though the bearing capacity of DDA model using regular icosagon blocks is much smaller than analytical results using expanded hexagon blocks or regular hexagon blocks, the bearing capacities in case of expanded hexagon blocks and regular hexagon blocks are approximately the same, regardless of grain size distribution and ballast depth. In terms of the mechanics of granular materials, the difference between expanded hexagon blocks and regular hexagon blocks is the sphericity of DDA blocks, while the difference between regular hexagon blocks and regular icosagon blocks is the angularity of DDA blocks. Accordingly, these results indicate that the angularity of ballast blocks has a greater effect on the bearing capacity of railroad ballast in discontinuous analysis than the sphericity.



(a)A ballast (b)B ballast

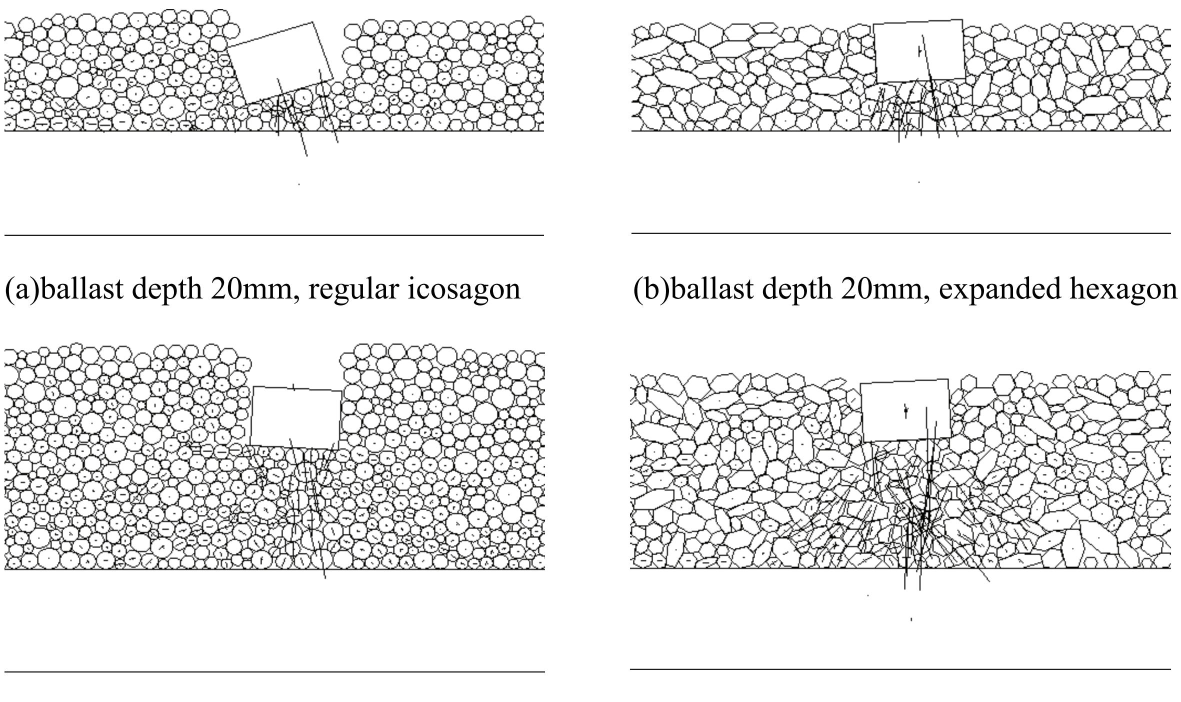
Figure10: Comparison of bearing capacity.

Next, discussed is the difference in the load-displacement relations due to the difference in the grain size distributions of model ballast. Comparing Figure 8 with Figure 9 at respective particle shapes of a ballast block under the same ballast depth condition, in case of DDA model using regular icosagon, the bearing capacity of DDA models for A ballast is larger than that for B ballast. However, in case of the other shapes, the clear tendency cannot be recognized. These results indicate that the difference in the grain size distributions of model ballast has a little influence on the bearing capacity of railroad ballast in discontinuous analysis compared with the influence of particle shape. The same is true for Figure 10.

Second, the validity of modeling methods of railroad ballast is examined. At the beginning, discussed is the difference in the load-displacement relations due to the difference in the particle shape of a ballast block. Comparing analytical results with experimental results at the same ballast depth in Figure 8, it is recognized that the load-displacement relations of DDA model using regular icosagon blocks has much smaller stiffness than experimental results. While, the load-displacement relations of DDA model using expanded hexagon or regular hexagon blocks resembles experimental results for A ballast at every ballast depth. Moreover, Figure 10a compares the vertical displacement at *P*=5kN of each DDA models with experimental results. In Figure 10, though the bearing capacity of DDA model using regular icosagon blocks is much smaller than that of experimental results, the bearing capacity of DDA model using expanded hexagon blocks or regular hexagon blocks is approximately equal to experimental results at every ballast depth. From these results, it is considered that the usage of expanded hexagon or regular hexagon blocks is appropriate of simulating the mechanical behavior of railroad ballast with DDA. As it is said that a ballast particle has at most six distinctive angles (Kono et al, 2000), it is interesting that the above-mentioned results were obtained, and this indicates that the imitation of the shape of real ballast particles makes the reliability of analytical results improved.

4.2 Mechanical Behavior of Railroad Ballast

First, the influence of the particle shape of ballast blocks on the mechanical behavior of railroad ballast is examined. Figure 11 shows the distribution of principal stress for DDA blocks at *P*=5kN. In Figure 11, it is recognized that the analytical results of DDA models using regular icosagon blocks differ from those of DDA models using expanded hexagon in the distribution of principal stress. For example, comparing Figure 11c with Figure 11d, in case of regular icosagon, there are many ballast blocks underneath the sleeper block which principal stress turns toward horizontal direction. Whereas, in case of expanded hexagon blocks, the principal stress of the ballast blocks mainly turns toward vertical direction. Besides, in Figure 11d, it



(a)ballast depth 20mm, regular icosagon

(b)ballast depth 20mm, expanded hexagon

(c)ballast depth 80mm, regular icosagon

(d)ballast depth 80mm, expanded hexagon

Figure11: Distribution of principal stress.

can be observed that high principal stress appears underneath the sleeper block. However, principal stress observed in Figure 11c is smaller compared with Figure 11d on the whole. The reason for this is that the DDA model using regular icosagon blocks cannot sustain vertical loads by the ballast blocks under the sleeper block alone as it has very low bearing capacity, though DDA models using expanded hexagon blocks can sustain it due to their high bearing capacity. Consequently, the lateral flow occurs in railroad ballast around the sleeper block in Figure 11c, and it leads to large settlement of the sleeper block. These results indicate that the particle shape of a ballast block is one of the dominant factors to determine the mechanical behavior of railroad ballast in numerical simulations, and that the mobility of individual particles is concerned with the mechanical properties of railroad ballast.

Second, the influence of ballast depth on the mechanical behavior of railroad ballast in discontinuous analysis is examined. Comparing Figure 11d with Figure 11b, it is recognized that in case of ballast depth of 80 mm, high ballast pressure suffered from a sleeper at train passages gradually spreads inside railroad ballast with the increment of ballast depth, though, in case of ballast depth of 20 mm, it does not spread well. These tendencies are in fair agreement with the experimental results. This indicates that discontinuous analysis is an effective method to simulate the mechanical behavior of railroad ballast in bearing capacity tests if an element of discontinuous analysis imitates the shape of real ballast particles well. However, according to Figure 8, though in case of the ballast depth of 50mm or 80 mm the load-displacement relations of analytical results (seeing the relations of expanded hexagon or regular hexagon) agree well with experimental results, in case of the ballast depth of 20 mm the analytical results have much harder stiffness than experimental results. This phenomenon seems to be mainly caused by the reason that the DDA models used in this study are two-dimensional while the laminated state of experimental specimens is three-dimensional. In case the total number of ballast blocks is small, it is considered that a problem in modeling real phenomenon with two-dimensional analysis tends to become evident as free motion of individual particles is strongly restricted.

5 CONCLUSIONS

The following conclusions can be obtained;

1. In DDA, the difference in the particle shape of a ballast block has a greater influence on the bearing capacity of railroad ballast than the grain size distributions of model ballast. Especially, the angularity of a ballast block has a considerable influence on the bearing capacity of railroad ballast in discontinuous analysis, compared with the sphericity.
2. Discontinuous analysis is an effective method to simulate the mechanical behavior of railroad ballast in bearing capacity tests, and the imitation of the shape of real ballast particles makes the reliability of analytical results improved.
3. The particle shape of a ballast block is one of the dominant factors to determine the mechanical behavior of railroad ballast in numerical simulations, and the mobility of individual particles is concerned with the mechanical properties of railroad ballast.
4. In case the total number of ballast blocks is small, a problem in modeling real phenomenon with two-dimensional analysis tends to become evident as free motion of individual particles is strongly restricted.

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