Sensurveiledning vår 2024

Dette er ikke et eksempel på en god besvarelse, kun en veiledning til sensor.

SØK3007 Skatt, beslutningsatferd og økonomisk politikk: Eksamen vår 2024

Bokmål

Eksamen består av to oppgaver som begge skal besvares. Ved sensuren vil de to oppgavene telle likt. Gode forklaringer og tolkninger belønnes på alle oppgaver.

Oppgave 1

a) Betrakt et marked hvor det er perfekt konkurranse, perfekt elastisk tilbud og en fallende etterspørselskurve. Anta at det innføres en avgift t per enhet av godet. Vis at effektivitetstapet ved beskatning (DWL) kan skrives som

$$DWL = \frac{1}{2}|\varepsilon|\frac{X^0}{p}t^2,$$

der ε er etterspørselens priselastisitet, X^0 er omsatt kvantum før skatt og p er produsentprisen. Tolk uttrykket for effektivitetstapet.

b) Analyser hvordan avgiftssystemet bør utformes når myndighetene kun tar hensyn til effektivitet.

c) Diskuter konflikter mellom hensynene til effektivitet og fordeling i utforming av avgiftssystemet.

Delspørsmål a): Pensumdekning er kap. 15.2 i læreboka, se vedlegg.

Delspørsmål b): Her kan studentene velge mellom to modeller (begge er vedlagt), invers elastisitetsregel (kap. 15.5.1) eller Ramsey (kap. 15.5.2). Ramsey er mer generell (tillater kryssprisеffekter) og gir derfor høyere uttelling enn invers elastisitetsregel.

Delspørsmål c): Både invers elastisitetsregel og Ramsey impliserer at det bør legges høyere avgift på nødvendighetsgoder som har prisuelastisk etterspørsel. Dette vil isolert sett ha uheldige fordelingseffekter. På den andre siden kan det argumenteres for at fordelingshensyn bedre kan ivaretas gjennom (progressiv) inntektsskatt.

Oppgave 2

Betrakt en økonomi hvor velgernes preferanser er gitt ved $U_i = x_i + b(G)$. U_i er nyttenivået til velger *i*, x_i er privat konsum for velger *i* og *G* er et kollektivt gode. Anta at $b'(G) > 0$ og $b''(G) < 0$. Det er *N* velgere i økonomien med ulik inntekt Y_i. Beslutningen om *G* tas ved flertallsvalg.

a) Anta at det kollektive godet finansieres ved en kopp-skatt som er lik for alle velgere. Finn og tolk betingelsen for ønsket produksjon av det kollektive godet for velger *i*.
b) Begrunn at forutsetningene for å bruke medianvelgerteoremet er oppfylt i dette tilfellet og finn den politiske likevekten.
c) Gjenta a) når det antas at det kollektive godet finansieres ved en proporsjonal inntektsskatt.
d) Diskuter hvilket skattesystem, kopp-skatt eller proporsjonal inntektsskatt, som gir det samfunnsøkonomisk beste utfallet.

Delspørsmål a): Den private budsjettbetingelsen er gitt ved $x_i + T = Y_i$ og den offentlige kan skrives som $G = NT$ (antar prisene på det private godet og det kollektive godet er normalisert til 1). Ved å kombinere disse og sette inn i nyttefunksjonen får vi $U_i = Y_i - \frac{G}{N} + b(G)$. Maksimering mhp *G* gir følgende førsteordensbetingelse $b'(G) = \frac{1}{N}$.
Tolkningen er at marginal betalingsvillighet for det kollektive godet skal være lik skatteprisen (skatteøkningen for den enkelte velger som følge av at det tilbys en enhet ekstra av det kollektive godet). Alle velgere ønsker samme omfang av det kollektive godet.

Delspørsmål b): Forutsetningene er endimensjonalt beslutningsproblem og entoppede preferanser. Begrunnelsen for at beslutningsproblemet endimensjonalt er at hvis tilbudet av det kollektive godet er bestemt, så følger kopp-skatten av den offentlige budsjettbetingelsen. Entoppede preferanser kan enklest begrunnes ved at nyttefunksjonen er konkav i *G*,
$\frac{\partial^2 U_i}{\partial G^2} = b''(G) < 0$.

Delspørsmål c): Ved proporsjonal inntektsskatt er budsjettbetingelsene gitt ved hhv. $x_i + tY_i = Y_i$ (privat) der *t* er inntektsskattesatsen og $G = tN\overline{Y}$ (offentlig) der $\overline{Y}$ er gjennomsnittlig inntekt. Ved å kombinere den private og offentlige budsjettbetingelsen og så sette inn i nyttefunksjonen får vi $U_i = Y_i - \frac{Y_i}{N\overline{Y}}G + b(G)$. Maksimering mhp G gir følgende førsteordensbetingelse $b'(G) = \frac{Y_i}{N\overline{Y}}$. Tolkningen er at marginal betalingsvillighet for det kollektive godet skal være lik skatteprisen, men merk at proporsjonal inntektsskatt gir et annet uttrykk for skatteprisen enn koppskatt. Velgere med høy inntekt har høyere skattepris enn velgere med lav inntekt, noe som innebærer at ønsket tilbud av det kollektive godet er avtakende i velgerens inntekt.

Delspørsmål d): Starter med å sammenlikne de politiske likevektene. Ved koppskatt er beslutningsproblemet trivielt siden alle ønsker samme tilbud av det kollektive godet, og den politiske likevekten er kjennetegnet ved $b'(G^K) = \frac{1}{N}$ der G^K er den politiske likevekten ved koppskatt.

Kan benytte medianvelgerteoremet også ved proporsjonal inntektsskatt. I motsetning til ved koppskatt er velgerne uenige om hvor mye som bør tilbys av det kollektive godet. Gitt at ønsket tilbud er avtakende i inntekt, er det velgeren med median inntekt (Y^m) som er avgjørende. Den politiske likevekten er da kjennetegnet ved $b'(G^I) = \frac{Y^m}{N\overline{Y}}$ der G^I er den politiske likevekten ved proporsjonal inntektsskatt.

Med kopp-skatt vil den politiske likevekten alltid være samfunnsøkonomisk effektiv. Dette begrunnes enklest ved å appellere til Lindahl-løsningen (ingen uenighet) som er samfunnsøkonomisk effektiv (tilfredsstiller Samuelson-betingelsen), se vedlegg.

Ved proporsjonal inntektsskatt er det flere muligheter:

i) $Y^m = \overline{Y} \Rightarrow G^I = G^K$ Samfunnsøkonomisk effektiv

ii) $Y^m < \overline{Y} \Rightarrow G^I > G^K$ Overprovision med inntektsskatt

iii) $Y^m > \overline{Y} \Rightarrow G^I < G^K$ Underprovision med inntektsskatt

Det trekker opp dersom det påpekes at de fleste inntektsfordelinger er høyreskjeve ($Y^m < \overline{Y}$). Det er relevant å diskutere fordeling. Påpekning av at inntektsskatt gir jevnere fordeling enn koppskatt trekker opp.

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this is provided. The extension to many consumers is then made and the resolution of the equity–efficiency trade-off is emphasized. This is followed by a review of some numerical calculations of optimal taxes based on empirical data.

dweight Loss

Lump-sum taxation was described as the perfect tax instrument because it does not cause any distortions. The absence of distortions is due to the fact that a lump-sum tax is defined by the condition that no change in behavior can affect the level of the tax. Commodity taxation does not satisfy this definition. It is always possible to change a consumption plan if commodity taxation is introduced. Demand can shift from goods subject to high taxes to goods with low taxes, and total consumption can be reduced by earning less or saving more. It is these changes at the margin, which we call *substitution effects*, that are the tax-induced distortions.

The introduction of a commodity tax raises tax revenue but causes consumer welfare to be reduced. The *deadweight loss* of the tax is the extent to which the reduction in welfare exceeds the revenue raised. This concept is illustrated in figure 15.1. Before the tax is introduced, the price of the good is p and the quantity consumed is X^0. At this price the level of consumer surplus is given by the triangle *abc*. A specific tax of amount t is then levied on the good, so the price rises to $q = p + t$ and quantity consumed falls to X^1. This fall in consumption together with the price increase

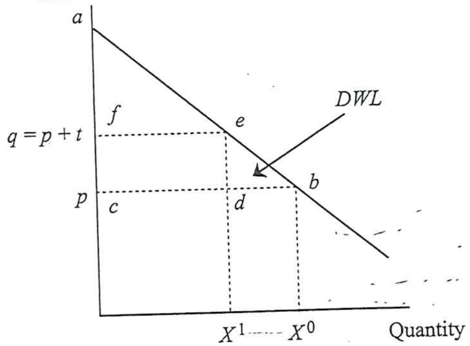

Figure 15.1
Deadweight loss

reduces consumer surplus to *aef*. The tax raises revenue equal to tX^1, which is given by the area *cdef*. The part of the original consumer surplus that is not turned into tax revenue is the deadweight loss, *DWL*, given by the triangle *bde*.

It is possible to provide a simple expression that approximates the deadweight loss. The triangle *bde* is equal to $\frac{1}{2}tdX$, where dX is the change in demand $X^0 - X^1$. This formula could be used directly, but it is unusual to have knowledge of the level of demand before and after the tax is imposed. Accepting this, it is possible to provide an alternative form for the formula. This can be done by noting that the elasticity of demand is defined by $\varepsilon^d = \frac{p}{X}\frac{dX}{dp}$, so it implies that $dX = \varepsilon^d \frac{X^0}{p} dp$. Substituting this into deadweight loss gives

$$DWL = \frac{1}{2}\left|\varepsilon^d\right| \frac{X^0}{p} t^2, \qquad (15.1)$$

since the change in price is $dp = t$. The measure in (15.1) is approximate because it assumes that the elasticity is constant over the full change in price from p to $q = p + t$.

The formula for deadweight loss reveals two important observations. First, deadweight loss is proportional to the square of the tax rate. The deadweight loss will therefore rise rapidly as the tax rate is increased. Second, the deadweight loss is proportional to the elasticity of demand. For a given tax change the deadweight loss will be larger the more elastic is demand for the commodity.

An alternative perspective on commodity taxation is provided in figure 15.2. Point a is the initial position in the absence of taxation. Now consider the contrast between a

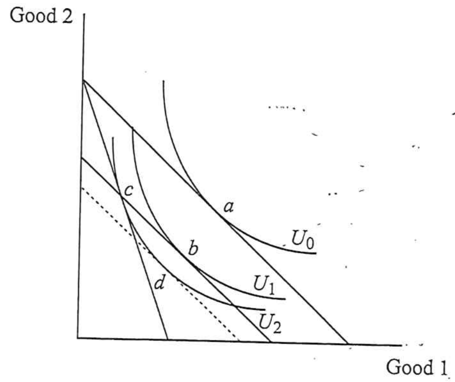

Figure 15.2
Income and substitution effects

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15.5.1 The Inverse Elasticity Rule

Figure 15.6 shows some of the features that the optimal set of commodity taxes will have. What the single-good formulation cannot do is give any insight into how that tax burden should be spread across different goods. For example, should all goods have the same rate of tax or should taxes be related to the characteristics of the goods? The first tax rule considers a simplified situation that delivers a very precise answer to this question. This answer, the *inverse elasticity rule*, provides a foundation for proceeding to the more general case. The simplifying assumption is that the goods are independent in demand so that there are no cross-price effects between the taxed goods. This independence of demands is a strong assumption, so it is not surprising that a clear result can be derived. The way the analysis works is to choose the optimal allocation and infer the tax rates from this. This was the argument used in the diagram when the intersection of the offer curve and the frontier of the production set was located and the tax rate derived from the implied budget constraint.

Consider a consumer who buys the two taxed goods and supplies labor. The consumer's preferences are described by the utility function $U(x_0, x_1, x_2)$, and his budget constraint is $q_1x_1 + q_2x_2 = x_0$. The utility-maximizing consumption levels of the two consumption goods are described by the first-order conditions $U_i' = \alpha q_i$, $i = 1, 2$, where U_i' is the marginal utility of good i and α is the marginal utility of income. The choice of labor supply satisfies the first-order condition $U_0' = -\alpha$.

With taxes t_1 and t_2 the government revenue constraint is $R = t_1x_1 + t_2x_2$. Since producer and consumer prices are related by $t_i = q_i - p_i$, this can be written as

$$q_1x_1 + q_2x_2 = R + p_1x_1 + p_2x_2. \tag{15.5}$$

The optimal tax rates are inferred from an optimization whereby the government chooses the consumption levels to maximize the consumer's utility while meeting the revenue constraint. This problem is summarized by the constrained maximization

$$\max_{\{x_1, x_2\}} L = U(x_0, x_1, x_2) + \lambda\left[q_1x_1 + q_2x_2 - R - p_1x_1 - p_2x_2\right]. \tag{15.6}$$

In this maximization the quantity of labor supply, x_0, is determined endogenously by x_1 and x_2 from the consumer's budget constraint, $x_0 = q_1x_1 + q_2x_2$.

The basic assumption that the demands are independent can be used to write the (inverse) demand function $q_i = q_i(x_i)$. Using these demand functions and the consumer's budget constraint to replace x_0, we write the first-order condition for the quantity of good i:

$$U_i' + U_0'\left[q_i + x_i\frac{\partial q_i}{\partial x_i}\right] + \lambda\left[q_i + x_i\frac{\partial q_i}{\partial x_i} - p_i\right] = 0. \tag{15.7}$$

The conditions $U_i' = \alpha q_i$ and $U_0' = -\alpha$ can be used to write this as

$$-\alpha x_i\frac{\partial q_i}{\partial x_i} + \lambda t_i + \lambda x_i\frac{\partial q_i}{\partial x_i} = 0. \tag{15.8}$$

where $t_i = q_i - p_i$. Now note that $\frac{x_i}{q_i}\frac{\partial q_i}{\partial x_i} = \frac{1}{\varepsilon_i^d}$, where ε_i^d is the elasticity of demand for good i. The first-order condition can then be solved to write

$$\frac{t_i}{p_i + t_i} = -\left[\frac{\lambda - \alpha}{\lambda}\right]\frac{1}{\varepsilon_i^d}. \tag{15.9}$$

Equation (15.9) is the inverse elasticity rule. This is interpreted by noting that α is the marginal utility of another unit of income for the consumer and λ is the utility cost of another unit of government revenue. Since taxes are distortionary, $\lambda > \alpha$. Since ε_i^d is negative, this makes the tax rate positive.

The inverse elasticity rule states that the proportional rate of tax on good i should be inversely related to its price elasticity of demand. Furthermore the constant of proportionality is the same for all goods. Recalling the discussion of the deadweight loss of taxation, it can be seen that this places more of the tax burden on goods where the deadweight loss is low. Its implication is clearly that necessities, which by definition have low elasticities of demand, should be highly taxed. It is this latter aspect that emphasizes the fact that the inverse elasticity rule describes an efficient way to tax commodities but not an equitable way. Placing relative high taxes on necessities will result in lower income consumers bearing relatively more of the commodity tax burden than high-income consumers.

15.5.2 The Ramsey Rule

The inverse elasticity rule is restricted by the fact that the demand for each good depends only on the price of that good. This rules out all cross-price effects in demand, meaning that the goods can be neither substitutes nor complements. When this restriction is relaxed, a more general tax rule is derived. The general result is called the *Ramsey rule*, and it is one of the oldest results in the theory of optimal taxation. It provides a description of the optimal taxes for an economy with a single consumer and with no equity considerations.

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To derive the Ramsey rule, it is necessary to change from choosing the optimal quantities to choosing the taxes. Assume that there are just two consumption goods in order to simplify the notation, and let the demand function for good i be $x_i = x_i(q)$ where $q = q_1, q_2$. The fact that the prices of all the commodities enter this demand function shows that the full range of interactions between the demands and prices are allowed. Using these demand functions, the preferences of the consumer can be written as

$$U = U(x_0(q), x_1(q), x_2(q)). \tag{15.10}$$

The optimal commodity taxes are those that give the highest level of utility to the consumer, while ensuring that the government reaches its revenue target of $R > 0$. The government's problem in choosing the tax rates can then be summarized by the Lagrangean

$$\max_{\{t_1, t_2\}} L = U(x_0(q), x_1(q), x_2(q)) + \lambda\left[\sum_{i=1}^{2} t_i x_i(q) - R\right], \tag{15.11}$$

where it is recalled that $q_i = p_i + t_i$. Differentiating (15.11) with respect to the tax on good k, we have the first-order necessary condition

$$\frac{\partial L}{\partial t_k} \equiv \sum_{i=0}^{2} U_i' \frac{\partial x_i}{\partial q_k} + \lambda\left[x_k + \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial q_k}\right] = 0. \tag{15.12}$$

This first-order condition needs some manipulation to place it in the form we want. The first step is to note that the budget constraint of the consumer is

$$q_1 x_1(q) + q_2 x_2(q) = x_0(q). \tag{15.13}$$

Any change in price of good k must result in demands that still satisfy this constraint so that

$$q_1 \frac{\partial x_1}{\partial q_k} + q_2 \frac{\partial x_2}{\partial q_k} + x_k = \frac{\partial x_0}{\partial q_k}. \tag{15.14}$$

In addition the conditions for optimal consumer choice are $U_0' = -\alpha$ and $U_i' = \alpha q_i$. Using these optimality conditions and (15.14), we rewrite the first-order condition for the optimal tax, (15.12), as

$$\alpha x_k = \lambda\left[x_k + \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial q_k}\right]. \tag{15.15}$$

Notice how this first-order condition involves quantities rather than the prices that appeared in the inverse-elasticity rule. After rearrangement, (15.15) becomes

$$\sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial q_k} = -\left[\frac{\lambda - \alpha}{\lambda}\right] x_k. \tag{15.16}$$

The next step in the derivation is to employ the Slutsky equation, which breaks the change in demand into the income and substitution effects. The effect of an increase in the price of good k upon the demand for good i is determined by the Slutsky equation as

$$\frac{\partial x_i}{\partial q_k} = S_{ik} - x_k \frac{\partial x_i}{\partial I}, \tag{15.17}$$

where S_{ik} is the substitution effect of the price change (the move around an indifference curve) and $-x_k \frac{\partial x_i}{\partial I}$ is the income effect of the price change (I denotes lump-sum income). Substituting from (15.17) into (15.16) gives

$$\sum_{i=1}^{2} t_i \left[S_{ik} - x_k \frac{\partial x_i}{\partial I}\right] = -\left[\frac{\lambda - \alpha}{\lambda}\right] x_k. \tag{15.18}$$

Equation (15.18) is now simplified by extracting the common factor x_k, which yields

$$\sum_{i=1}^{2} t_i S_{ik} = -\left[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial I}\right] x_k. \tag{15.19}$$

The substitution effect of a change in the price of good i on the demand for good k is exactly equal to the substitution effect of a change in the price of good k on the demand for good i because both are determined by movement around the same indifference curve. This symmetry property implies $S_{ki} = S_{ik}$, which can be used to rearrange (15.19) to give the expression

$$\sum_{i=1}^{2} t_i S_{ki} = -\theta x_k, \tag{15.20}$$

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where $\theta = \left[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial I}\right]$ is a positive constant. Equation (15.20) is the Ramsey rule describing a system of optimal commodity taxes and an equation of this form must hold for all goods, $k = 1, \ldots, n$.

The optimal tax rule described by (15.20) can be used in two ways. If the details of the economy are specified (the utility function and production parameters), then the actual tax rates can be calculated. Naturally the precise values would be a function of the structure chosen. Although this is the direction that heads toward practical application of the theory (and more is said later), it is not the route that will be currently taken. The second use of the rule is to derive some general conclusions about the determinants of tax rates. This is done by analyzing and understanding the different components of (15.20).

To proceed with this, the focus on the typical good k is maintained. Recall that a substitution term measures the change in demand with utility held constant. Demand defined in this way is termed *compensated demand*. Now begin in an initial position with no taxes. From this point the tax t_i is the change in the tax rate on good i. Then $t_i S_{ki}$ is a first-order approximation to the change in compensated demand for good k due to the introduction of the tax t_i. If the taxes are small, this will be a good approximation to the actual change. Extending this argument to take account of the full set of taxes, it follows that $\sum_{i=1}^{2} t_i S_{ki}$ is an approximation to the total change in compensated demand for good k due to the introduction of the tax system from the initial no-tax position. In employing this approximation, the Ramsey rule can be interpreted as saying that the optimal tax system should be such that the *compensated demand for each good is reduced in the same proportion* relative to the before-tax position. This is the standard interpretation of the Ramsey rule.

The importance of this observation is reinforced when it is set against the alternative, but incorrect, argument that the optimal tax system should raise the prices of all goods by the same proportion in order to minimize the distortion caused by the tax system. This is shown by the Ramsey rule to be false. What the Ramsey rule says is that it is the distortion in terms of quantities, rather than prices, that should be minimized. Since it is the level of consumption that actually determines utility, it is not surprising that what happens to prices is secondary to what happens to quantities. Prices only matter so far as they determine demands.

Although the actual tax rates are only implicit in the Ramsey rule, some general comments can still be made. By the approximation interpretation, the rule suggests that as the proportional reduction in compensated demand must be the same for all goods, and those goods whose demand is unresponsive to price changes must bear higher taxes in order to achieve the same reduction. Although broadly correct, this statement can only be completely justified when all cross-price effects are accounted for. One simple case that overcomes this difficulty is that in which there are no cross-price effects among the taxed goods. This is the special case that led to the inverse elasticity rule.

Returning to the general case, goods that are unresponsive to price changes are typically necessities such as food and housing. Consequently using the Ramsey rule leads to a tax system that bears most heavily on necessities. In contrast, the lowest tax rates would fall on luxuries. If put into practice, such a tax structure would involve low-income consumers paying disproportionately larger fractions of their incomes in taxes relative to high-income consumers. The inequitable nature of this is simply a reflection of the single-consumer assumption: the optimization does not involve equity and the solution reflects only efficiency criteria.

The single-consumer framework is not accurate as a description of reality, and it leads to an outcome that is unacceptable on equity grounds. The value of the Ramsey rule therefore arises primarily through the framework and method of analysis it introduces. This can easily be generalized to more relevant settings. It shows how taxes are determined by efficiency considerations and hence gives a baseline from which to judge the effects of introducing equity.

15.6 Equity Considerations

The lack of equity in the tax structure determined by the Ramsey rule is inevitable given its single-consumer basis. The introduction of further consumers who differ in incomes and preferences makes it possible to see how equity can affect the conclusions. Although the method that is now discussed can cope with any number of consumers, it is sufficient to consider just two. Restricting the number in this way has the merit of making the analysis especially transparent.

Consider then an economy that consists of two consumers. Each consumer h, $h = 1, 2$, is described by their (indirect) utility function

$$U^h = U^h\left(x_0^h(q), x_1^h(q), x_2^h(q)\right). \tag{15.21}$$

These utility functions may vary between the consumers. Labor remains the untaxed numéraire, and all consumers supply only the single form of labor service.

The government revenue constraint is now given by

$$R = \sum_{i=1}^{2} t_i x_i^1(q) + \sum_{i=1}^{2} t_i x_i^2(q), \tag{15.22}$$

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Table 6.1
Prices and quantities

	Private good	Public good
Price	Same	Different
Quantity	Different	Same

The idea of personalized pricing can be captured by assuming that the government announces the share of the cost of the public good that each consumer must bear. For example, it may say that each of two consumers must pay half the cost of the public good. Having heard the announcement of these shares, the consumers then state how much of the public good they wish to have supplied. If they both wish to have the same level, then that level is supplied. If their wishes differ, the shares are adjusted and the process repeated. The adjustment continues until shares are reached at which both wish to have the same quantity. This final point is called a *Lindahl equilibrium*. It can easily be seen how this mechanism overcomes the two sources of inefficiency. The fact that the consumers only pay a share of the cost reduces the perceived unit price of the public good. Hence the private cost appears lower, and the consumers increase their demands for the public good. Additionally the shares can be tailored to match the individual valuations.

To make this reasoning concrete, let the share of the public good that has to paid by consumer h be denoted τ^h. The scheme must be self-financing, so, with two consumers, $\tau^1 + \tau^2 = 1$. Now let G^h denote the quantity of the public good that household h would choose to have provided when faced with the budget constraint

$$x^h + \tau^h G^h = M^h. \tag{6.11}$$

The Lindahl equilibrium shares $\{\tau^1, \tau^2\}$ are found when $G^1 = G^2$. The reason why efficiency is attained can be seen in the illustration of the Lindahl equilibrium in figure 6.7. The indifference curves reflect preferences over levels of the public good and shares in the cost. The shape of these captures the fact that each consumer prefers more of the public good but dislikes an increased share. The highest indifference curve for consumer 1 is to the northwest and the highest for consumer 2 to the northeast. Maximizing utility for a given share (which gives a vertical line in the figure) achieves the highest level of utility where the indifference curve is vertical. Below this point the consumer is willing to pay a higher share for more public good, and above it is just the other way around. Hence the indifference curves are backward-bending. The *Lindahl*

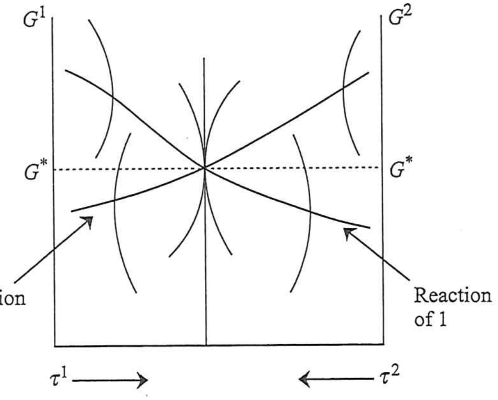

Figure 6.7
Lindahl equilibrium

reaction functions are then formed as the loci of the vertical points of the indifference curve. The equilibrium requires that both consumers demand the same level of the public good; this occurs at the intersection of the reactions functions. At this point the indifference curves of the two consumers are tangential and the equilibrium is Pareto-efficient.

To derive the efficiency result formally, note that utility is given by the function $U^h\left(M^h - \tau^h G^h, G^h\right)$. The first-order condition for the choice of the quantity of public good is

$$\frac{U_G^h}{U_x^h} = \tau^h, \qquad h = 1, 2. \tag{6.12}$$

Summing these conditions for the two consumers yields

$$\frac{U_G^1}{U_x^1} + \frac{U_G^2}{U_x^2} \equiv MRS_{G,x}^1 + MRS_{G,x}^2 = \tau^1 + \tau^2 = 1. \tag{6.13}$$

This is the Samuelson rule for the economy, and it establishes that the equilibrium is efficient. The personalized prices equate the individual valuations of the supply of public goods to the cost of production in a way that uniform pricing cannot. They also correct for the divergence between private and social benefits.