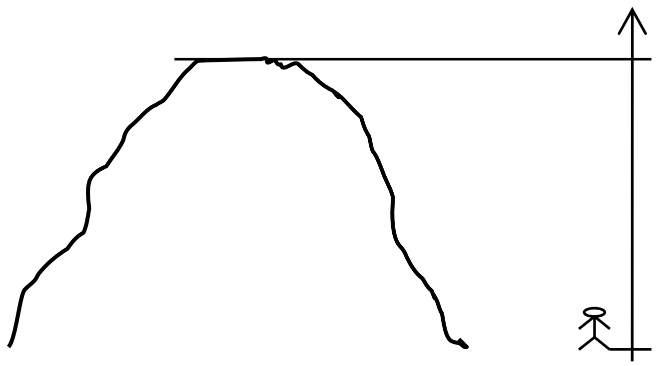
EXAM IN TFY 4300 Energy and environmental physics - Suggested solution
Tuesday December 19th 2006 Corrected 06.12.07

1. *Weather and climate*
   a) *Lapse rates*
   You are planning to climb to the top of Mountain Gråkallen (552 m above sea level) from "Ravnkloa" at sea-level (0 m) where the temperature is 25°C. What temperature should you expect at the top? Consider dry air and that c_p is constant. The dry adiabatic lapse rate is Γ_d=0.01°C/m. What would the temperature at the top be if the air was saturated with water vapour? The saturated adiabatic lapse rate is taken to be Γ_s=0.006°C/m. Why is $\Gamma_s<\Gamma_d$?



The vertical temperature gradient for dry air is given by (the dry adiabatic lapse rate):

$$\frac{\partial T}{\partial z} = -\Gamma_d = -0.01^\circ \text{ C/m}$$

The temperature thus decreases linearly with height, expressed as $T(z) = T_0 - \Gamma_d \cdot z$

At the base camp z=z_1 we have the temperature T_1: $T_1 = T(z_1) = T_0 - \Gamma_d \cdot z_1$,

and at the top z=z_t, and the temperature is T_t, given by $T_t = T(z_t) = T_0 - \Gamma_d \cdot z_t$.

By subtracting T_t from T_1, we get: $T_t - T_1 = T(z_t) - T(z_1) = -\Gamma_d \cdot (z_t - z_1)$,

which can be solved for T_t:

$$\underline{\underline{T_t}} = T_1 - \Gamma_d \cdot (z_t - z_1) = \left[25 - 0.01 \cdot (552 - 0)\right]{}^\circ\text{C} = \underline{\underline{19.5^\circ\text{C}}}$$

With saturated air, we use Γ_s instead of Γ_d and get a top temperature of

$$\underline{\underline{T_t}} = T_1 - \Gamma_s \cdot (z_t - z_1) = \left[25 - 0.006 \cdot (552 - 0)\right]{}^\circ\text{C} = \underline{\underline{21.7^\circ\text{C}}}$$

Γ_s is smaller than Γ_d i.e. the temperature drops more slowly with altitude if the air contains water vapour. This is because the water will release it's latent heat when it condenses from water vapour into water drops. This latent heat heats up the air as it rises, so that the cooling rate smaller than for dry air.

b) The oceans

Explain why the oceans are an important part of the climate system.

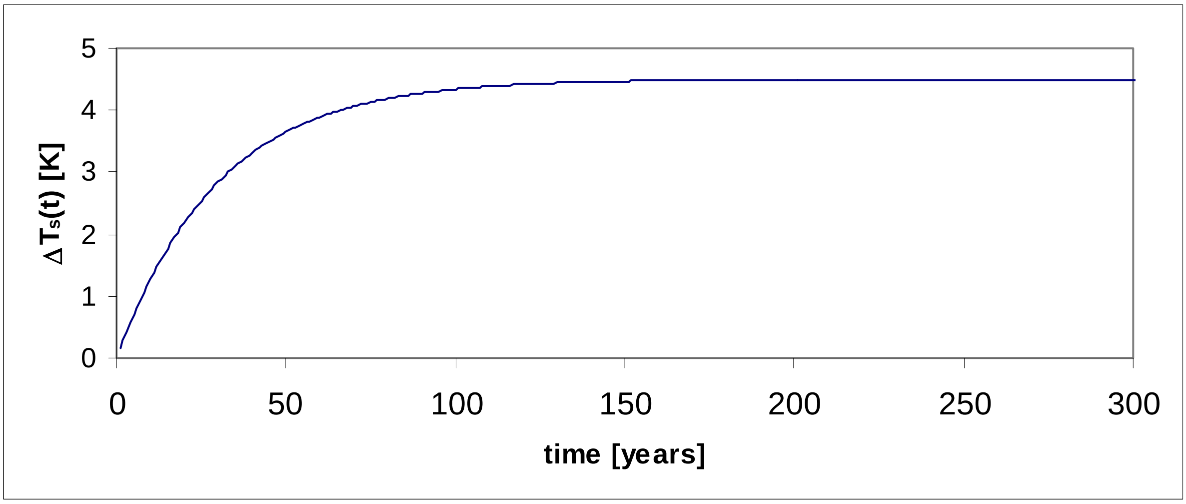
The oceans are an important part of the climate system since they

- cover ca 70% of the surface of the earth,
- absorb much of the solar energy entering,
- store a lot of heat due to the large heat capacity and mixing of the top layer,
- transport a lot of heat in the big ocean currents, and
- evaporate easily to form clouds and transport latent heat to other locations on earth.
- take up a lot of CO_2 in various ways.

c) Delay.

Assume a constant radiative forcing of $\Delta I = 5 W/m^2$, a gain of $G_f = 0.9 Km^2/W$ and a time lag of $\tau_e = 30$ years. What is the final value for ΔT_s, e.g. what is $(\Delta T_s)_{ss}$? Plot how ΔT_s increases with time. How many years does it take for ΔT_s to reach half of its final value?

The final value for ΔT_s is $(\Delta T_s)_{ss} = G_f \Delta I = 0.9 \times 5.0$ K = 4.5K



Let t* be the time after t=0 when ΔT_s reaches half its final value. We then have from $\Delta T_s(t) = (\Delta T_s)_{ss}(1 - e^{-t/\tau})$:

$$\frac{(\Delta T_s)_{ss}}{2} = (\Delta Ts)_{ss} \cdot (1 - e^{-t^*/\tau}) \Leftrightarrow \frac{1}{2} = (1 - e^{-t^*/\tau}) \Leftrightarrow e^{-t^*/\tau} = \frac{1}{2}$$

$$\Leftrightarrow \frac{-t^*}{\tau} = \ln\frac{1}{2} \Leftrightarrow t^* = \tau \ln 2 = (30 \cdot \ln 2) \text{ years} = 20.8\text{years}$$

d) El Niño

Explain what the "El niño Southern Oscillation" is and how it occurs. Where and when does it take place?

The "El niño Southern Oscillation" is a change in the atmosphere and ocean circulation patterns in the (Southern) Pacific ocean that happens with an irregular period of 2 to 7 years. The normal situation is shown in the figure below.

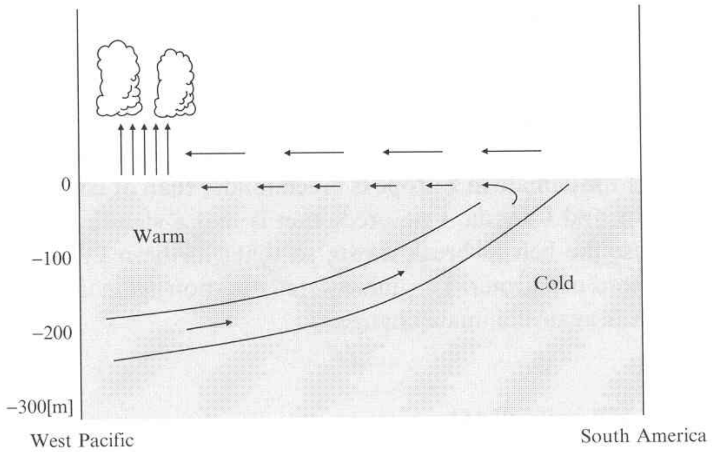


Figure 2.19 The Pacific Ocean at the Equator in its normal mode. Westward winds induce westward surface currents, which push down the thermocline. An eastward undercurrent (a cold tongue) supplies the necessary water. Evaporation in the west (Indonesia and Australia) produces clouds and precipitation

The westward trade winds (blowing from the east to the west) are caused by the hot air rising at the equator being replaced by colder air coming from the north and south of the equator. When this air approaches the equator, the increasing rotational speed of the earth (closer to the equator) causes the air to move westward (easterly/from the east) relative to the surface of the earth. The westward winds induce westward surface currents in the South Pacific Ocean which piles up warm water in the West Pacific (the water is pushed up 40cm). The mixed layer thickness increases and pushes down the thermocline where the water gets colder. The water that is pushed away from South America, is replaced by this cold water from the West Pacific as indicated in the figure. In a normal situation we thus have hot water and hot air in the West Pacific, and cold water and cold air at the coast of South America. This temperature difference causes a westwards air flow due to the pressure differences.

Disturbances to this normal circulation pattern of water and air, might cause the cold water from the West Pacific to rise to the ocean surface further away from the South American coast. Then the thermocline at the South American coast will be lower and the mixed layer warmer. The temperature difference between East and West will now be smaller, leading to smaller pressure difference, less westward wind and less movement of ocean water. This will lower the temperature in the West Pacific, and reinforce the decreased cooling of the water outside South America. This positive feedback maintains the El niño phenomenon for approximately one year, when the conditions go back to the normal situation.

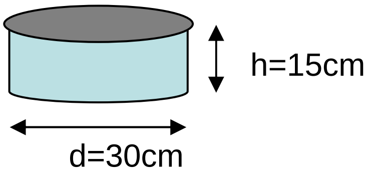
2. Heat transport

a) Why is heat transport an important issue in energy physics/technology?

Since heat is a form of energy, the optimization of heat gain and/or minimizing heat loss in any energy converting system is important. Heat gain and heat loss are governed by heat transport, so understanding and predicting heat transport is important for optimization of energy efficiencies and energy conservation.

b) Convective cooling of a cooking pot

A metal cooking pot with shiny outside surface, of dimensions shown below, is filled with food and water and placed on a cooking stove. Assume that the lid is tight and has the same diameter as the pot it self. Also neglect heat loss by radiation, and assume that the outer surface of the pot and the lid has a surface temperature of 100C. The surroundings have a temperature of 16C. What is the minimum energy required to maintain it at boiling temperature for 30 minutes if it is sheltered from the wind?



Solution: we look at the lid and the pot separately.

For free convection we need to calculate the Rayleigh number from $\mathcal{A} = \frac{g\beta X^3 \Delta T}{\kappa \nu}$.

We need to choose a temperature to evaluate the air parameters at, and choose the average temperature of the pot and the surroundings i.e. (100+16)C/2=58C. From the table in the appendix we find $\mathcal{A} / X^3 \Delta T = 0.58 \cdot 10^8 m^{-3} K^{-1}$ and k =2.88 x 10^{-2} $Wm^{-1}K^{-1}$ for air at 60C.

For the lid X=d=30cm, so that

$\mathcal{A} = 0.58 \cdot 10^8 m^{-3} K^{-1} \times X^3 \Delta T = 0.58 \cdot 10^8 m^{-3} K^{-1} \times (0.3m)^3 (100-16)K = 1.3 \cdot 10^8 > 10^5$,

giving turbulent flow, and $\mathcal{N} = 0.14 \cdot \mathcal{A}^{0.33} = 0.14 \cdot \left(1.3 \cdot 10^8\right)^{0.33} = 66.9$.

The convective heat loss from the lid is thus, with $A = \pi r^2$:

$$P_{v,lid} = A\mathcal{N} \frac{k(T_s - T_f)}{X} = \pi \left(\frac{0.3m}{2}\right)^2 \cdot 66.9 \cdot \frac{2.88 \cdot 10^{-2}\ Wm^{-1}K^{-1} \cdot (100-16)K}{0.3m} = 38.1W .$$

For the pot X=h=15cm, and

$\mathcal{A} = 0.58 \cdot 10^8 m^{-3} K^{-1} \times X^3 \Delta T = 0.58 \cdot 10^8 m^{-3} K^{-1} \times (0.15m)^3 (100-16)K = 1.6 \cdot 10^7 > 10^5$,

giving laminar flow, and $\mathcal{N} = 0.56 \cdot \mathcal{A}^{0.25} = 0.56 \cdot \left(1.6 \cdot 10^7\right)^{0.25} = 35.7$.

For the pot the area equals $A = 2\pi rh$, so that

$$P_{v,pot} = A\mathcal{N} \frac{k(T_s - T_f)}{X} = 2\pi \left(\frac{0.3m}{2}\right) \cdot 0.15m \cdot 66.9 \cdot \frac{2.88 \cdot 10^{-2}\ Wm^{-1}K^{-1} \cdot (100-16)K}{0.15m} = 81.3W .$$

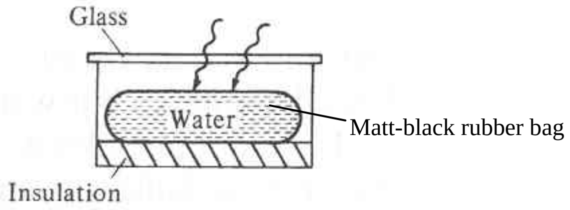
Hence the total heat loss is

$P_{tot} = P_{lid} + P_{pot} = 38.1W + 81.3W = 119.4W$.

The energy required to compensate for this heat loss for 30 minutes (Δt=1800s) is:

<u>$E = P_{tot} \times \Delta t = 119.4\ J/s \times 1800\ s = 2.1\ 10^5\ J = 0.21MJ$</u>

c) *Explain how the solar water heater in the figure on the below works. Give details about the typical properties of the various parts. What measures have been taken to optimize the heat gain and minimize the heat loss compared to a solar water heater consisting of an open water container on the ground?*



The solar water heater works as follows:
In the morning the water will be heated gradually, and with the temperature increase the heat losses also increase. Eventually the water reaches a maximum temperature, the stagnation temperature, and at this temperature the heat losses are maximal and equal the heat gain, so that the temperature will stay constant. When the sun sets, the water will start cooling, since the heat loss will be larger than the heat gain, and the temperature will decrease until it reaches the temperature of the ambient.

Solar radiation enters the solar water heater through the glass layer. This glass should have a high transmittance for the solar radiation (typically is around 90%). The remaining solar radiation reaches the water container; a black rubber bag. This bag should have a high absorbance for the solar spectrum, and ideally a low emittance for the thermal radiation emitted by the bag. A matt-black bag has absorbance near 90%. The rubber heats up due to this absorption of the solar radiation, and this heat is conducted to the water inside the bag. The thermal resistance of the rubber is normally very small, so that one can assume that the water reaches the same temperature as the bag. The water is kept inside a rubber bag to eliminate heat loss due to evaporation, and the rubber bag is kept inside a container with thermal insulation in the bottom, to minimize heat loss due to convection from the top and sides, and due to conduction from the bottom of the container. (For an open water container on the ground, all these loss mechanisms would have been severe.) The insulation at the bottom should have very low thermal conductivity. To minimize the heat loss due to radiation the long wavelength emittance should be as low as possible, but typically it is near 90%

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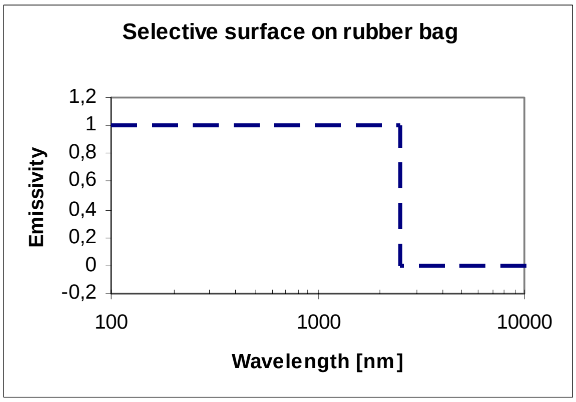
d) *You are supposed to coat the upper glass sheet and the rubber bag in the drawing above with (ideal) coatings that optimize the heat gain and minimize the heat loss for the solar water heater. Should the two coatings/surfaces have the same optical properties? Why/why not? Explain what a selective surface is. Draw curves for the (ideal) wavelength dependence for the emittance $\varepsilon(\lambda)$ for the coating/selective surface on the glass and on the rubber bag. (Indicate the important wavelength ranges, with typical values.)*

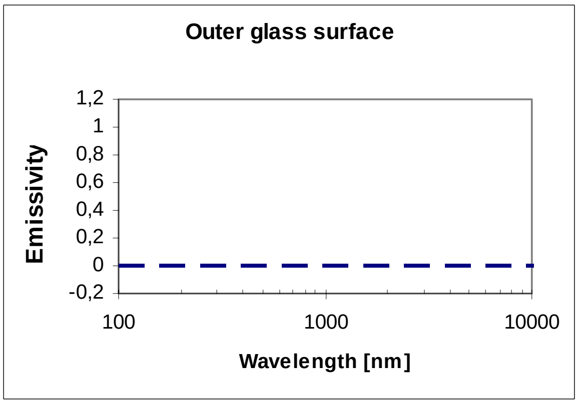
The rubber bag and the glass surface should have different optical properties:
For the rubber bag we want the absorbance in the spectral range of the solar radiation (300-2500nm) to be high, while the emittance (and thus the absorbance) should be low for the thermal wavelengths where the solar collector is emitting. For a solar collector at 100-300C, the maximum black-body radiation is for wavelengths in the region (using λ_{max}=2898/T

μm/K) 7.7- 5.0 μm. The emittance should thus be as low as possible in this wavelength region.

For the glass we want a high transmittance, i.e. low absorbance, for the λ< 2500nm, and low emittance for the thermal wavelengths as well.

A selective surface has a wavelength dependence for the absorbance/emittance [ε(λ) = α(λ)], that is optimized for a certain application. The term is normally used for solar collector surfaces that need maximum absorption of the solar spectrum and minimal emission of thermal radiation.





3. Wind energy

   a) *Give an expression for the power generated by a wind turbine in a wind of speed u_o and air density ρ, intercepting a cross-section A of the wind front. What does the power coefficient C_P tell us? What is the maximal value of C_P? Why is it not 100%? (i.e. Why can we not extract 100% of the energy in the wind?)*

An expression for the power delivered by the turbine is

$P_T = \frac{1}{2} C_p \rho A u_0^3 = C_p P_0$, where P_0 is the (available) power in the wind.

$C_p = 4a(1-a)^2$ tells us how much of the incoming power in the wind P_0 that can be extracted by a wind turbine. The maximum for C_p is reached for a=1/3 and equals C_p^{max}= 0.59. C_p can not be 100% since that would imply that all the power is taken out of the wind, and the wind speed at the down-steam side u_2 would equal zero and the air at the down-stream side will not have any energy to move away from the turbine region.

   b) *The electricity consumption in Norway is ca 110TWh per year. How many windmills with a diameter of 80m are needed to supply 25% of this consumption by wind energy? Assume that the wind blows at 13m/s 1/3 of the time (and that there is no wind at other times), an air density of 1.2 kgm^{-3}, and that the efficiency of the windmill is 50% of the maximum theoretical value.*

A wind mill of diameter 80m has an area of A= $\pi\ 40^2\ m^2$= 5024 m^2, and can collect wind energy over this area.

The energy in the wind with speed u_0, through a cross-section A is

$$P_T = \frac{1}{2}\rho A u_0^3 \cdot C_p$$

The air density is taken to be ρ = 1.2 kgm^{-3}, and C_p is said to be 50% of the maximum value of 59%, i.e. C_p =0.50 x 0.59 = 0.295.

Thus for a wind speed of 13 m/s

$$P_T = \frac{1}{2}\rho A u_0^3 \cdot C_p = \frac{1}{2} 1.2 kgm^{-3} \cdot 5024 m^2 \cdot 13^3 m^3 s^{-3} \cdot 0.295 = 2.0 MW$$

The wind is blowing only 1/3 of the time, so the generated power from one turbine is:

$$P_T = 2.0/3\ MW = 0.65 MW$$

In one year one turbine will produce E_T = 0.65MW x (24 x 365)h = 5.7GWh, so we need 0.25x110TWh/5.7GWh = <u>4820 turbines</u> to supply 25% of the total *electric energy* needed for one year.

c) *If the same amount of electricity should be provided by a typical coal fuelled power plant, approximately how much thermal energy should the coal provide (in kWh per year)?*

Assuming that a typical coal fired power plant has a thermal efficiency of 33%, we need three times as much thermal energy from the coal combustion, i.e. 3x110TWh = 330TWh.

If the efficiency is higher, say 45%, then 1/0.45 x 110TWh = 244Wh is needed.

4. *The hydrogen society*

a) *Sketch and explain how a fuel cell works. Write down the reaction in the fuel cell. What are the advantages by using hydrogen as a fuel?*

A fuel cell is shown in the figure below. It works like an ordinary battery, by converting chemical energy (stored in the hydrogen) into electrical energy. As long as hydrogen and oxygen are supplied the fuel cell will produce electric energy, while an ordinary battery will be exhausted for chemical energy during operation and needs to be recharged.

Hydrogen and oxygen enters at different locations in the cell. The hydrogen enters an electrolyte, which is a liquid in which ions can move. In the electrolyte the H-atoms looses an electron to the anode, and H+ ions are formed. The H+ ions move in the electrolyte from the anode to the cathode, while <u>the electrons set up a current in the external circuit</u>. At the cathode, the electrons, the hydrogen ions and oxygen atoms combine to form water, as described in the equation below.

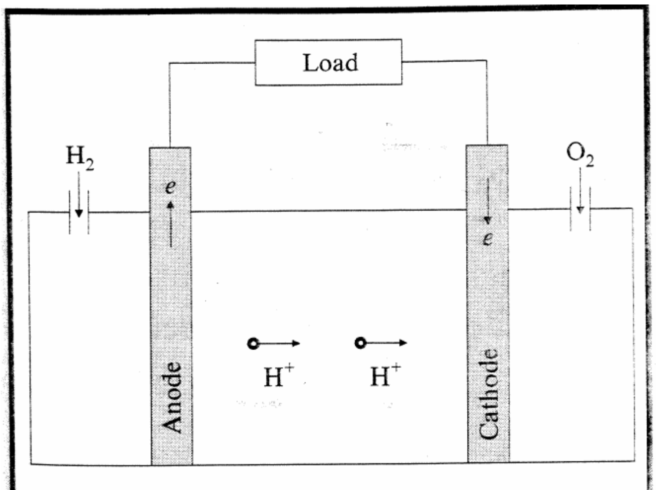


Figure 4.20 Fuel cell. Hydrogen enters at the anode and produces electrons and ions. They combine with oxygen at the cathode to form water

The basic reaction of the hydrogen fuel cell is

$$2H_2 + O_2 \rightarrow 2H_2O$$

Advangtages:

1) Environment:

- Exhaust= water
- No local emissions (NOx under some conditions)
- No CO_2 emissions CAN be achieved (dependent on energy/hydrogen source and technology)

2) Energydistribution

- a possible energy carrier – along with electricity –when fossil energy ”runs out”
- flexible production – from renewable and fossil energy sources
- Flexible use – can be converted into (electrical) energy
  - In fuel cells (not limited by the Carnot cycle)
  - By combustion (engines, turbines, catalytic…)
- Simpler storage than electricity

b) Give at least two examples on how hydrogen can be produced and two examples of how it can be stored.

Production

- From fossil fuels
- From water (electrolysis)
- From biomass and waste

Storage:

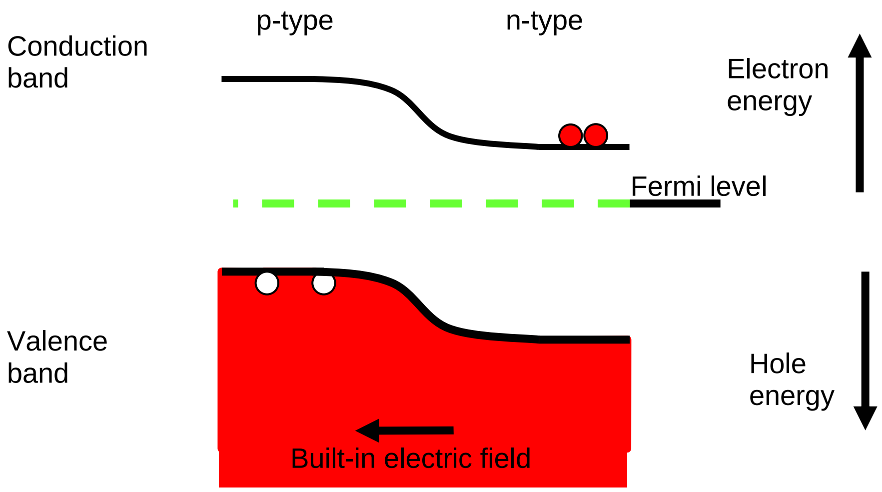
- Compressed hydrogen gas
- Liquid hydrogen (20 K)
- Chemically bound in a (liquid) energy-containing compound
- Absorbed or adsorbed in a solid

5. Silicon solar cells

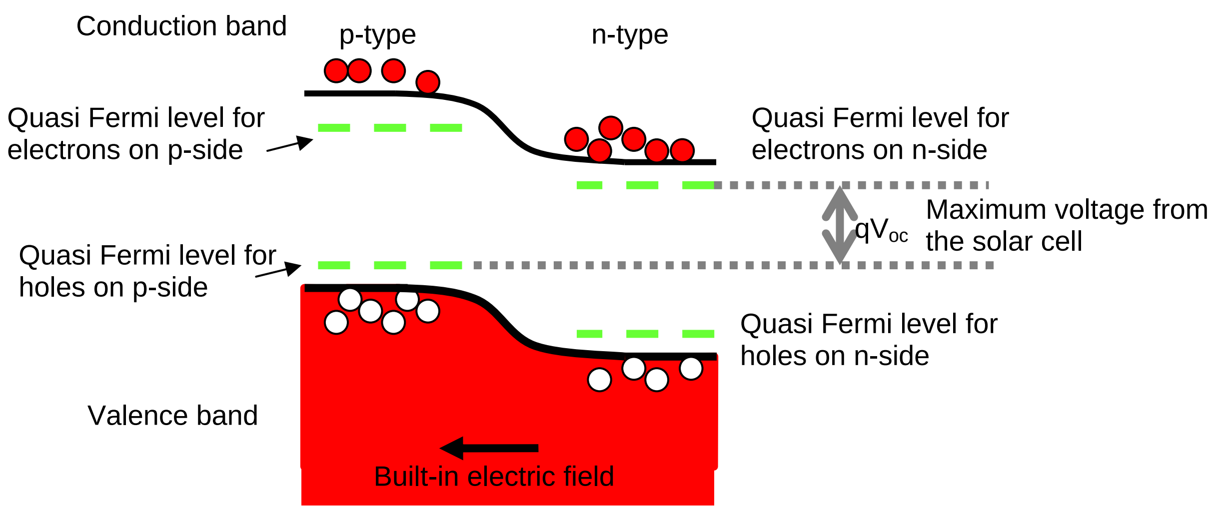
a) *What is the basic working principle of a typical solar cell? Draw energy band diagrams for the cell in the dark and under illumination. Indicate the Fermi level (cell in the dark) and the Fermi level splitting (leading to Quasi Fermi levels) caused by the photogenerated carriers.*

In a solar cell solar radiation is converted to electric energy by absorption in a semiconductor. In the absorption process an electron is excited from a state in the valence band (where the electron takes part in the bonding of the semiconductor) to a state in the conduction band (where the electron is able to conduct a current through the semiconductor). The empty state left behind in the valence band will also contribute to the photo-generated current, so each incoming photon, with energy larger than the band gap of the semiconductor will crate one electron-hole pair. The photo-generated electron-hole pair will move in opposite directions in the built-in electric field that (normally) is produced by forming a pn-junction in the semiconductor. (The electrons move from the p-side to the n-side.) Outside the pn-junction, the electrons and holes will diffuse to the external contacts on the solar cell.

In the dark the solar cell behaves like a diode, and if nothing is connected to the contacts, the band diagram will look like indicated below (constant Fermi level):

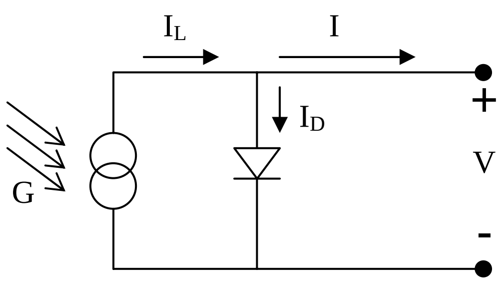


Under illumination, excess electrons and holes are photo-generated throughout the solar cell, and as a result the Fermi level splits into two quasi Fermi levels on each side of the pn-juntion, as indicated in the energy diagram below:



The voltage over the solar cell is determined by the splitting of the Quasi Fermi level for electrons on the n-side and holes on the p-side.

b) *Make a drawing of an equivalent circuit of the solar cell under illumination, with the photogenerated current I_L indicated as a current source. Indicate the currents that flow in the circuit when the cell is under illumination, and write down an expression for the current that can be extracted from the solar cell. Use this expression find an expression for the short circuit current and for the open circuit voltage. What do these parameters represent?*



The current that can be extracted for the solar cell can be expressed as:

$$I(V) = I_L - I_D = I_L - I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

where $I_D = I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$ is the dark current.

The short circuit current I_{sc} represents the largest current that the solar cell can deliver, but only to a short circuited load, i.e. when the voltage over the cell is zero. We find an expression for I_{sc} by setting V=0 and I_{sc}=I(0) in the expression for I(V):

$$I_{sc} = I(0) = I_L - I_0\left[\exp\left(\frac{e\cdot 0}{kT}\right) - 1\right] = I_L - I_0[1-1] = I_L$$

The short circuit current is thus equal the photo generated current I_L.

The open circuit voltage represents the largest voltage over the solar cell, and occurs when there is no load connected to the solar cell, i.e. when the current equals zero. We find an expression for Voc by setting $V=V_{oc}$ and I=0, in the expression for I(V):

$$I(V) = 0 = I_L - I_0\left[\exp\left(\frac{eV_{oc}}{kT}\right) - 1\right]$$

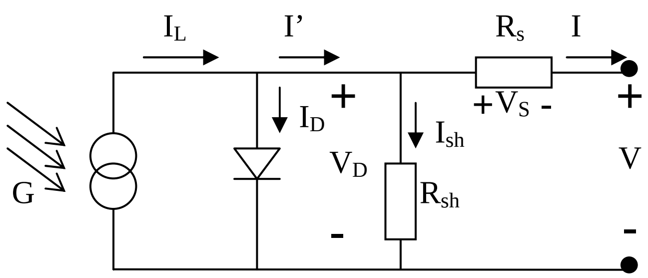
$$\Leftrightarrow I_L = I_0\left[\exp\left(\frac{eV_{oc}}{kT}\right) - 1\right]$$

$$\Leftrightarrow \frac{I_L}{I_0} + 1 = \exp\left(\frac{eV_{oc}}{kT}\right)$$

$$\Leftrightarrow \ln\left(\frac{I_L}{I_0} + 1\right) = \frac{eV_{oc}}{kT}$$

$$\Leftrightarrow V_{oc} = \frac{kT}{e}\ln\left(\frac{I_L}{I_0} + 1\right)$$

c) *Next make a new equivalent circuit drawing and include series resistance and shunt resistance in the solar cell. From the circuit derive expressions for the current and voltage delivered by the solar cell during operation. What is the effect of a large series resistance and a low shunt resistance on the efficiency of the solar cell?*



The current from the solar cell is now reduced due to the series resistance R_s and the shunt resistance R_{sh}. The current I delivered by the cell is now

$I = I' - I_{sh} = (I_L - I_D) - I_{sh}$

I_{sh} is the current flowing through the shunt resistance (ideally this current should be zero), and from Ohms law (V=RI), this current equals

$I_{sh} = V_D / R_{sh}$

V_D can be related to V, by noting that $V_D = V_S + V$, where V_S is the voltage drop over the series resistance. Again we use Ohm's law to find: $V_S = R_S I$, and $V_D = R_S I + V$

Finally,

$$I = (I_L - I_D) - \frac{V_D}{R_{sh}} = I_L - I_D - \left(\frac{R_s I + V}{R_{sh}}\right)$$

i.e. the current from the solar cell is reduced from the ideal $I' = I_L - I_D$.

For the voltage over the solar cell we get:
$V = V_D - R_S I$
i.e. the voltage is reduced from the ideal V_D.

A large series resistance and a small shunt resistance reduces both the current and the voltage delivered by the cell during operation, thus the efficiency given by $\eta = \frac{I \times V}{P_{solar}}$ is reduced.