Evaluation of Different Procedures and Models for the Construction of Dynamic Modulus Master Curves of Asphalt Mixtures

Silvia Angelone, Manuel Borghi, Marina Cauhape Casaux & Fernando Martinez
Road Laboratory, School of Engineering, University of Rosario, Rosario, Argentina

ABSTRACT: The dynamic modulus |E*| is the primary input material property of asphalt mixtures for the asphalt pavement design procedures based on mechanistic principles. Among other factors, this dynamic modulus is a function of temperature and loading frequency. In order to model the effects of these factors, the dynamic modulus of the asphalt mixture is described using a master curve constructed at a given reference temperature on the basis of the principle of frequency-temperature superposition for thermo-rheologically simple materials. The amount of shifting at each temperature is given by a shift factor that describes the temperature dependency of the material. Different methods and mathematical functions have been proposed to model these shift factors and the resulting dynamic modulus master curve. The objective of this paper is to evaluate these various methods and mathematical equations that can be satisfactorily used for modelling the dynamic modulus |E*| master curves, as a function of frequency at a reference temperature for six different asphalt mixtures used in Argentina. The methodology for evaluating these models was based on using the same laboratory test data and comparing the resulting curves through correlation analysis. The experimental results and a description of the considered procedures and models used to develop the master curves are presented followed by a comparative analysis and a synthesis of the obtained findings.

KEY WORDS: Dynamic modulus, asphalt mixtures, master curves, shifts factors.

1 INTRODUCTION

Pavement design is moving towards more mechanistic-based methodologies where the characterization of mechanical properties of paving materials plays a vital role in the determination of pavement structure responses. For the asphalt mixtures, the dynamic modulus |E*| is the primary input material property and it is defined as the absolute value of the maximum (peak-to-peak) stress divided by the maximum recoverable (peak-to-peak) axial strain for a material subjected to a sinusoidal loading.

Among other factors, this dynamic modulus is a function of temperature and loading frequency. In order to model the effects of these factors, the dynamic modulus of the asphalt mixture is described using a master curve constructed at a given reference temperature on the basis of the principle of frequency-temperature superposition for thermo-rheologically simple materials. It has been shown that this frequency-temperature superposition principle may hold even if the linear viscoelasticity conditions are violated (Chehab et al., 2002).

In the construction of master curves, dynamic modulus values obtained at multiple temperatures are shifted by applying a multiplier to the frequency at which the measurements are taken so that the individual results combine to form a single smooth curve of variation of the dynamic modulus versus the frequency. The amount of shifting at each temperature is given by a shift factor aT that describes the temperature dependency of the material and it is defined as:

$$aT = \frac{f_R}{f} \qquad (1)$$

where:

f_R : reduced frequency (loading frequency at the reference temperature)

f : loading frequency

aT : shift factor

The obtained master curve could be used to estimate or interpolate the dynamic modulus at different temperatures and loading frequencies of interest from a limited set of laboratory test data (Medani et al., 2004).

Several mathematical functions have been proposed to model the dynamic modulus master curve to be used in pavement design softwares but in general, the whole modulus curve has a S-shaped form with the modulus tending to a limited high modulus (glassy modulus) for large frequencies and to a small constant value (equilibrium modulus) when the frequency tends to zero (Pronk, 2004). Also for the shift factor aT, several functions describing the temperature dependency of this parameter have been proposed.

The objective of this paper is to evaluate these various methods and mathematical functions that can be satisfactorily used for modelling the dynamic modulus |E*| master curves, as a function of frequency at a reference temperature for six different asphalt mixtures used in Argentina. The methodology for evaluating these models was based on using the same laboratory test data and comparing the resulting curves through correlation analysis. The experimental results and a description of the considered procedures and models used to develop the master curves are presented followed by a comparative analysis and a synthesis of the obtained findings.

2 MODELS FOR THE DYNAMIC MASTER CURVES

Three different models were evaluated describing the dynamic modulus master curves of asphalt mixtures:

- a symmetrical or standard logistic sigmoidal model
- a non-symmetrical or generalized logistic sigmoidal model
- a power model

The methodology for evaluating these three methods was based on using the same laboratory test data to generate the |E*| master curves as a function of the Reduced frequency (f_R) at a reference temperature (TR) of 20ºC and then comparing the resulting curves through correlation analysis.

2.1 The symmetrical or standard logistic sigmoidal model (SLSM)

The functional form of this model is:

$$\log\left|E^*\right| = \delta + \frac{(\alpha - \delta)}{1 + e^{(\beta + \gamma \cdot \log f_R)}} \qquad (2)$$

where:

|E*| : dynamic modulus (MPa)

δ : logarithm of the minimum value of |E*| (|E*|min, equilibrium modulus)

α : logarithm of the maximum value of |E*| ((|E*|max, glassy modulus)

β, γ : parameters describing the shape of the sigmoidal function and the location of the inflection point

This type of function has been adopted in the Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures developed under the project NCHRP 9-19 (NCHRP, 2004) and by other different authors (Witczak & Sotil, 2004; Witczak et al., 2004).

2.2 The non-symmetrical or generalized logistic sigmoidal model (GLSM)

The functional form of this model is represented by the following equation:

$$\log\left|E^*\right| = \delta + \frac{(\alpha-\delta)}{\left[1+\lambda e^{(\beta+\gamma\cdot\log f_R)}\right]^{(1/\lambda)}} \tag{3}$$

where:

λ : additional parameter producing the non symmetrical shape.

In this equation |E*|, f_R, δ, α, β and γ have the same meaning as in equation (2). The difference between equation (2) and (3) is the introduction of the lambda parameter, λ, which allows the curve to have a non-symmetric shape; when $\lambda = 1$, the equation reduces to the standard sigmoidal format as represented by equation (2) (Richards, 1959; Rowe et al, 2009).

2.3 The power model

The power model is defined by the equation:

$$\log\left|E^*\right| = A - B\frac{f_R{}^C}{f_R{}^C + D} \tag{4}$$

where:

A, B, C and D: parameters defining the form of the function

The parameter A defines the logarithm of the maximum |E*| value ((|E*|max, glassy modulus) for the high frequencies; when the frequency tends to zero, the logarithm of the minimum |E*| value ((|E*|min, equilibrium modulus) tends to (A-B) (Pronk, 2004).

3. EQUATIONS FOR THE SHIFT FACTORS aT

Three conventional forms for shift factors used for asphalt material analysis have been considered in this paper:

- an Arrhenius equation
- a Williams-Landel-Ferry (WLF) equation
- a second order polynomial equation

3.1 The Arrhenius equation

The Arrhenius function for calculating the shift factor aT is presented in equation (5):

$$\log(aT) = C_A\left(\frac{1}{T_i} - \frac{1}{T_R}\right) \tag{5}$$

with:

C_A : Arrhenius material constant

T_i : test temperature of interest (ºK)

T_R : reference temperature (ºK)

This equation has been adopted by different researchers to calculate the shift factor aT as a function of the temperature (Bonaquist, 2003; Picado et al. 2003; Bennert et al., 2004).

3.2 The Williams-Landel-Ferry (WLF) equation

Equation (6) is the WLF model for calculating the shift factor aT:

$$\log(aT) = \frac{-C_1\cdot(T_i - T_R)}{(C_2 + T_i - T_R)} \tag{6}$$

with:

C_1 and C_2 : WLF material constants
Several authors used this classical equation to shift the dynamic modulus data for the construction of master curves (Wu et al., 2006; Di Benedetto et al., 2010).

3.3 The second order polynomial equation

Equation (7) is the second order polynomial equation for evaluating the shift factor aT:

$$\log(aT) = a + b \cdot T_i + c \cdot T_i^2 \tag{7}$$

with:
T_i : test temperature of interest (ºC)

If the test temperature of interest T_i is equal to the reference temperature T_R, equation (7) must be conditioned in order to give shift factors aT equal to the unity. Different authors have proposed a second order polynomial equation to model the variation of the shift factor as a function of the testing temperature (Kim et al., 2005; Kalousch et al., 2002, Al-Khateeb et al., 2006)

4 PROCEDURES AND MATERIALS

4.1 Testing Procedures

In this paper, uniaxial compression tests with sinusoidal loads (haversine) were performed using a servo-pneumatic machine, developed at the Road Laboratory of the University of Rosario. The test frame is enclosed into a temperature chamber where the temperature control system is able to achieve the required testing temperatures ranging from 0 °C to 50 °C. The measurements of the dynamic modulus in uniaxial compression were carried out following a procedure similar as it is described in the AASHTO TP-62 Standard Method of Test for Determining Dynamic Modulus of Hot-Mix Asphalt Concrete Mixtures. Two samples of each asphalt mixtures were tested and the obtained results were averaged. Tall samples with 100 mm diameter and a relationship height/diameter equal to 1.5 were used for the determination of the dynamic modulus in uniaxial compression. The |E*| results were determined for 7 frequencies (5, 4, 2, 1, 0.5, 0.25 and 0.10 Hz) and 4 temperatures (10, 20, 30 and 40 °C) in order to have a full viscoelastic characterization of the asphalt mixtures.

4.2 Asphalt Mixtures

Six different asphalt mixtures were considered in this study identified as A, B, H, I, J and K. A brief description of each one is presented in Table 1. It should be noted that the mixtures H and I are two different non conventional asphalt mixtures formulated with a local calcareous soil (named “Tosca” in Argentina), natural sand and asphalt bitumen (Laboratory of Road Research, 1972). Due to the geographical and geological characteristic of some regions of Argentina where the absence of good quality rock aggregates requires carrying this kind of materials from distances sometimes over 350 kilometres, this sub normal aggregate is used as a substitute in asphalt base layers in order to reduce costs and preserving the natural resources. These resulting asphalt mixtures have remarkable rheological properties as are shown later herein.

5 OBTAINED RESULTS

The average dynamic modulus results for the 6 asphalt mixtures at the 7 testing frequencies and the 4 temperatures are listed in Table 2.

Table 1: Asphalt mixtures considered

Id.	Description	Asphalt content by weight (%)	Bitumen type	Air Voids (%)	Maximum aggregate size (mm)
A	Dense asphalt concrete with basaltic aggregates	4.7	Conventional AC30	4.9	19.0
B	Dense asphalt concrete with basaltic aggregates	5.1	Conventional AC30	4.1	19.0
H	Non conventional asphalt mixture with calcareous soil and natural sand	9.0	Conventional AC70-100	3.2	37.5
I	Non conventional asphalt mixture with calcareous soil and natural sand	9.3	Conventional AC70-100	3.1	25.0
J	Dense asphalt concrete with granitic aggregates	4.5	Polymer modified PmB-III	4.2	19.0
K	Dense asphalt concrete with granitic aggregates	4.8	Polymer modified PmB-III	3.9	19.0

It should be noted the reduced temperature susceptibility of the non conventional asphalt mixtures H and I with higher |E*| values at 30 and 40ºC and at all frequencies compared to the other conventional asphalt mixtures. Based on the three models for the construction of the dynamic modulus master curves and the three equations for calculating the shift factors aT, the results listed in Table 2 were used to develop nine different master curves for each mixture. The parameters of the models for the dynamic modulus master curves for the different mixes and the parameters involved in the shift factor equations were obtained simultaneously by minimising the sum of the square of the errors of the experimental and model values (in both, logarithmic and arithmetic spaces) using the Solver function in the Excel spreadsheet. In this paper, Solver was used because it is a simple, robust and worldwide used optimization method. In order to illustrate the different |E*| behaviour of the tested mixtures, Figure 1 presents a comparison of the six master curves constructed with this procedure using the GLSM model and the WLF equation adjusted in logarithmic space.

Tables 3 to 8 present a summary of the main results obtained using the three models and the three equations for the six mixtures considered in this paper (Arr: Arrhenius; WLF: Williams-Landel-Ferry; Poly: Polynomial; Av: Average; St.Dv: Standard Deviation). In these tables, the maximum and the minimum |E*| values (glassy and equilibrium modulus respectively), the resulting shift factors at 10, 30 and 40ºC, the correlation coefficient R^2 and the relationship between the standard error of estimate values and the standard deviation of measured values Se/Sy, are presented. These two last parameters are a good indicator of the goodness-of-fit of the models: higher R^2 and lower Se/Sy values indicate better fitting of the master curve to the considered data (Witczak et al., 2002).

6 ANALYSIS OF RESULTS

The analysis of the obtained results shows that the nine possibilities of models and equations considered in this paper, in logarithmic and arithmetic spaces, provide excellent fitting between measured and modelled values for the six analyzed asphalt mixtures. In all cases, the R^2 values were grater than 0.98 and Se/Sy values smaller than 0.12 showing the excellent goodness-of-fit. From the point of view of their simplicity, the best models are the SLSM and the Power models using the Arrhenius equation, because only five parameters must be adjusted during the optimizing process, followed by the same models but with the WLF equation with six parameters to be adjusted.

Table 2: Average dynamic modulus |E*| for the six tested mixtures (MPa)

Temperature (ºC)	Frequency (Hz)	Asphalt mixture					
		A	B	H	I	J	K
10	5.00	18005	11641	12154	4678	10628	8380
10	4.00	17127	11188	11872	4512	9964	8090
10	2.00	15493	9603	11500	4316	8883	7090
10	1.00	13600	8322	10373	4163	7626	6008
10	0.50	11919	7024	9880	4038	6663	5239
10	0.25	10222	5899	9266	3926	5574	4432
10	0.10	7983	4605	8028	3710	4399	3458
20	5.00	9405	5666	8619	3818	5161	3707
20	4.00	8880	5372	8170	3808	4991	3495
20	2.00	6903	4326	7307	3613	3943	2706
20	1.00	5565	3246	6392	3453	3093	2151
20	0.50	4249	2428	5795	3341	2440	1717
20	0.25	3196	1932	4939	3183	1911	1398
20	0.10	2162	1209	4190	2964	1393	1005
30	5.00	3298	2136	4638	2852	1692	1737
30	4.00	2901	1956	4474	2776	1600	1660
30	2.00	2165	1405	3774	2650	1071	1315
30	1.00	1581	1042	3228	2504	845	970
30	0.50	1162	750	2756	2311	649	797
30	0.25	854	543	2301	2182	502	676
30	0.10	557	395	1812	2014	393	576
40	5.00	954	680	2815	2399	854	674
40	4.00	819	561	2646	2350	783	620
40	2.00	648	437	2243	2225	742	514
40	1.00	479	379	1845	2013	553	496
40	0.50	375	330	1483	1841	498	475
40	0.25	299	298	1255	1676	402	286
40	0.10	239	196	956	1498	298	248

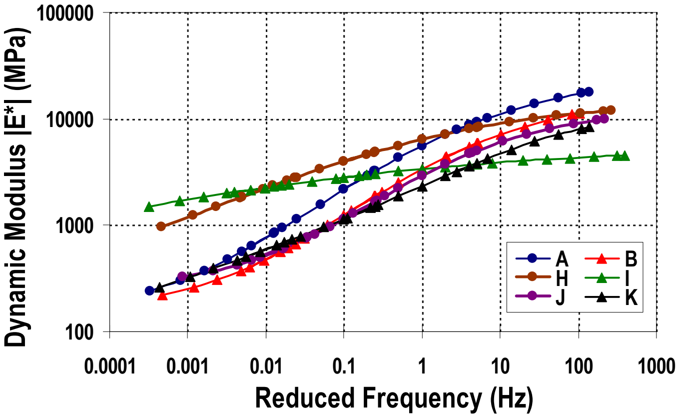

Figure 1: |E*| master curves for the 6 tested mixtures (GLSM, WLF equation)

In logarithmic space, for the mixtures A and B, conventional asphalt mixtures with conventional bitumen, the maximum and minimum |E*| values (glassy and equilibrium modulus) modelled with the nine possibilities investigated in this study show almost the same results with coefficients of variation (COV) lower than the commonly used 20% threshold, indicating that the models and equations are not statistically different. However in arithmetic space, significant differences were observed for the minimum |E*| values with COV greater than 48%.

Table 3: Summary of obtained results for the mixture A

		Model	SLSM			GLSM			Power			Av.	St.Dv.	COV
		Equation	Arr.	WLF	Poly.	Arr.	WLF	Poly.	Arr.	WLF	Poly.	(MPa)	(MPa)	(%)
Logarithmic		R^2	0.9996	0.9997	0.9997	0.9996	0.9997	0.9997	0.9996	0.9997	0.9997	----	----	----
		Se/Sy	0.021	0.018	0.018	0.021	0.016	0.018	0.021	0.018	0.018	----	----	----
		|E*|max	27376	26532	26703	27977	28764	27032	27367	26519	26743	27224	750	2.8%
		|E*|min	96	93	93	105	133	99	96	93	93	100	13	13.0%
	log (aT)	10°C	1.388	1.431	1.421	1.388	1.443	1.422	1.388	1.431	1.423	1.415	0.021	1.5%
		30°C	-1.297	-1.302	-1.306	-1.297	-1.304	-1.306	-1.297	-1.302	-1.306	-1.302	0.004	0.3%
		40°C	-2.511	-2.492	-2.496	-2.511	-2.488	-2.495	-2.511	-2.492	-2.496	-2.499	0.009	0.4%
Arithmetic		R^2	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	----	----	----
		Se/Sy	0.018	0.017	0.017	0.017	0.017	0.016	0.018	0.017	0.017	----	----	----
		|E*|max	28131	28182	28277	29299	28160	28408	28119	28152	28156	28321	378	1.3%
		|E*|min	83	66	54	179	71	60	83	60	61	80	38	48.3%
	log (aT)	10°C	1.431	1.438	1.441	1.432	1.439	1.442	1.431	1.441	1.435	1.437	0.004	0.3%
		30°C	-1.337	-1.312	-1.301	-1.338	-1.306	-1.300	-1.337	-1.302	-1.305	-1.315	0.017	1.3%
		40°C	-2.588	-2.515	-2.463	-2.590	-2.496	-2.460	-2.588	-2.484	-2.480	-2.518	0.055	2.2%

Table 4: Summary of obtained results for the mixture B

		Model	SLSM			GLSM			Power			Av.	St.Dv.	COV
		Equation	Arr.	WLF	Poly.	Arr.	WLF	Poly.	Arr.	WLF	Poly.	(MPa)	(MPa)	(%)
Logarithmic		R^2	0.9987	0.9986	0.9987	0.9989	0.9990	0.9990	0.9987	0.9986	0.9986	----	----	----
		Se/Sy	0.037	0.037	0.037	0.033	0.032	0.032	0.037	0.037	0.037	----	----	----
		|E*|max	17317	17123	17239	18506	18515	18591	17309	17116	17234	17661	661	3.7%
		|E*|min	126	125	125	146	151	149	126	125	125	133	12	9.0%
	log (aT)	10°C	1.292	1.307	1.299	1.292	1.315	1.305	1.292	1.307	1.299	1.301	0.008	0.6%
		30°C	-1.207	-1.207	-1.210	-1.207	-1.208	-1.210	-1.207	-1.207	-1.210	-1.208	0.002	0.1%
		40°C	-2.336	-2.325	-2.330	-2.336	-2.321	-2.326	-2.336	-2.325	-2.330	-2.330	0.006	0.2%
Arithmetic		R^2	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9994	----	----	----
		Se/Sy	0.024	0.023	0.023	0.024	0.022	0.022	0.024	0.024	0.026	----	----	----
		|E*|max	23776	23932	24246	23866	24930	24597	23751	27711	25716	24725	1294	5.2%
		|E*|min	38	30	19	44	30	24	38	5	5	26	14	54.0%
	log (aT)	10°C	1.340	1.346	1.352	1.340	1.354	1.352	1.340	1.350	1.399	1.353	0.018	1.3%
		30°C	-1.252	-1.235	-1.212	-1.252	-1.203	-1.211	-1.252	-1.237	-1.196	-1.228	0.022	1.8%
		40°C	-2.424	-2.372	-2.285	-2.424	-2.280	-2.281	-2.424	-2.374	-2.189	-2.339	0.084	3.6%

Table 5: Summary of obtained results for the mixture H

		Model	SLSM			GLSM			Power			Av.	St.Dv.	COV
		Equation	Arr.	WLF	Poly.	Arr.	WLF	Poly.	Arr.	WLF	Poly	(MPa)	(MPa)	(%)
Logarithmic		R^2	0.9953	0.9979	0.9983	0.9965	0.9980	0.9983	0.9953	0.9979	0.9983	----	----	----
		Se/Sy	0.070	0.047	0.041	0.060	0.046	0.042	0.070	0.047	0.042	----	----	----
		|E*|max	20004	15852	15814	14991	14686	16155	19980	15842	16473	16644	1975	11.9%
		|E*|min	215	176	164	1	1	142	217	177	119	135	82	60.8%
	log (aT)	10°C	1.345	1.751	1.739	1.338	1.728	1.743	1.345	1.751	1.744	1.609	0.200	12.4%
		30°C	-1.256	-1.312	-1.362	-1.249	-1.308	-1.365	-1.256	-1.312	-1.367	-1.310	0.048	3.7%
		40°C	-2.432	-2.333	-2.348	-2.419	-2.332	-2.353	-2.432	-2.332	-2.358	-2.371	0.044	1.8%
Arithmetic		R^2	0.9979	0.9982	0.9988	0.9982	0.9987	0.9988	0.9979	0.9986	0.9979	----	----	----
		Se/Sy	0.047	0.043	0.035	0.044	0.037	0.035	0.047	0.038	0.046	----	----	----
		|E*|max	16033	15988	15799	13981	14686	15801	16030	15709	15665	15521	709	4.6%
		|E*|min	625	560	326	1	13	322	626	393	594	384	246	63.9%
	log (aT)	10°C	1.499	1.526	1.599	1.492	1.573	1.600	1.499	1.589	1.436	1.535	0.058	3.8%
		30°C	-1.400	-1.385	-1.330	-1.393	-1.325	-1.331	-1.400	-1.321	-1.309	-1.355	0.038	2.8%
		40°C	-2.710	-2.648	-2.392	-2.698	-2.457	-2.392	-2.711	-2.437	-2.491	-2.548	0.140	5.5%

Table 6: Summary of obtained results for the mixture I

	Model	SLSM			GLSM			Power			Av.	St.Dv.	COV
	Equation	Arr.	WLF	Poly.	Arr.	WLF	Poly.	Arr.	WLF	Poly.	(MPa)	(MPa)	(%)
Logarithmic	R^2	0.9889	0.9891	0.9892	0.9891	0.9891	0.9897	0.9891	0.9891	0.9897	----	----	----
	Se/Sy	0.107	0.106	0.106	0.106	0.106	0.103	0.106	0.106	0.103	----	----	----
	|E*|max	7911	5431	4970	6904	5435	5211	6862	5423	5198	5927	1029	17.4%
	|E*|min	84	335	460	267	355	214	305	341	201	285	109	38.2%
	log (aT) 10ºC	1.445	1.895	2.134	1.437	1.895	2.122	1.436	1.895	2.121	1.820	0.302	16.6%
	30ºC	-1.350	-1.408	-1.554	-1.342	-1.408	-1.555	-1.342	-1.408	-1.554	-1.436	0.093	6.5%
	40ºC	-2.613	-2.496	-2.528	-2.598	-2.496	-2.543	-2.597	-2.496	-2.541	-2.545	0.047	1.8%
Arithmetic	R^2	0.9896	0.9899	0.9911	0.9896	0.9902	0.9911	0.9896	0.9900	0.9894	----	----	----
	Se/Sy	0.104	0.102	0.096	0.104	0.100	0.096	0.104	0.101	0.104	----	----	----
	|E*|max	6242	6156	5835	6166	5775	5834	6264	6040	6393	6078	220	3.6%
	|E*|min	542	476	209	538	492	218	528	509	376	432	133	30.9%
	log (aT) 10ºC	1.522	1.571	1.753	1.521	1.651	1.753	1.522	1.583	1.445	1.591	0.107	6.7%
	30ºC	-1.421	-1.422	-1.432	-1.421	-1.401	-1.432	-1.421	-1.416	-1.336	-1.411	0.030	2.1%
	40ºC	-2.752	-2.716	-2.544	-2.750	-2.604	-2.544	-2.752	-2.690	-2.562	-2.657	0.093	3.5%

Table 7: Summary of obtained results for the mixture J

	Model	SLSM			GLSM			Power			Av.	St.Dv.	COV
	Equation	Arr.	WLF	Poly.	Arr.	WLF	Poly.	Arr.	WLF	Poly.	(MPa)	(MPa)	(%)
Logarithmic	R^2	0.9973	0.9924	0.9875	0.9963	0.9928	0.9886	0.9973	0.9924	0.9875	----	----	----
	Se/Sy	0.053	0.088	0.112	0.062	0.086	0.107	0.053	0.088	0.112	----	----	----
	|E*|max	14645	12509	11946	12359	13151	13270	14640	12506	11941	12996	1038	8.0%
	|E*|min	287	231	199	219	262	282	287	231	199	244	36	14.9%
	log (aT) 10ºC	1.311	1.626	1.755	1.310	1.642	1.794	1.311	1.626	1.757	1.570	0.204	13.0%
	30ºC	-1.224	-1.179	-1.252	-1.223	-1.180	-1.263	-1.224	-1.179	-1.252	-1.220	0.033	2.7%
	40ºC	-2.370	-2.073	-2.000	-2.368	-2.069	-1.996	-2.370	-2.073	-2.000	-2.147	0.170	7.9%
Arithmetic	R^2	0.9983	0.9983	0.9983	0.9982	0.9986	0.9986	0.9983	0.9983	0.9983	----	----	----
	Se/Sy	0.042	0.043	0.041	0.044	0.039	0.039	0.042	0.041	0.042	----	----	----
	|E*|max	16749	16763	16709	16239	20268	20196	16740	16685	17034	17487	1570	9.0%
	|E*|min	208	191	242	113	440	439	208	246	190	253	112	44.4%
	log (aT) 10ºC	1.422	1.426	1.413	1.422	1.422	1.421	1.422	1.412	1.443	1.423	0.009	0.6%
	30ºC	-1.328	-1.302	-1.379	-1.328	-1.364	-1.369	-1.328	-1.385	-1.330	-1.346	0.029	2.1%
	40ºC	-2.572	-2.496	-2.725	-2.572	-2.675	-2.687	-2.572	-2.743	-2.548	-2.621	0.088	3.3%

Table 8: Summary of obtained results for the mixture K

	Model	SLSM			GLSM			Power			Av.	St.Dv.	COV
	Equation	Arr.	WLF	Poly.	Arr.	WLF	Poly.	Arr.	WLF	Poly.	(MPa)	(MPa)	(%)
Logarithmic	R^2	0.9934	0.9968	0.9958	0.9958	0.9975	0.9958	0.9934	0.9968	0.9958	----	----	----
	Se/Sy	0.084	0.058	0.067	0.066	0.051	0.067	0.084	0.058	0.067	----	----	----
	|E*|max	62716	42799	50809	9451	11223	46912	62395	42610	50334	42139	19442	46.1%
	|E*|min	51	44	43	1	1	37	52	45	43	35	20	56.5%
	log (aT) 10ºC	1.373	1.493	1.451	1.362	1.441	1.450	1.373	1.493	1.451	1.432	0.051	3.5%
	30ºC	-1.283	-1.271	-1.285	-1.272	-1.258	-1.285	-1.282	-1.270	-1.285	-1.277	0.009	0.7%
	40ºC	-2.484	-2.365	-2.404	-2.462	-2.367	-2.404	-2.483	-2.364	-2.403	-2.415	0.049	2.0%
Arithmetic	R^2	0.9970	0.9974	0.9984	0.9971	0.9987	0.9984	0.9970	0.9988	0.9970	----	----	----
	Se/Sy	0.055	0.052	0.041	0.055	0.036	0.041	0.055	0.036	0.056	----	----	----
	|E*|max	19078	19349	19153	18465	17370	21603	19002	19242	16352	18846	1446	7.7%
	|E*|min	233	209	96	224	106	50	235	68	214	159	77	48.6%
	log (aT) 10ºC	1.528	1.540	1.564	1.527	1.569	1.575	1.528	1.580	1.415	1.536	0.050	3.3%
	30ºC	-1.427	-1.397	-1.210	-1.427	-1.141	-1.207	-1.427	-1.134	-1.246	-1.291	0.127	9.8%
	40ºC	-2.763	-2.669	-2.066	-2.762	-2.008	-2.046	-2.762	-1.989	-2.323	-2.376	0.358	15.1%

For the non conventional asphalt mixtures H and I, significant lower minimum |E*| values have been obtained in some cases in both, logarithmic and arithmetic space. Also for the mixture K, great differences result for the glassy and equilibrium modulus using the GLSM model and the Arrhenius and the WLF equations compared to the others. In general, the greatest differences were observed in the minimum |E*| values and particularly, when the optimization procedure is considered in arithmetic space.

For the testing conditions of temperatures and frequencies used in this study, the analysis of the aT values show that all the |E*| master curves (at the reference temperature of 20ºC) are located in a range of the reduced frequency fR between 0.0002 and 400 Hz. The glassy modulus physically represents the limiting |E*| value attained at very high frequency (fR→∞) and low temperature. On the other hand, the equilibrium modulus physically represents the limiting |E*| value attained at very low frequency (fR→0) and high temperature. In consequence, it must be pointed out that the extrapolation of |E*| values beyond the range of temperatures and frequencies covered in the experimental determinations could give unrealistic estimations.

For the six mixtures and the nine possibilities of models and equations obtained in logarithmic and arithmetic space, the shift factors at 10, 30 and 40ºC give coefficients of variations (COV) lower than 17% showing that the three equations for the calculating of the aT values are almost equivalent for the range of testing temperatures and frequencies used in this study. However, a more detailed analysis shows that in general, the shift factors at 10 and 30ºC calculated with the Arrhenius equation are smaller than those calculated with the WLF and the polynomial equations and hence, the resulting dynamic modulus master curves are shifted towards the low frequencies compared with the other two procedures.

Finally, it was also observed that in some cases, the obtained parameters defining the |E*| master curves are strongly dependent of the seed values used in the adjustment procedures. This condition is most critical when the adjustment procedure is developed in the arithmetic space.

7 SUMMARY OF FINDINGS

In this paper, various methods and mathematical functions that can be satisfactorily used for modelling the dynamic modulus E* master curves as a function of frequency at a reference temperature for six different asphalt mixtures used in Argentina, has been evaluated. The nine considered possibilities of models and equations adjusted in both logarithmic and arithmetic spaces provide excellent fitting between measured and modelled values with very high R^2, greater than 0.98 in all cases. The SLSM and the Power models using the Arrhenius equation are the most convenient ones because they have only five parameters to be adjusted during the optimizing process. In some cases, the obtained parameters defining the |E*| master curves with the different models and equations are strongly dependent of the seed values used in the adjustment procedures and then, it could result in very different values for the glassy and equilibrium modulus. So, the extrapolation of |E*| values beyond the range of temperatures and frequencies covered in the experimental determinations could give unrealistic estimations. Finally, the master curves modelled with any of the procedures investigated in this paper could be used as input in pavement design softwares for practical purposes.

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