

A Theoretical Approach to the Measured Deflections in Railway Slab Track

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ABSTRACT: The results of a theoretical investigation, performed during the design of the section of the High-Speed line ($V > 200$ km/h) of the Greek Railways (OSE) between Korinthos – Patras, for the use of the German type Rheda 2000 type Slab Track are presented in this paper. The behavior of the Rheda 2000 Slab Track is compared to the behavior of the Rheda Sengeberg type that is in operation in the Athens – Thessaloniki line, with published measurements for the German railway network. A parametric analysis of the stiffness variation is performed that is dependent on the subgrade quality while at the same time its participation in the configuration of the total coefficient of the track stiffness static and dynamic is also examined. The measured deflections on slab track sections with Rheda Sengeberg laid in the German railway network are simulated with the proposed theoretical method. An estimate of the probability of occurrence used as input in the theoretical calculations is presented for the evaluation of the slab track behavior in Greece. Conclusions are drawn in relation to the stiffness variation of the subgrade, and its influence on deflection.

KEY WORDS: Railway track, superstructure, resilient fastening, actions, stiffness, deflection, theoretical approach, dimensioning, probability of occurrence.

1 INTRODUCTION

The railway track superstructure is the equivalent of the road pavement and as in the case of pavements there are flexible and rigid track superstructures. The main difference between the two types of infrastructure is that the loads in railways are applied in two discrete locations along the rails, whereas in road pavements the location of the load application is random. The track superstructure is a multilayered system: (a) in the case of the classic ballasted track the superstructure also includes the ballast, the equivalent of a flexible pavement, and the blanket layer (sub-ballast) consisting of compacted sand and gravel, which further distributes the loads and protects the substructure from the penetration of crashed ballast particles, mud ascent and pumping (where substructure is the term for the layers under the ballast) and (b) in the case of the Ballastless or Slab Track (Figure 1) the superstructure also includes the concrete slab or Continuously Reinforced Concrete Pavement (CRCP) which is the equivalent of a rigid pavement. The concrete slab seats on a series of successive bearing layers with a gradually decreasing modulus of elasticity: the Cement Treated Base (CTB), underlain by the Frost Protection Layer (FPL) and the foundation or prepared subgrade. The three layers under the concrete slab (i.e. CTB, FPL and foundation) constitute

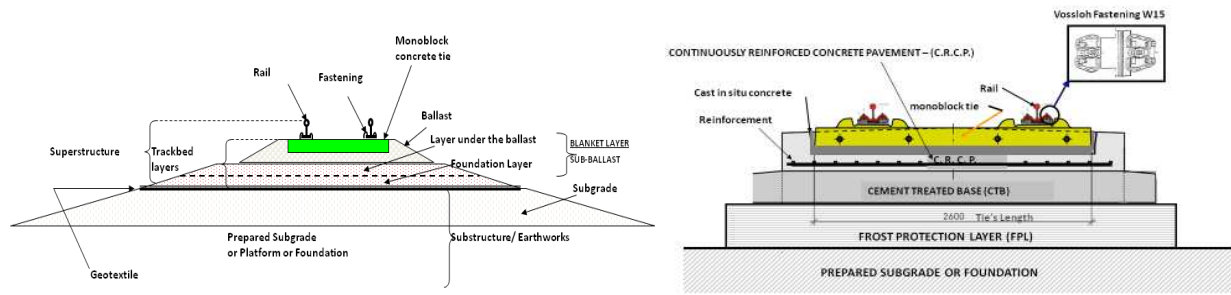


Figure1: Greek network: (left) cross section of classic Ballasted Track with monoblock sleepers (terminology UIC, code 719R, 1994), and (right) cross section of Rheda Sengeberg type Slab Track system as in Tempì tunnel (Tsoukantas et al., 2006).

the substructure of the Slab Track. The Slab Track is typically used in High Speed lines ($V > 200$ km/h) of mixed passenger and freight traffic with maximum axle load of 22.5 t.

In order to adopt the Slab Track technology, the Greek Railways performed an extensive investigation program that studied Slab Track systems laid in High-Speed tracks ($V_{\max} > 200$ km/h) under operation, mainly in Germany. Two types of Slab Track systems were considered: Rheda Classic, which was the first application of Slab Track that took place in 1972, and Rheda2000, which is the most up-to-date, modern and most technologically advanced type of the Rheda Slab Track "family". Based on the findings of the research program these two types of Slab Track were selected for the Greek railway network and applied in the High-Speed Line between Patras - Athens/Piraeus – Thessaloniki.

The deflection y and the mean stress \bar{p} on the ballast-bed play key roles in the design and maintenance of High Speed railway tracks and their values mainly depend on the track stiffness coefficient (Giannakos, 2011a). However, there is a lack of data in the international literature correlating the magnitude of the deflection and stress on the ballast to the track stiffness coefficient. The research performed for the Greek Railway network (Giannakos, 2008, 2009, Tsoukantas, 1999) addressed this issue.

Due to older problems of cracking in concrete sleepers a new method for the calculation of loads and stresses on a railway track had been developed as a result of extensive research (Giannakos, 2004, Giannakos&Loizos, 2010a). During the recent research (Giannakos, 2009, 2008) this method was used to calculate the theoretically the deflection of the slab track and the results were compared to the measurements on track performed in Germany. The comparison is presented to the present paper.

2 STIFFNESS AND ITS INFLUENCE ON THE ACTIONS ON TRACK

It must be noted here that in all four calculation methods the total static stiffness coefficient of the track ρ_{total} (spring constant as it is denoted in French and Greek literature, in German literature it is denoted as c) is of decisive importance for the calculation of the action/reaction on each tie. In general:

$$\frac{1}{\rho_{\text{total}}} = \sum_{i=1}^v \frac{1}{\rho_i} \quad (1)$$

where i are the layers that constitute the multilayered structure "Track", and ρ_{total} the total static stiffness coefficient of track, which must be calculated for each case.

2.1 Method cited in the French literature

Given by (Alias, 1984, Prud'homme & Eriau, 1976, RGCF, 1973):

$$R_{total} = \left(Q_{wheel} + Q_{\alpha} + 2 \cdot \sqrt{\left[\sigma^2(\Delta Q_{NSM}) \right] + \left[\sigma^2(\Delta Q_{SM}) \right]} \right) \cdot \bar{A}_{stat} \cdot 1.35 \quad (2)$$

Where R denotes the action/ reaction on each support point of the rail on a sleeper, and covers a probability of occurrence P=95.5% (for probability of occurrence see Giannakos and Loizos, 2010a), where: Q_{wheel} = the static load of the wheel, Q_{α} = load due to cant (superelevation) deficiency, $\sigma(\Delta Q_{NSM})$ = standard deviation of the Non-Suspended Masses of vehicle, $\sigma(\Delta Q_{SM})$ = standard deviation of the Suspended Masses of vehicle, \bar{A}_{stat} = reaction coefficient of the tie which is equal to:

$$\bar{A}_{stat} = \frac{1}{2\sqrt{2}} \cdot 4 \sqrt{\frac{\rho_{total} \cdot \ell^3}{E \cdot J}} \quad (3)$$

and ρ_{total} = coefficient of total static stiffness (elasticity) of track, ℓ = distance between sleepers, E, J = Modulus of Elasticity and Moment of Inertia of the rail

2.2 Method cited in the German literature

The action on the sleeper is estimated (Eisenmann, 1981, 2004, et al., 1994):

$$R_{max} = \left(1 + 0.9 \cdot \left(1 + \frac{V_{max} - 60}{140} \right) \right) \cdot \bar{A}_{stat} \cdot Q_{wheel} \quad (4a)$$

for $V_{max} \leq 200$ km/h (124.30 mi/h), with probability of occurrence P=99.7%, where \bar{A}_{stat} is calculated through equation (3).

Prof. Eisenmann in 1993 for speeds above 200 km/h proposed:

$$R_{max} = \left(1 + 0.9 \cdot \left(1 + \frac{V - 60}{380} \right) \right) \cdot \bar{A}_{stat} \cdot Q_{wheel} \quad (4b)$$

2.3 Method cited in the American literature

According to AREMA 2005 (Hay, 1982, Selig & Waters, 2000, AREMA, 2010):

$$R_{max} = \bar{A}_{stat} \cdot \left(1 + \frac{D_{33} \cdot V}{D_{wheel} \cdot 100} \right) \cdot Q_{wheel} \quad (5)$$

where $D_{33}=33$, is the diameter of a wheel of 33 inches, D_{wheel} the wheel diameter of the vehicle examined in inches, p_{max} the maximum pressure per unit length of the track under the sleeper, and \bar{A}_{stat} is the same as in equation (3).

2.4 Giannakos (2004) method

The actions on the track panel are calculated through the following equation (Giannakos, 2004, 2012a):

$$R_{max} = \frac{1}{2\sqrt[8]{2}} \cdot 16 \sqrt{\left(\frac{\rho_{total} \cdot \ell^3}{E \cdot J} \right)^3} \cdot (Q_{wheel} + Q_{\alpha}) + \sqrt[3]{ \underbrace{\left(k_{\alpha} \cdot V \cdot \sqrt[8]{2^6 \cdot (m_{NSM-vehicle} + m_{TRACK})^4 \cdot E \cdot J \cdot \left(\frac{\rho_{total}}{\ell} \right)^3} \right)^2}_{\sigma(\Delta Q_{NSM})} + \underbrace{\left(\frac{V - 40}{1000} \cdot N_L \cdot Q_{wheel} \right)^2}_{\sigma(\Delta Q_{SM})} } \quad (6)$$

The result of equation (6) in kN, for a probability of occurrence 99.7 %, where Q_{wheel} = the static wheel load in kN, Q_{α} = the load due to superelevation deficiency in kN, $m_{\text{NSM-vehicle}}$, the mass in tonnes (1t = 2204.62 pounds) of the Non Suspended Masses of the vehicle, m_{TRACK} , the track mass participating in their motion in tonnes also (Giannakos, 2010b), ρ_{total} in kN/mm, ℓ the distance between the sleepers in mm, V in km/h, NL fluctuating between 0.7 and 1.5 dependent on the track leveling defaults and k_{α} coefficient of the condition of the rail running table (it is the trace of the running wheel of a “sinusoidal form” on the railhead’s surface), fluctuating from $38942.43 \cdot 10^{-7}$ for ground rail running table to $155769.73 \cdot 10^{-7}$ for non-ground rail running table, for tracks of good condition and maybe till $324520.28 \cdot 10^{-7}$ for secondary lines with rail running table in a very bad condition (as it is derived from measurements in railway networks see Giannakos and Loizos, 2010a), E the modulus of elasticity [kN/mm²], J the moment of inertia of the rail [mm⁴], $\sigma(\Delta Q_{\text{NSM}})$ = the standard deviation of the dynamic load due to Non Suspended Masses and $\sigma(\Delta Q_{\text{SM}})$ = the standard deviation of the dynamic load due to Suspended Masses (Giannakos, 2012b).

According to this method the dynamic stiffness coefficient of the track, as a function of the static stiffness coefficient of the track ρ_{total} is given by the equation (Giannakos, 2004):

$$\rho_{\text{dynam}} = h_{\text{TR}} = \frac{1}{2 \cdot \sqrt{2}} \cdot \sqrt[4]{E \cdot J \cdot \frac{\rho_{\text{total}}}{\ell}} = \frac{2 \cdot L}{\ell} \cdot \frac{1}{\sum_{i=1}^v \frac{1}{\rho_i}} = \frac{2 \cdot L}{\ell} \cdot \rho_{\text{total}} \quad (7)$$

$$\text{Where, } L = \sqrt[4]{\frac{4EJ\ell}{\rho_{\text{total}}}} \quad (8)$$

2.5 Comparison of theoretical calculation of sleeper loading with ballasted track observations

For the verification of the validity of the four methods in relation to the real conditions observed on track, we compare the calculations’ results to the real situation (appearance of cracks) in the Greek network. The appearance of the cracks led in the replacement of the sleepers from track, thus in a reduced Life-Cycle. The extended cracking, over the failure threshold (R3 region), observed in the Greek railway network at a percentage of more than 60 % of the U2/U3 sleepers laid on track, was not justified by the methods cited in the international literature at that time (French, German, American). The cracking was observed on the twin-block concrete sleepers of French technology, namely Vagneux U2, U3 with RN fastenings, for tracks designed for $V_{\text{max}} = 200$ km/h and temporary operational speed $V_{\text{operation}} = 140$ km/h. The calculations performed by the three methods did not provide results over the failure threshold (140–170 kN) and they were predicting no cracking at all. After an extensive research that included collaboration among various universities and railway organizations in Europe, the Giannakos (2004) method was developed whose results successfully predicted the extended cracking of the U2/U3 sleepers (Giannakos, 2011a, Giannakos & Loizos, 2010a), calculating actions over the cracking threshold and in almost all cases over the failure threshold. This method was derived from theoretical analyses and/or measurements from laboratory tests performed in Greece, Austria, France, and Belgium and observations from real on-track experience. Moreover, International Federation of Concrete (fib) has adopted this method for precast concrete railway track systems (fib, 2006).

The conditions of the Greek network between the 1980s and the beginning of 1990s, consisted of very compacted, polluted ballast bed (Giannakos & Loizos, 2010a) and stiff support ($\rho_{\text{ballast}} = 380$ kN/mm) and substructure classified according to the fluctuation of coefficient ρ_{subgrade} for the seating of the track from (a) $\rho_{\text{subgrade}} = 40$ kN/mm for pebbly substructure to the most adverse conditions of either (b) $\rho_{\text{subgrade}} = 100$ kN/mm, which

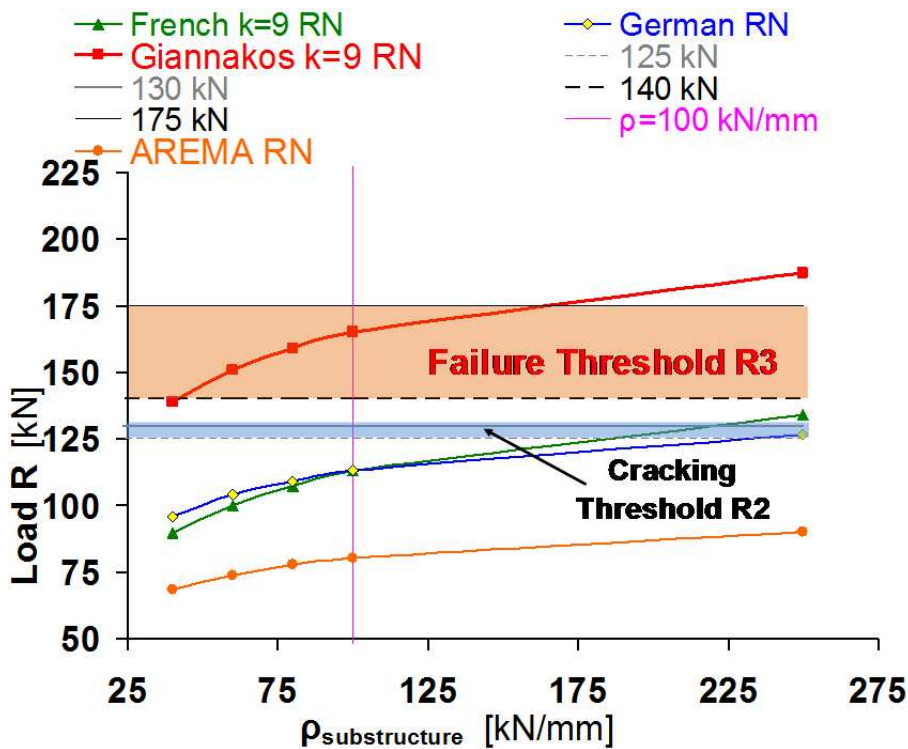


Figure2: Calculation of actions on U2/U3 twin-block ties with the four methods.

corresponds to frozen ballast bed and substructure or approximately the rigidity of Newly Constructed Lines 1 (NBS1) of the DB – German Railways (107 kN/mm) [Eisenmann et al., 1994], or (c) $\rho_{\text{subgrade}} = 250 \text{ kN/mm}$ for stiff (rigid) subgrade at the bottom of a tunnel or on a concrete bridge with small height of ballast-bed. The calculations according to the three aforementioned methods were programmed in a computer code and parametric investigations were performed varying the stiffness of the substructure as described in Giannakos (2004, Giannakos & Loizos, 2010a). The results are depicted in Figure 2, with $\rho_{\text{total}}=100 \text{ kN/mm}$ the most characteristic value.

In the following paragraphs Giannakos (2004) method is applied on a slab track case study for the estimation of the deflection/ subsidence and the results are compared against the measured values.

3 DEFLECTION VALUES MEASURED IN A SLAB TRACK SECTION

In the article *Entwicklung, Bemessung und Erforschung des schotterlosen Oberbaues "Rheda"*, the measurements on a "Rheda" type slab track are cited by professor Eisenmann et al., (1979), for static axle load 20 t (ICE 1). The fastening system used at that era was Ioarv 108 with pad Zwp55. The static stiffness coefficient of this pad was approximately 35 kN/mm (personal communication with Mr. W. Boesterling, Vossloh Gmbh in an e-mail of Feb. 2007).

The measured mean values of deflection of the rail running table for an axle-load of 20t are cited in Table 1 below (Eisenmann et al., 1979, p.34):

Table 1

	Measurement July 1972	Measurement December 1974	Measurement June 1977
Deflection of rail running table	1.49 mm	1.27 mm	1.31 mm
Deflection of concrete slab	0.37 mm	0.27 mm	0.23 mm
Relative deflection of rail vs slab	1.12 mm	1.00 mm	1.08 mm

In the paper measurements of the deflection of the concrete slab beneath each axle of a three axle bogie are also cited. The values presented in Table 1 are used in Giannakos (2004) method for the rail running table's deflection.

4 VERIFICATION OF GIANNAKOS (2004) METHOD FOR SLAB TRACK

The method uses equation (1) to calculate ρ_{total} from five parameters of the ballasted track layers: rail, fastening's pad, sleeper, ballast and subgrade. In the case of slab track the five (or four or three) layers are: rail, fastening pad 1, fastening pad 2 (if it exists, depending on the fastening type), concrete sleeper (if it exists, depending on the slab track type) and concrete slab comprising the total structure of slab track (CRCP, CTB, subgrade). The investigation was performed for the High Speed network of Greek Railways (Giannakos, 2009, 2008).

In order to verify Giannakos (2004) method for slab track these five layers were used for the 1979 Rheda type slab track: rail with $\rho_{rail}=75.000$ kN/mm, fastening pad 1 (Zwp55) with $\rho_{pad}=35$ kN/mm, fastening pad 2 (Zw687) with $\rho_{pad}=450$ kN/mm, concrete sleeper with average $\rho_{sleeper}=13.500$ kN/mm and concrete slab with $\rho_{slab}=\rho_{subgrade}$ ranging from 86 kN/mm, 100 kN/mm, 114 kN/mm, 171 kN/mm, 250 kN/mm (Giannakos, 2011b), to 425 kN/mm.

The subsidence y_{total} , of the multi-layered structure track, is calculated by the following equation (Giannakos, 2010c, 2004):

$$y_{total} = \bar{A}_{subsidence} \cdot (Q_{wheel} + Q_{\alpha}) + \frac{\mu \cdot \left(\sqrt{[\sigma(\Delta Q_{NSM})]^2 + [\sigma(\Delta Q_{SM})]^2} \right)}{h_{TR}} \quad (9)$$

$$\text{where: } \bar{A}_{subsidence} = \frac{1}{2\sqrt{2}} \cdot \sqrt[4]{\frac{\rho^3}{E \cdot J \cdot h_{TR}^3}} \quad (10)$$

and $h_{TR}=\rho_{dynam}$ is given by the equation (7), and the probability of occurrence ranges from 68.3 % (coefficient $\mu=1$) to 95.5 % (coefficient $\mu=2$).

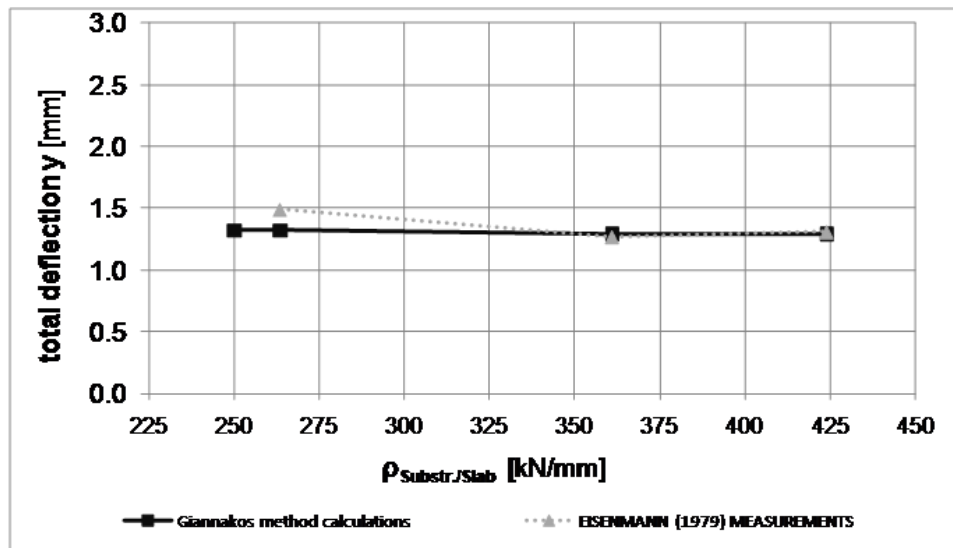


Figure3: Theoretical results of deflection in slab track Rheda type calculated by the Giannakos (2004) method and probability of occurrence 95.5 % vs measurements as in Eisenmann et al., 1979.

The calculations were programmed in a computer code and parametric investigations were performed varying the stiffness of the substructure as described in Giannakos (2004, Giannakos & Loizos, 2010a) and the probability of occurrence between 68.3 % and 95.5 %, with linear extrapolation for coefficient μ . The results are depicted in Figures 3 and 4 and are presented as a function of the total static track stiffness, for the Rheda Sengeberg type Slab Track already applied in High-speed lines under operation in the Greek network.

In Figure 3 the calculated values of deflection were derived for a probability of occurrence 95.5 %. The difference is very small, however a further investigation was performed for a range of the probabilities of occurrence. The results are depicted in Figure 4.

The results of a probability of occurrence of 80 % ($\mu=1,43$) are plotted inside the area of measured values for a range of $\rho_{\text{substructure}}$ from 86 to 425 kN/mm. This proves that the method is also applicable for slab track.

The method was also used to calculate the deflection and the actions on the slab track. For Rheda 2000 the deflections are almost the same as in the case of Rheda Sengeberg, since the total stiffness coefficients are almost identical. Rheda 2000 has a negligibly lower stiffness coefficient due to the absence of concrete sleeper ($\rho_{\text{sleeper}}=1/13,500$ equals to 0.00127 – 0.00151 of the ρ_{total} for a fluctuation of ρ_{subgrade} from 86 to 425 kN/mm). The influence of the stiffness variation of the subgrade (from 86 kN/mm to 425 kN/mm) on the total deflection y_{total} fluctuates approximately around a percentage of 7 %.

The application of Slab Track technology in a railway network creates the need for Transition Zones, which serve as interfaces between the ballastless and ballasted track sections, where significant and abrupt change in stiffness occurs. The Transition Zones guarantee a smooth stiffness transition between slab track and ballasted track, resulting in a smooth variation of the forces that act on the track (Giannakos et al., 2011b).

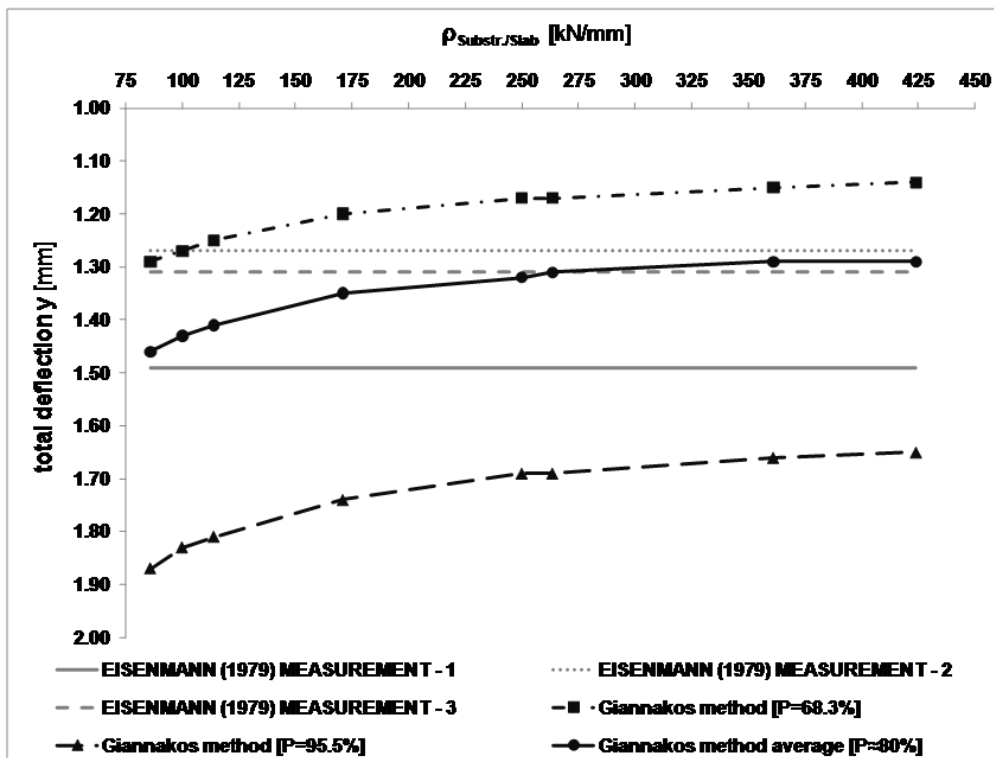


Figure4: Theoretical results of deflection in slab track Rheda type calculated by the Giannakos (2004) method and probability of occurrence 68.3 %, 80 % and 95.5 % vs the three measurements as in Eisenmann et al., 1979.

5 CONCLUSIONS

The parametric analysis performed with Giannakos (2004) method, gave results in agreement with the measurements on Rheda Sengeberg slab track in Germany. For a probability of occurrence of 80% (coefficient $\mu=1.43$), Giannakos (2004) method gives results that are the same as the measured values. In general the deflections reduce with the increase in the subgrade stiffness. The elasticity in the slab track structure is mainly provided by the elastic pad of the fastening.

REFERENCES

- Alias J., 1984. *La voie ferree*. IIème edition, Eyrolles, Paris.
- AREMA, 2010, *Manual for Railway Engineering*.
- Eisenmann J., 2004. *Die Schiene als Tragbalken*. Eisenbahningenieur, 5/2004
- Eisenmann J., Duwe B., Lempe U., Leykauf G., Steinbeisser L., 1979. *Entwicklung, Bemessung und Erforschung des schotterlosen Oberbaues "Rheda"*. AET (34), p.23-41.
- Eisenmann J., Leykauf G., Mattner L., 1994. *Vorsläge zur Erhöhung der Oberbauelastizität*. ETR 43 H 7/8.
- Eisenmann J., 1981. *The Rail as support and Roadway, theoretical principles and practical examples - part 2*. In Fastenrath Fritz, *Railroad Track - Theory and Practice*, Frederic Ungar Pub.Co., New York.
- fib (Federation Internationale du beton), 2006, *Precast concrete railway track systems*. With the participation of the author.
- Giannakos K., 2012a. *Selected Topics on Railways*. University of Thessaly Greece, Department of Civil Engineering, Volos, Greece, in greek, www.uth.gr.
- Giannakos K., 2012b. *Modern Railway Infrastructure: Resilient Fastenings improve Track's Life-Cycle*. TRA2012, Athens, 23-26 April, proceedings.
- Giannakos K., 2011a. *Heavy Haul Railway Track Maintenance and Use of Resilient versus Stiff Fastenings*. TRR Volume 2261/2011, p.155-162.
- Giannakos K., Tsoukantas S., Sakareli A., Zois C., 2011b. *Schwankung des Steifigkeit zwischen Fester Fahrbahn und Schottergleis - Beschrieben wird die parametrische Analyse der Schwankung des Steifigkeitsfaktors im Übergangsbereich zwischen Fester Fahrbahn und Schottergleis*. EI - Eisenbahningenieur, Germany, February, p. 12-19.
- Giannakos K., Loizos A., 2010 a. Evaluation of actions on concrete sleepers as design loads–Influence of fastenings. International Journal of Pavement Engineering (IJPE), Volume 11, Issue 3 June pages 197 – 213.
- Giannakos K., 2010 b. *Theoretical calculation of the track-mass in the motion of unsprung masses in relation to track dynamic stiffness and damping*. International Journal of Pavement Engineering (IJPE), Special Rail Issue High-Speed Railway Infrastructure: Recent Developments and Performance, volume 11, number 4, August 2010, p. 319-330.
- Giannakos K., 2010 c. *Interaction between Superstructure and Substructure in Railways*. Fifth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in honor of professor I. M. Idriss, San Diego, CA - USA, May 24-29 proceedings.
- Giannakos K., 2009. *Calculation of Stiffness coefficients, ρ , in the Transition Zone between Embedded Track and Ballasted Track - Design of the New Railway Terminal at New Ikonion port of Piraeus*. Obermeyer Planen + Beraten GmbH, for ERGA-OSE, Athens.
- Giannakos K., 2008. *Calculation of Stiffness coefficients, ρ , in the Transition Zone between Slab Track and Ballasted Track – Slab Track Rheda 2000 type Design of the New Railway Track of Normal Gauge in the Kiato – km 2+000 position (Rododaphne)*. Obermeyer Planen + Beraten GmbH, for ERGA-OSE, Athens.
- Giannakos K., 2004. *Actions on Railway Track*. Papazissis publications, Athens, English edition, www.papazisi.gr.

- Hay W., 1982, *Railroad Engineering*, John Wiley & Sons
- Prud'homme A., Eriau, J., 1976. *Les nouvelles traverses en beton de la SNCF*. RGCF-2.
- R.G.C.F. (Revue Generale des Chemins de Fer), Comite de redaction, 1973. *Sollicitations de la Voie, du Ballast et de la Plate-forme*. Mai.
- Selig E., Waters J., 1994/2000. *Track Geotechnology and Substructure Management*. Thomas Telford, 1994, reprinted 2000.
- Tsoukantas S., “*Investigation of Problems concerning the Slab Track application in Greece in tunnels, plain line and bridges*”, (Research Program of OSE-ERGOSE)”, Athens, 1999.
- Tsoukantas S., Giannakos K., Topintzis T., Zois H., 2006. *System Rheda 2000 from the point of view of a Structural Engineer – the most modern evolution of superstructure technology in Railway projects*. 15th Greek Conference of Concrete, Technical Chamber of Greece, Greek Department of Concrete, (Member of FIB – RILEM), Alexandroupolis, 24-27 October, Proceedings.
- U.I.C. (Union International des Chemins de Fer), fiche UIC (Code) 719R/1-1-1994 “*Earthworks and track-bed layers for railway lines*”.