

# EFFECT OF NONLINEAR TEMPERATURE GRADIENT ON RESPONSES OF JOINTED CONCRETE PAVEMENTS

M. A. Zokaei Ashtiani, C. Carrasco & S. Nazarian

*Department of Civil Engineering, University of Texas at El Paso, El Paso, Texas, USA*

**ABSTRACT:** Since the initial development of pavement analysis software in the early 1970's, significant improvements have been conducted to enhance the capabilities of the finite element based jointed pavement analysis tools. A series of software development efforts have culminated in the production of NYSLAB, a jointed pavement analysis tool that has the capability to predict the complete thermo-mechanical responses. This paper presents a series of studies developed in NYSLAB to model nonlinear thermal gradients in concrete pavements. Nonlinear temperature gradients can produce slab expansion and contraction that lead to the generation of frictional tractions between slabs and foundation. The results presented here highlight the importance of considering nonlinear thermal gradient and the effect of frictional tractions in the analysis of jointed concrete pavements since they have a significant impact on PCC slabs bending stresses.

**KEY WORDS:** Rigid pavement, temperature gradient, traffic load, friction

## 1 INTRODUCTION

Seasonal and daily temperature variations in concrete pavements and resulting slab shapes play a crucial role in the magnitude of stresses and deflections imposed by the traffic loads. The accurate prediction of thermo-mechanical responses of jointed plain concrete pavements (JPCP) due to the combined effects of environmental and traffic loads are of primary importance in a mechanistic-empirical pavement design procedure.

It is well-established that the actual temperature gradient through the depth of pavement slabs is nonlinear. This nonlinear thermal gradient can produce stresses in slabs due to their external and internal restraints (Ioannides *et al.*, 1998). Subgrade reaction, edge contact between adjacent slabs and slab-foundation friction are the external restraints that can produce stresses in slabs due to thermal curling and thermal expansion or contraction. Also, the restraining interaction of surrounding layers across the depth of slabs, which resists against the distortion of the slab as a consequence of nonlinear thermal gradient, can produce additional internal stresses in slabs. Since the initial work by Westergaard (1927) several closed form solutions have been proposed to calculate strains and stresses in concrete slabs due to nonlinear thermal effects (Choubane *et al.*, 1992, Ashraf *et al.*, 1996). However, temperature induced curling and expansion or contraction in JPCP significantly influenced by the contact conditions along the slab-foundation interface. These contact conditions (subgrade reaction and interface friction) significantly impact the

mechanical behavior of the pavement and complicate JPCP analysis because it introduces some nonlinearity to the problem. A sophisticated modeling such as finite element (FE) modeling is required to more accurately consider all the possible loads and environmental conditions.

In 2D FE-based analysis tools such as ILLI-SLAB and JSLAB2004, determining thermal loads effects was limited to linear thermal gradient. Also, the effect of contact condition was limited to the modeling of separation between slabs and foundation as a consequence of positive and negative temperature gradient and modeling the horizontal interaction of slab-foundation was not possible. Harik *et al.* (1996) proposed a method to superimpose the effect of the nonlinear temperature gradient to stresses caused by linear temperature gradient obtained from the finite element solution of the 2D analysis tools. A 3D FE rigid pavement analysis tool, such as EverFE can consider linear or nonlinear temperature gradient.

NYSLAB, a jointed pavement analysis tool has been developed recently at the University of Texas at El Paso to enhance the capabilities of the JPCP analysis tools. Nonlinear temperature gradient can be considered in that program. The slab and the top foundation layer are connected through interface elements to better model slab curling as well as the interactions between slabs and foundation (Carrasco *et al.*, 2010 and 2011).

This paper presents the procedure of modeling nonlinear temperature gradient in the finite element formulation of NYSLAB. Also results of a series of studies that highlight NYSLAB's capabilities to predict pavement response under nonlinear temperature gradient are presented.

## 2 MODELING OF NONLINEAR TEMPERATURE GRADIENT

The “first order shear deformation laminated plate theory” or “Mindlin laminated plate theory” are employed in NYSLAB to model multiple bonded slab layers with different material properties and thicknesses. Mindlin theory allows for accounting shear deformations that become significant for relatively thick plates. It is assumed that the in-plane strains are linear through the laminate thickness in that theory,

$$\varepsilon = \varepsilon_0 + z \varepsilon_1 \quad (1)$$

where  $\varepsilon_0$  is related to in-plane stretching of mid-plane and  $\varepsilon_1$  is the curvature of deformed plate.  $z$  is the distance from the mid-plane of laminate.

Nine-node isoparametric elements are used to discretize the pavement slabs based on the formulation proposed by Reddy (2004). Slab elements have five degrees of freedom per node; two horizontal or in-plane displacements in longitudinal and transverse directions, vertical deflection and two rotations about longitudinal and transverse axes. Equation 2 describes the finite element formulation for each slab element,

$$[K]^e \{U\}^e = [F]^e \quad (2)$$

where  $[K]^e$  and  $[F]^e$  are the stiffness matrix and force vector for each element (Zokaei *et al.*, 2011).  $\{U\}^e$  is the displacement vector in each element according to the five mentioned degrees of freedom. The elements of equilibrium equation 2 are expanded in equation 3 as,

$$\begin{bmatrix} \begin{bmatrix} K^{11} \\ K^{12} \\ K^{13} \\ K^{14} \\ K^{15} \end{bmatrix}^T & \begin{bmatrix} K^{12} \\ K^{22} \\ K^{23} \\ K^{24} \\ K^{25} \end{bmatrix}^T & \begin{bmatrix} K^{13} \\ K^{23} \\ K^{33} \\ K^{34} \\ K^{35} \end{bmatrix}^T & \begin{bmatrix} K^{14} \\ K^{24} \\ K^{34} \\ K^{44} \\ K^{45} \end{bmatrix}^T & \begin{bmatrix} K^{15} \\ K^{25} \\ K^{35} \\ K^{45} \\ K^{55} \end{bmatrix}^T \end{bmatrix} \begin{Bmatrix} u^e \\ v^e \\ w^e \\ \phi_x^e \\ \phi_y^e \end{Bmatrix} = \begin{Bmatrix} F^1 - F_T^1 \\ F^2 - F_T^2 \\ F^3 \\ F^4 - F_T^4 \\ F^5 - F_T^5 \end{Bmatrix} \quad (3)$$

$F_T$  is the contribution of thermal loads added to the force vector,

$$F_T^1 = \int_s \frac{\partial N_i}{\partial x} N_T dx dy \quad F_T^2 = \int_s \frac{\partial N_i}{\partial y} N_T dx dy \quad F_T^4 = \int_s \frac{\partial N_i}{\partial x} M_T dx dy \quad F_T^5 = \int_s \frac{\partial N_i}{\partial y} M_T dx dy \quad (4)$$

where,  $N_i$  is the interpolation function used to interpolate the responses inside each element in terms of their values at nodal points.  $N_T$  and  $M_T$  are the resultant thermal force and thermal moments which can be calculated for each laminate,

$$N_T = \sum_{k=1}^N \int_{Z_k}^{Z_{k+1}} Q^{(k)} \alpha^{(k)} \Delta T dz \quad M_T = \sum_{k=1}^N \int_{Z_k}^{Z_{k+1}} Q^{(k)} \alpha^{(k)} \Delta T z dz \quad (5)$$

where  $Q$  and  $\alpha$  are the stiffness components and the coefficient of thermal expansion for each layer in the laminate.  $\Delta T$  is the temperature gradient (TG) through the thickness of the laminated slabs,

$$Q_{ij}^k = \frac{E^{(k)}}{1-\nu^{(k)2}} \begin{bmatrix} 1 & \nu^{(k)} & 0 \\ \nu^{(k)} & 1 & 0 \\ 0 & 0 & \frac{1-\nu^{(k)}}{2} \end{bmatrix} \quad \alpha^{(k)} = \begin{bmatrix} \alpha \\ \alpha \\ 0 \end{bmatrix} \quad (6)$$

where,  $E^{(k)}$ ,  $\nu^{(k)}$  are the modulus of elasticity and Poisson's ratio for  $k^{\text{th}}$  layer of the laminate. In Equations 4 and 5, temperature variation with any polynomial equation can be integrated within each layer and its resultant force vector can be added to the element force vector.

The bottom slab layer and the top foundation layer in NYSLAB are connected to one another through interface elements. Interface elements have the characteristic of being active in each node when in compression and inactive when in tension to model the interface separation produced during curling. Also, each node of the interface elements has the ability to define the state of contact during sliding as a consequence of thermal expansion and contraction; whether in slip mode or stick mode (Urzua *et al.*, 1977). The active/inactive and slip/stick states are determined through an iterative process. The use of interface elements facilitates the modeling of the loss of contact between layers when thermal curling occurs. Also, calculating the frictional or shear stress at the slab-foundation interface is possible by applying an appropriate constitutive friction law (Barbero *et al.*, 1995). For each node in contact, normal and shear stresses are related through the isotropic Mohr-Coulomb friction law,  $F_t = \mu F_n$ , where,  $F_t$  is the tangential or shear stress,  $F_n$  is the normal stress, and  $\mu$  is the coefficient of friction. Shear stress at each node of

interface element is calculated based on the normal stress at that point, which can be affected by temperature curling and traffic loads.

A thorough explanation of the finite element formulation and underlying theory can be found in Zokaei *et al.* (2011) and is not repeated here for the sake of brevity. The mathematical model of the entire pavement section in the formulation of NYSLAB is shown in Figure 1.

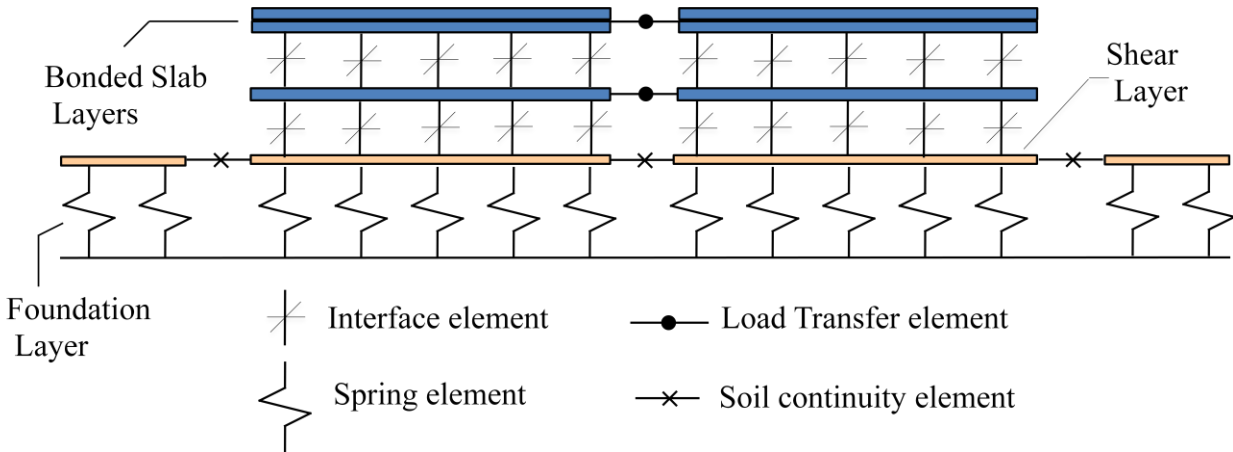


Figure1: Jointed pavement section as modeled in NYSLAB (Zokaei *et al.*, 2011).

### 3 PARAMETRIC STUDIES

A series of parametric studies was carried out to evaluate the performance of NYSLAB in calculating stresses and deflections in Portland cement concrete (PCC) slabs due to nonlinear temperature gradient and to better understand the impact of thermal loads together with superimposed traffic loads in JPCP responses. For this purpose, a three by two jointed PCC slab in longitudinal and transverse direction, respectively, resting on a Winkler foundation with modulus of subgrade reaction of 200 psi/in was modeled (see Figure 2). Each slab was 15 ft long, 14 ft wide and 10 in. thick. The modulus of elasticity of the PCC was set to 4,000 ksi with a Poisson's ratio of 0.15, coefficient of thermal expansion of  $6 \times 10^{-6}/^{\circ}\text{F}$  and a unit weight of 150 pcf. The space between the adjacent slabs was set to 0.25 in. in both directions. The slabs were connected by dowels (1.25 in. diameter and uniformly spaced at 1 ft intervals) and tie bars (0.75 in. diameter and spaced at 1 ft intervals) in the transverse and longitudinal joints, respectively. The coefficient of friction between PCC slabs and foundation was set to 0.3.

When applicable, the pavement was loaded with a certain truck configuration. Each tire had dimensions of 8 in. by 6 in., with a contact pressure of 90 psi. For the other parametric studies presented below, only thermal loads are considered together with the PCC slab self weight.

#### 3.1 Effect of Non-linear Temperature Gradient

Simulating thermal effects with a nonlinear temperature profile allows for a more realistic modeling of temperature variation through the thickness of the PCC slabs. In the previous section the process of including thermal loads to the finite element formulation of the pavement was described. The temperature profile through the thickness of the slab is expressed as a cubic function (Eq. 7) that can be fitted by considering the temperature at four different points through

the depth. This order of polynomial was selected because it is common for temperature to be measured at four points across the thickness of the slabs at pavement test sites (Yu *et al.*, 1998). Assuming the origin at the mid-plane of the slab, the temperature gradient is defined as:

$$\Delta T = a_0 + a_1z + a_2z^2 + a_3z^3 \quad (7)$$

The constant term  $a_0$  produces expansion and contraction in the mid-plane of the PCC slabs. The linear term  $a_1$  produces pure bending in the PCC slab due to the temperature difference between its top and bottom. The higher-order terms of temperature profile in Eq. 7 produce internal stresses in the PCC slab regardless of its external constraints (Ioannides *et al.*, 1998).

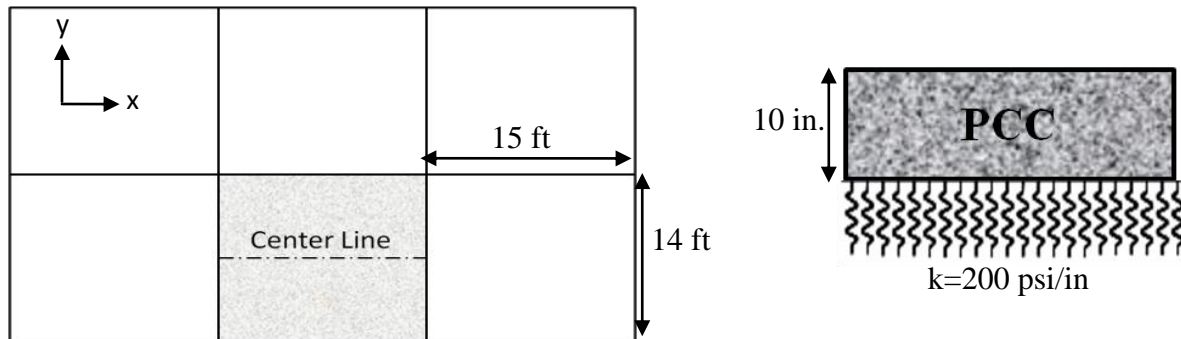


Figure2: Pavement structure and selected slab in the cases studied.

The effect of a non-linear temperature profile was studied by evaluating the stresses and displacements along a longitudinal section that passes through the center of the slabs as shown in Figure 2 (*center line*). Two sets of TG correspond to night-time (or negative TG) and day-time (or positive TG) were selected (Figure 3-1 and 3-2).  $N_0$  and  $P_0$  in those sets represent a linear thermal gradient that the mid-plane of the slabs has zero change in temperature and the difference between temperature at the top and bottom of the slab is  $-15^\circ\text{F}$  and  $15^\circ\text{F}$ , respectively. In the both night-time and day-time state, changing the temperature gradient from  $N_1$  to  $N_4$  or from  $P_1$  to  $P_4$  demonstrates the level of deviation from the linear temperature profile. In the night-time state from  $N_1$  to  $N_4$ , the temperatures at the top and bottom of slabs were maintained at  $45^\circ\text{F}$  and  $60^\circ\text{F}$ , respectively, while the set temperature was  $70^\circ\text{F}$ . This means that thermal contraction occurred throughout the depth of the PCC slab while the decrease in temperature at the top was greater than that at the bottom of the slab. Also, in the day-time state from  $P_1$  to  $P_4$ , the temperatures at the top and bottom of slabs were maintained at  $95^\circ\text{F}$  and  $80^\circ\text{F}$ , respectively, while the set temperature was  $70^\circ\text{F}$ . This is the indication of thermal expansion throughout the depth of the PCC slab while the increase in temperature at the top was greater than that at the bottom of the slab. The coefficients  $a_i$  (Eq. 7) for all the case studies are also shown in Figure 3-1 and 3-2.

Figure 4 shows the bending stresses in the longitudinal (x) direction at the top and bottom of the PCC slab along the center-line described above due to the negative TG. With a linear temperature profile ( $N_1$ ), the stresses at the top and bottom of the slab are almost equal in magnitude (top in tension and bottom in compression). The small difference between stresses at the top and bottom of PCC slab in this case is due to the effect of friction between the PCC slab and the foundation. However, as the temperature profile becomes more nonlinear, the stresses both at the top and bottom shift in the positive (tensile) value about 100 psi and 60 psi,

respectively. These results indicate that even though in all four cases the temperature-change at the top and bottom of the slabs are the same, the nonlinear thermal terms produce significant additional stresses. This translates into internal stresses in the PCC slabs due to the nonlinear temperature gradient.

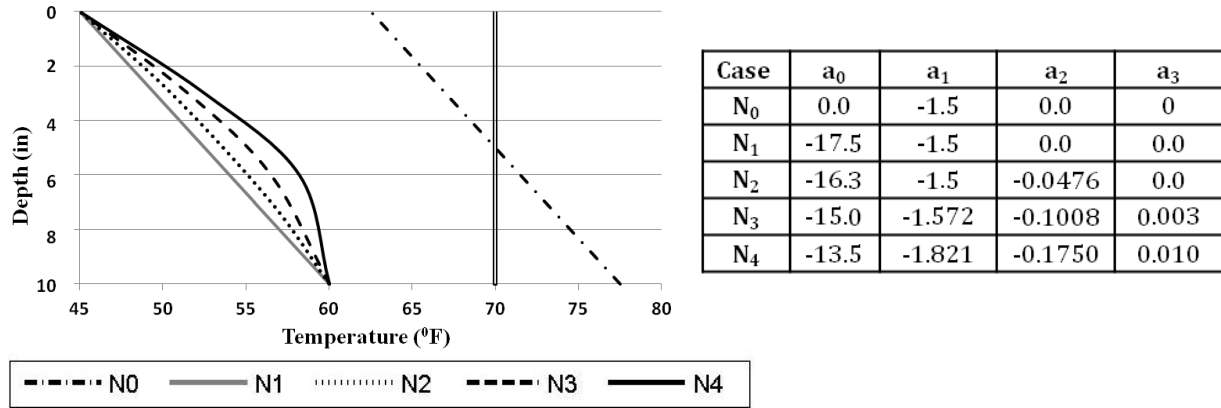


Figure3-1: Temperature-change profile and coefficients for temperature-variation for the five Negative Temperature gradient cases studied.

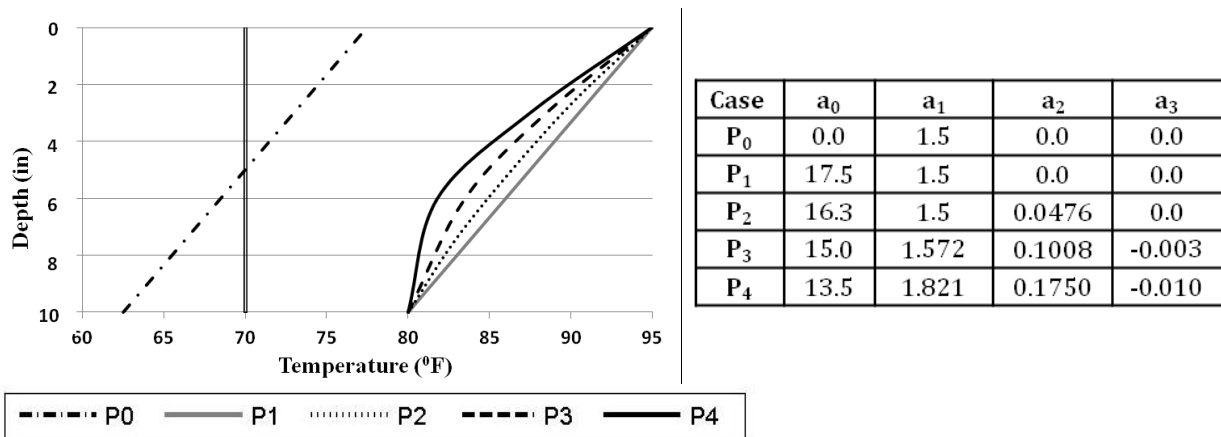


Figure3-2: Temperature-change profile and coefficients for temperature-variation for the five Positive Temperature gradient cases studied.

In the cases N<sub>1</sub> to N<sub>4</sub>, the uniform negative temperature change ( $a_0$ ) produces additional stresses in the PCC slabs because of the presence of friction. When the PCC slabs contract, the frictional resistance of the foundation layer produces a uniform tensile traction on the PCC slabs. In this case, the moments induced by the frictional resistance reduces the tensile stresses at the top and compressive stresses at the bottom of the PCC slabs. In case N<sub>0</sub>, the horizontal displacement at the mid-depth of slabs is zero; however, the horizontal displacements at the bottom surface of PCC slabs as a consequence of curling can produce frictional tractions. As the bottom surface of slabs expand, the compressive frictional tractions produce an additional positive moment in the PCC slabs, which can increase the compressive stress at the bottom and tensile stress at the top of the PCC slabs (Figure 4). It is worth to mention that if there was no

friction assumed between the PCC slabs and foundation, the stresses in both case  $N_0$  and case  $N_1$  would be identical.

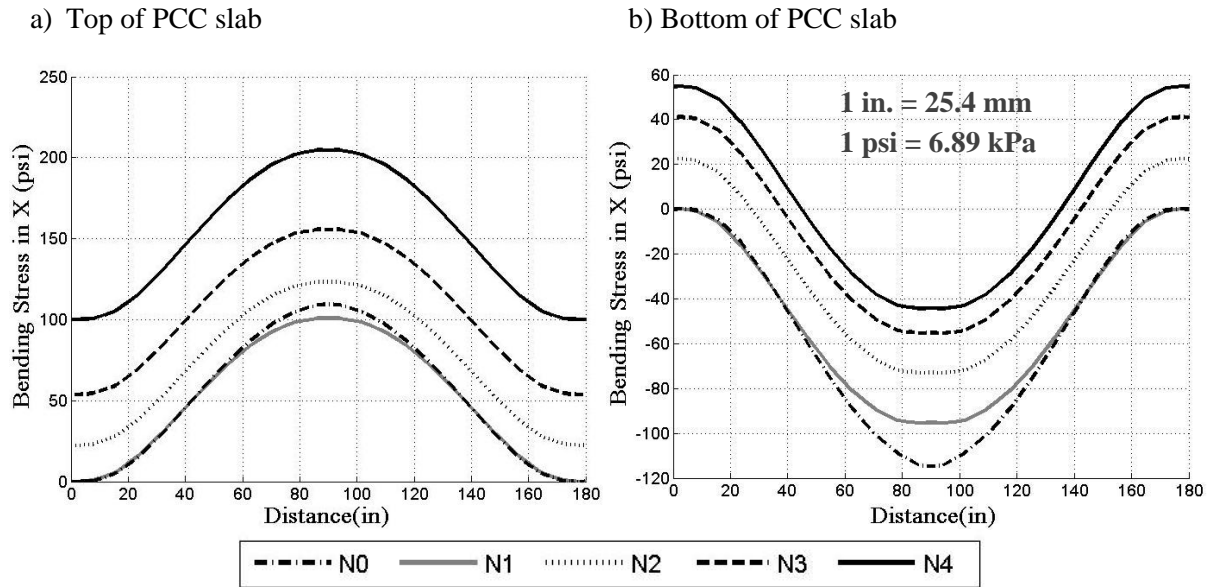


Figure4: longitudinal bending stress ( $\sigma_{xx}$ ) at the top and bottom of the selected PCC slab through the Center line for various negative temperature gradients.

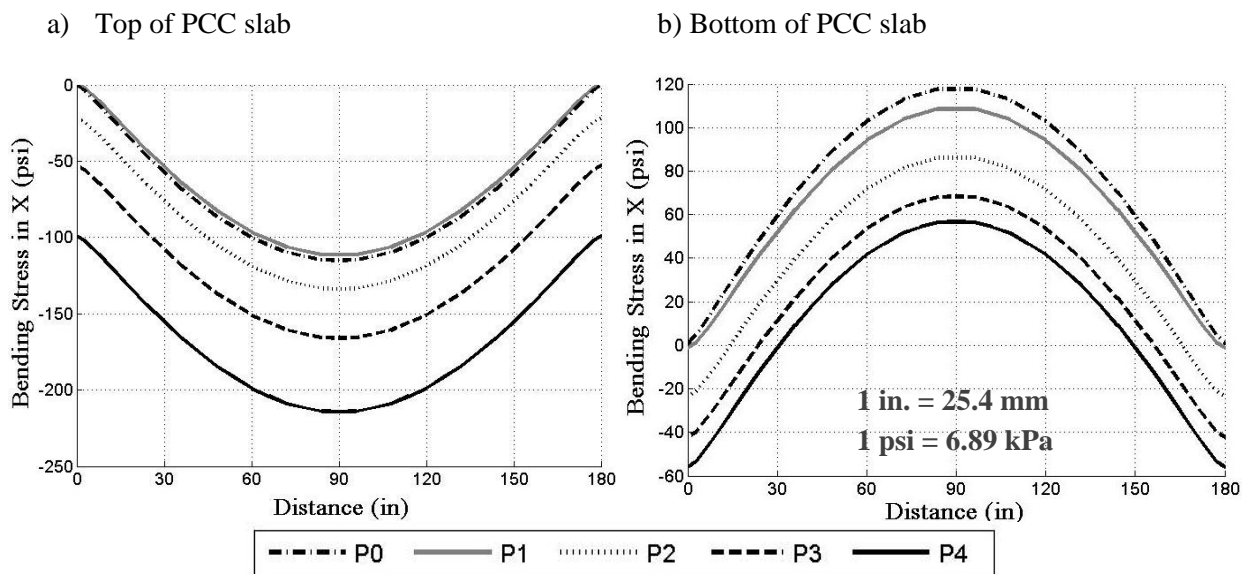


Figure5: longitudinal bending stress ( $\sigma_{xx}$ ) at the top and bottom of the selected PCC slab through the Center line for various positive temperature gradients. (**Note:** Tension is positive and Compression is negative)

The bending stresses for the positive TG are shown in Figure 5. In this state, as the temperature profile becomes more nonlinear (from  $P_1$  to  $P_4$ ), the stresses both at the top and bottom shift in the negative (compressive) value about 100 psi and 50 psi, respectively. In those

cases, the frictional resistance of the foundation layer produces a uniform compressive traction on the PCC slabs and the moments induced by that frictional resistance reduces the tensile stresses at the bottom and compressive stresses at the top of the PCC slabs. In the case  $P_0$ , the bottom surface of the slabs contract, thus the tensile frictional tractions produce an additional negative moment in the PCC slabs, which can increase the compressive stress at the top and tensile stress at the bottom of the PCC slabs (Figure 5).

Figure 6 shows the longitudinal displacement at the mid-plane of the PCC slab. As expected, longitudinal contraction occurs in the negative TG cases ( $N_1$  to  $N_4$ ) and longitudinal expansion occurs in the positive TG cases ( $P_1$  to  $P_4$ ) as a consequence of changes in temperature in each case from the set temperature. However, for all the temperature profiles, the increasing of non-linearity decreases the average temperature change leading to a decrease in the thermal contraction or expansion of the slab mid-plane. It is important to note that general conclusion cannot be drawn about the thermal contraction or expansion pattern since it greatly depends on the shape of the temperature profile. In the cases  $N_0$  and  $P_0$ , the longitudinal displacements at the mid-depth of PCC slab are zero since there is no temperature change at that point.

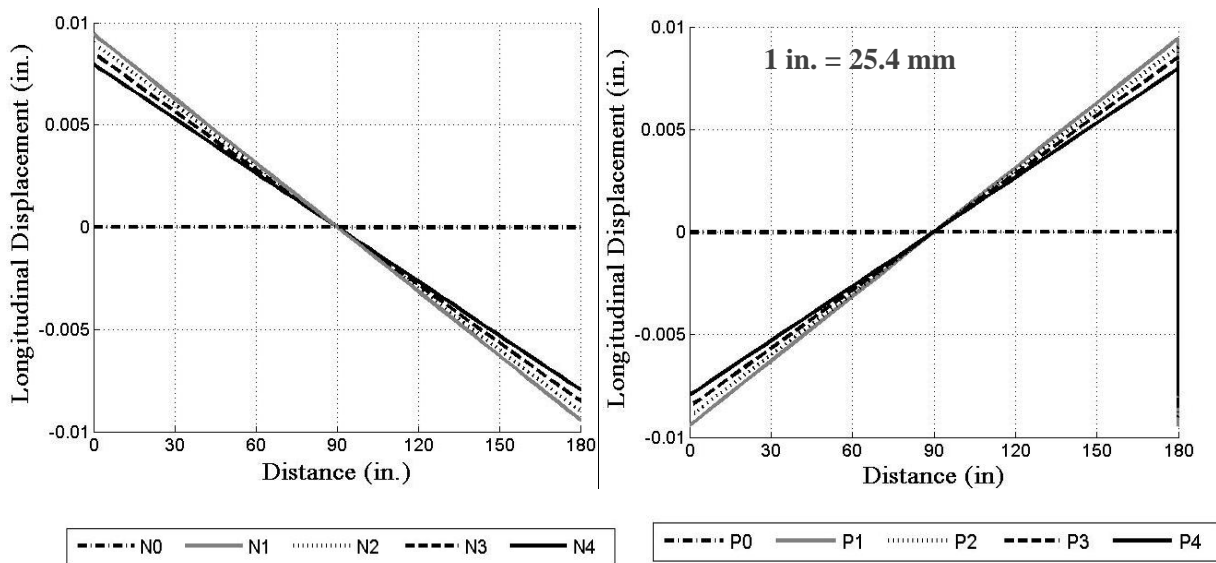


Figure6: Longitudinal displacement of mid-depth of the PCC slab for negative and positive temperature gradients.

### 3.2 Effect of Combination of Thermal and Traffic Loads

The behavior of concrete pavements due to simultaneous action of traffic and thermal loads was examined by performing a series of simulations. For this purpose the critical loading scenario that produce top-down or bottom-up cracking which results in fatigue damage in the PCC slab were analyzed. The critical loading case for top-down cracking scenario according to the Mechanistic-Empirical Pavement Design Guide (NCHRP 1-37A, 2004) involves a pavement section exposed to a negative temperature gradient and loaded with axle configuration shown in Figure 7 (Truck 1). In the bottom-up cracking scenario the pavement section is subjected to the simulations action of positive temperature gradient and the second truck case (Truck 2) shown in Figure 7.



Figure 8 shows the longitudinal stresses at the top of the PCC slab through Stress-line described in Figure 7 for the first scenario. The maximum tensile stress at the top of PCC slab due to truck load (Truck 1) in this case is 100 psi. The simultaneous action of truck and linear negative TG (Truck1+N<sub>1</sub>) and truck and nonlinear negative TG (Truck1+N<sub>4</sub>) will increase the maximum tensile stress to 240 psi and 350 psi, respectively. Figure 9 shows the longitudinal stresses at the bottom of the PCC slab through Stress-line described in Figure 7 for the second scenario. In this case the truck load produces tensile stress at the bottom of PCC slab with the maximum value about 150 psi. The maximum tensile stress due to the simultaneous action of Truck2 and linear positive TG (Truck2+P<sub>1</sub>) in this case is greater than that produced by the simultaneous action of Truck2 and nonlinear positive TG (Truck2+P<sub>4</sub>).

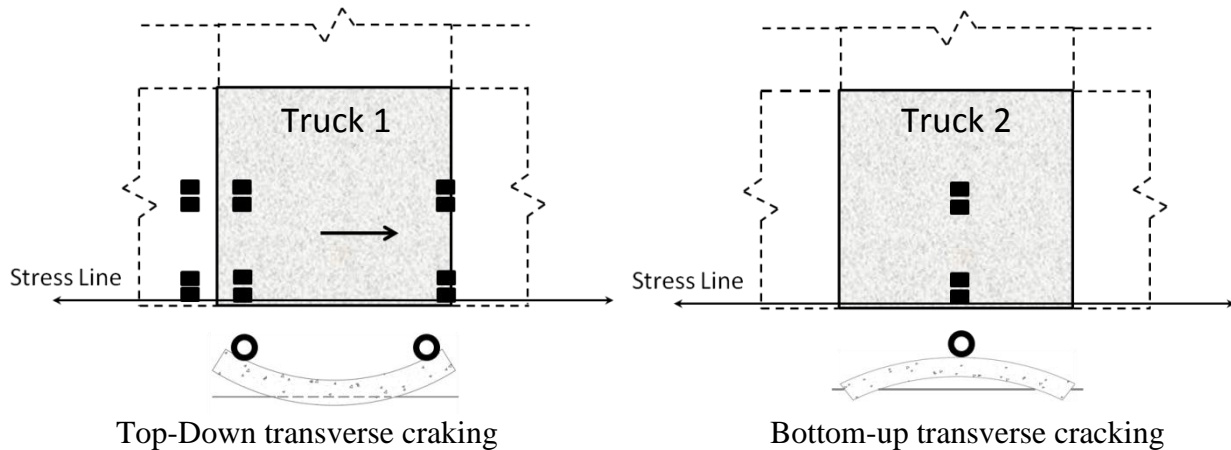


Figure7: Placement of truck for two scenarios to evaluate the simulation effect of thermal and traffic load.

#### 4 SUMMARY AND CONCLUSION

Pavement responses due to nonlinear temperature gradient within a PCC slab and the combined traffic and thermal loads were examined in this paper. The study employed improved finite element models that were incorporated in NYSLAB to calculate pavement stresses and deflections. Development to NYSLAB's interface elements and the procedure of applying nonlinear thermal gradient were summarized highlighting their capabilities in capturing the shear interaction in the slab-foundation interface and calculating the internal stresses.

A series of parametric studies were conducted that considered various temperature gradients and the superimposed truck load. From the results presented here, the following conclusions can be drawn:

- a) The nonlinear terms in the temperature gradient can produce additional internal stresses at the top and bottom of PCC slabs. This demonstrates that assuming a linear temperature gradient tends to under-estimate the stresses within the pavement slabs and may lead to their significant under-design.
- b) Increasing the level of deviation from linear temperature gradient lead to a significant increase in tensile stress in the night-time (negative) temperature-change state and decrease in tensile stress in the day-time (positive) temperature-change state.

c) The temperature variation through the slab depth and resulting slab shape can significantly increase the stresses in PCC slabs caused by traffic loads. The results from simultaneous truck and thermal loads demonstrate that eliminating this combined effect in the thermo-mechanical analysis of concrete pavements may lead to their inaccurate prediction of stresses in PCC slabs.

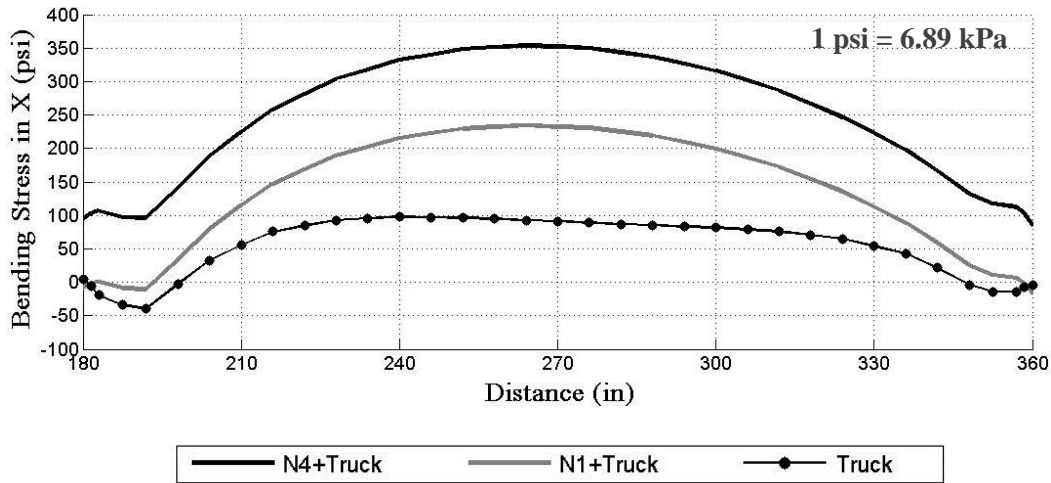


Figure8: Tensile stress at the top of PCC slabs in longitudinal direction ( $\sigma_{xx}$ ) through Stress-line for the first loading scenario (top-down cracking).

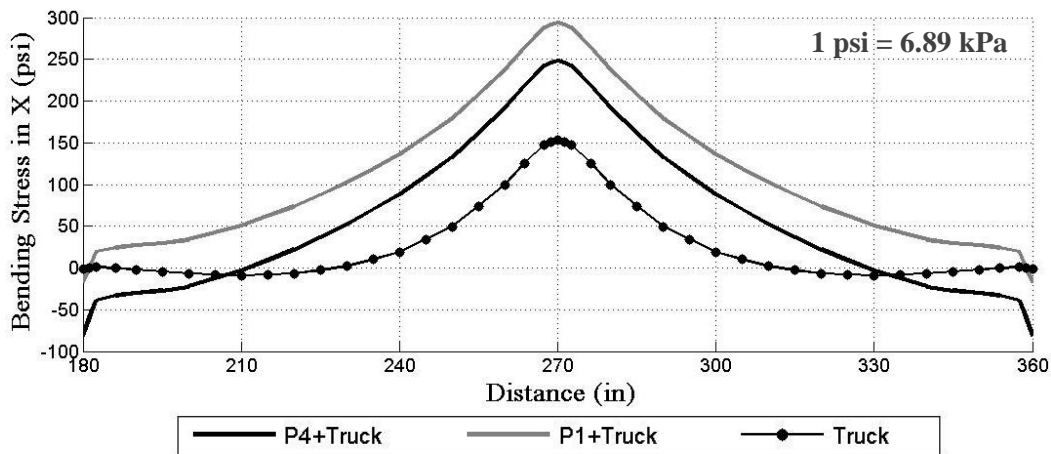


Figure9: Tensile stress at the bottom of PCC slabs in longitudinal direction ( $\sigma_{xx}$ ) through Stress-line for the second loading scenario (bottom-up cracking). (Note: Tension is positive and Compression is negative)

## REFERENCES

- Ashraf, M. and Hansen, W., 1996. *Prediction of Stresses in Concrete Pavements Subjected to Non-linear Gradients*. Cement and Concrete Composites, No. 18, 381-387.
- Barbero, E.J., Lucianof, R., and Sacco, E., 1995. *Three-Dimensional Plate and Contact/Friction Elements for Laminated Composite joints*. Computers & Structures, Vol. 54, No. 4.
- Carrasco, C., Limouee, M., Celaya, M., Abdallah, I., and Nazarian, S., 2010. *NYSLAB: A Software for Analysis of Jointed Pavements*. Publication FHWA-RD-07-1008-01. FHWA, U.S. Department of Transportation.
- Carrasco, C., Limouee, M., Nazarian, S., and Bendana, J., 2011. *Development of NYSLAB Improved Analysis Tool for Jointed Pavement*. Transportation Research Record, No. 2227.
- Choubane, B. and Tia, M., 1992. *Nonlinear Temperature Gradient Effect on Maximum Warping Stresses in Rigid Pavements*. Transportation Research Record, No. 1370, pp. 11–19.
- Harik, I. E., Jianping, P. , Southgate, H. and Allen, D., 1996. *Temperature Effects on Rigid Pavements*. Journal of Transportation Engineering, Vol. 120, No. 1.
- Ioannides, M. and Khazanovich, L., 1998. *Nonlinear Temperature Effects on Multilayered Concrete Pavements*. Journal of Transportation Engineering, Vol. 124, No. 2.
- NCHRP. *Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures*. Final report for Project 1-37A. Part 3, Chapter 4. National Cooperative Highway Research Program, Transportation Research Board, National Research Council, Washington, D.C., March 2004.
- Reddy, J. N., 2004. *Mechanics of Laminated Composite Plates and Shells Theory and Analysis*. CRC Press LLC, Second edition.
- Shoukry, S. N, William G. W., and Riad, M., 2003. *Nonlinear Temperature Gradient Effects in Dowel Jointed Concrete Slabs*. The International Journal of Pavement Engineering, Vol. 4.
- Urzua, J.L., Pecknold, D.A., Lopez, L.A., and Munse, W.H., 1977. *Analysis Procedure for Frictional Contact Problem Using Interface Finite Elements*. Structural research series NO. 438. Department of Civil Engineering University of Illinois at Urbana-Champaign.
- Westergaard, H.M., 1927. *Analysis of Stresses in Concrete Pavements Due to Variations of Temperature*. Proceedings of the Highway Research Board 6, National Research Council, Washington, D.C., pp. 201–215.
- Yu, H.T., Khazanovich, L., Darter, M.I., and Ardani, A., 1998. *Analysis of Concrete Pavement Responses to Temperature and Wheel Loads Measured from Instrumented Slabs*. Transportation Research Record, No. 1639, pp. 94–101.
- Zokaei, M.A., Tirado, C., Carrasco, C., and Nazarian, S., 2011. *Development and Improvement of Computer Software NYPAS*. Publication FHWA-RD-07-1008-03. FHWA, U.S.