① We are given $r_f = 0.04$, $\sigma_m = 0.2$, and $S_m = 0.3$.

a) $\sigma_p = a\,\sigma_m = \underline{\underline{0.2a}}$

b) We know that the slope of the capital market line is S_m and the intercept is r_f:

$$E[r_p] = r_f + S_m \cdot \sigma_p$$

$$= 0.04 + 0.3 \cdot 0.2a = \underline{\underline{0.04 + 0.06a.}} \quad ⊗$$

c) $$U = E[r_p] - \frac{5}{4}\sigma_p^2$$

$$= 0.04 + 0.06a - \frac{5}{4} \cdot 0.04a^2$$

$$= 0.04 + 0.06a - 0.05a^2$$

$$\frac{dU}{da} = 0.06 - 0.1a = 0 \Leftrightarrow a^* = \frac{0.06}{0.1} = \underline{\underline{0.6.}}$$

d) $$U^* = 0.04 + 0.06 \cdot 0.6 - 0.05 \cdot 0.6^2 = \underline{\underline{0.058}}$$

⊗ Alternatively: $S_m = \frac{E[r_m] - r_f}{\sigma_m} \Leftrightarrow E[r_m] = S_m \cdot \sigma_m + r_f$

$$= 0.3 \cdot 0.2 + 0.04$$

$$= 0.1.$$

$$E[r_p] = 0.1 \cdot a + 0.04(1-a) = \underline{0.04 + 0.06a.}$$

② a) We know that

$$f_{n-1,n} = \frac{(1+r_{0,n})^n}{(1+r_{0,n-1})^{n-1}} - 1 \iff (1+f_{n-1,n})(1+r_{0,n-1})^{n-1} = (1+r_{0,n})^n \iff$$

$$r_{0,n} = \left((1+f_{n-1,n})(1+r_{0,n-1})^{n-1}\right)^{\frac{1}{n}} - 1$$

Inserting for the forward rates, we get

$$r_{0,1} = \left((1+0.05)(1+r_{0,0})^0\right)^{\frac{1}{1}} - 1 = \underline{\underline{0.05}}$$

$$r_{0,2} = \left((1+0.060024)(1+0.05)^1\right)^{\frac{1}{2}} - 1 = \underline{\underline{0.055}}$$

$$r_{0,3} = \left((1+0.070071)(1+0.055)^2\right)^{\frac{1}{3}} - 1 = \underline{\underline{0.06}}$$

$$r_{0,4} = \left((1+0.080142)(1+0.06)^3\right)^{\frac{1}{4}} - 1 = \underline{\underline{0.065}}$$

b) $$P_0 = \sum_{t=1}^{4} \frac{6.42}{(1+r_{0,t})^t} + \frac{100}{(1+r_{0,4})^4}$$

$$= \frac{6.42}{1.05} + \frac{6.42}{(1.055)^2} + \frac{6.42}{(1.06)^3} + \frac{100+6.42}{(1.065)^4} = \underline{\underline{100}}$$

c) $$P_0^{ZCB} = \frac{100}{(1.065)^4} = \underline{\underline{77.73}}$$

d) We can use four ZCBs maturing at time 1, 2, 3, and 4.

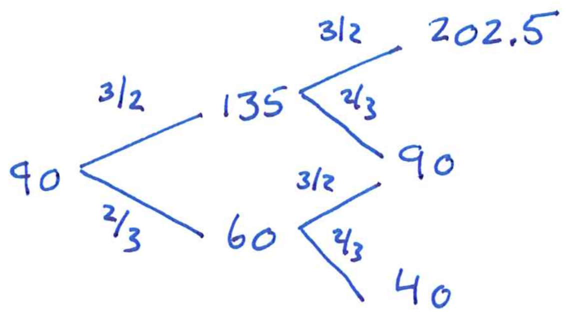
Maturity date	Payoff	Price
1	6.42	$\frac{6.42}{1.05} = 6.11$
2	6.42	$\frac{6.42}{(1.055)^2} = 5.77$
3	6.42	$\frac{6.42}{(1.06)^3} = 5.39$
4	106.42	$\frac{106.42}{(1.065)^4} = 82.72$

$$99.99 \approx 100$$

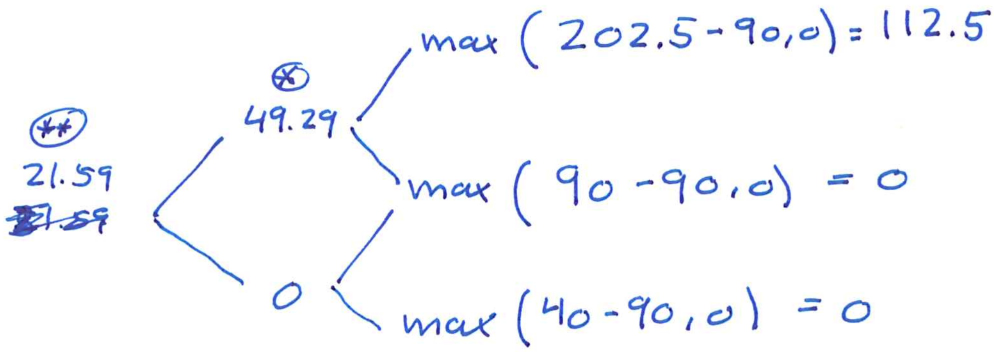
③ We can first find the risk-neutral probability

$$q = \frac{1+r_f - d}{u-d} = \frac{1.05 - 2/3}{3/2 - 2/3} = \underline{0.46}$$

Stock price tree



a) Use backward induction:

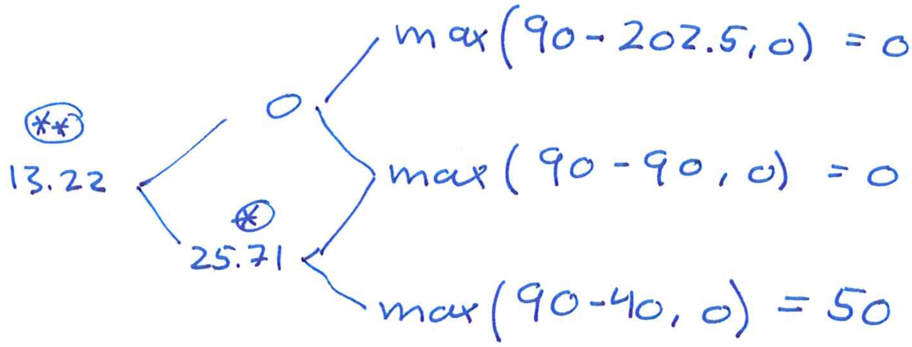


(*) $\frac{0.46 \cdot 112.5}{1.05} = 49.29$

(**) $\frac{0.46 \cdot 49.29}{1.05} = 21.59$

$\underline{\underline{C_0 = 21.59}}$

b)



(*) $\frac{(1-0.46)\cdot 50}{1.05} = 25.71$

(**) $\frac{(1-0.46)\cdot 25.71}{1.05} = 13.22$

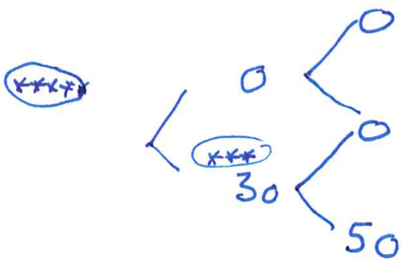
$\underline{\underline{P_0 = 13.22}}$

Alternatively, by using the put-call parity:

$P_0 + S_0 = C_0 + PV(X) \Leftrightarrow P_0 = C_0 + PV(X) - S_0 \Leftrightarrow$

$P_0 = 21.59 + \frac{90}{(1.05)^2} - 90 = \underline{\underline{13.22}}$

c)



(***) $\max(90-60, 25.71) = 30$

↑ from b)

(****) $\frac{(1-0.46)\cdot 30}{1.05} = 15.43$

$\underline{\underline{P_0^A = 15.43}}$

(4) We first find β:

$$\beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = \frac{\rho_{AM}\sigma_A\sigma_M}{\sigma_M^2} = \frac{\rho_{AM}\sigma_A}{\sigma_M} = \frac{0.625 \cdot 0.4}{0.2} = \underline{1.25}$$

The cost of equity capital is

$$k = r_f + (E[r_M] - r_f)\beta_A = 0.05 + (0.09 - 0.05)\cdot 1.25 = \underline{0.10}$$

The growth rate is given by

$$g = b \cdot ROE = 0.25 \cdot 0.2 = \underline{0.05}$$

The dividend payment is

$$D = E(1-b) = 100 \cdot 0.75 = \underline{75}$$

The stock price is

$$P_0 = \frac{75}{0.1 - 0.05} = \underline{1500}$$

With $b=0$, $g=0$ and $D=100$:

$$P_0 |_{b=0} = \frac{100}{0.1} = \underline{1000}$$

$$PVGO = 1500 - 1000 = \underline{\underline{500}}$$

Alternatively (using the formula from the last homework):

$$PVGO = \frac{E_1 b(ROE - k)}{k(k-g)} = \frac{100 \cdot 0.25 \cdot (0.2 - 0.1)}{0.1(0.1 - 0.05)} = \underline{\underline{500}}$$