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Exam FY3403: Particle Physics
Wednesday December 19th 2012
09.00-13.00
English

The exam consists of 3 problems. Each problem counts for a specific percentage of the total weight of the exam. This percentage is indicated for each problem on the following pages. Note that each sub-exercise **(a)**, **(b)**, etc. does not necessarily count equally.

Read each problem carefully in order to avoid unnecessary mistakes.

Allowed material to use at exam: C.

- Approved, simple calculator.
- K. Rottmann: Matematisk formelsamling.
- K. Rottmann: Mathematische Formelsammlung. Barnett & Cronin: Mathematical Formulae.

Also consider the Supplementary Material on the last page of this exam.

PROBLEM 1 (50%)

In all the problems below you may use words, equations, and figures in your response. Your response is expected to be detailed - short and inaccurate answers will be given a lower score.

(a) Describe the reason for why the following particles do not decay: neutrino, electron, photon, proton.

(b) Which interaction (weak, strong, electromagnetic) is responsible for the following decays and *why*:

- $\Sigma^- \to n + \pi^-$
- $\Sigma^- \to n + e^- + \bar{\nu}_e$
- $\Delta^{++} \to p^+ + \pi^+$.

You may use that the quark content of the particles above is: $n = udd$, $\Sigma^- = dds$, $\pi^- = d\bar{u}$, $\pi^+ = u\bar{d}$, $\Delta^{++} = uuu$, $p = uud$.

(c) Give an example of a particle that is (i) a boson with spin 0, (ii) a fermion with spin 3/2, (iii) a boson with spin 1. What kind of quantum field does the Klein-Gordon equation describe?

(d) Describe the meaning of quark confinement and how this is related to the color of a particle. Describe what is meant by asymptotic freedom in the context of the quark model.

(e) Describe the experiment conducted by Lee and Yang that demonstrated parity violation. Which interactions (weak, electromagnetic, strong) conserve parity?

(f) Describe what is meant by renormalization of the coupling constants in Feynman diagrams. Describe in particular the physical interpretation of the finite contributions to the renormalized coupling constant which makes it ”running”.

(g) Give an example of a process which can occur both via weak and electromagnetic interactions. Give an account for the conditions that must be present in order for the weak contribution to dominate the electromagnetic contribution and vice versa.

(h) Describe what is meant by an homomorphism in group theory. Provide an argument for whether or not SU(2) is isomorphic to SO(3). Is SU(2) a representation of SO(3)?

(i) The energy spectrum for positronium (e^-e^+) may be computed using the Schrödinger equation and perturbation theory. Consider now quarkonium ($q\bar{q}$ where q is a quark). Discuss if it would be possible to use the same treatment as for positronium to find the energy levels of quarkonium and specify any complications that would arise in such a treatment.

(j) Define what is meant by a gauge theory and spontaneous symmetry breaking. Describe how the physical (measurable) quantities in electromagnetic theory change for different choices of gauge. Describe the Higgs mechanism.

PROBLEM 2 (25%)

(a) Consider electron-electron scattering $e^- + e^- \to e^- + e^-$. Write down the two lowest order Feynman diagrams contributing to this process and their respective Feynman amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$. You only need to consider the contribution from photon-mediated scattering (i.e. you don't have to include the contribution from Z^0 mediated scattering).

(b) Assume that we are working in the high-energy limit so that the electron mass m_e can be set to zero.

Compute the spin-averaged total amplitude for this process $\langle|\mathcal{M}|^2\rangle$ where $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$. Express the answer solely in terms of the electromagnetic coupling constant g_e and the four-momenta p_i of the particles.

You can use as a known fact that:

$$\langle|\mathcal{M}_1|^2\rangle = \frac{8g_e^4}{(p_1 - p_3)^4}[(p_1p_2)(p_3p_4) + (p_1p_4)(p_2p_3)] \tag{1}$$

where p_1 and p_3 are the incoming and outgoing momenta of the external electron lines coupled at a vertex while p_2 and p_4 are the incoming and outgoing momenta of the external electron lines coupled at the other vertex.

Hint: Use Casimir's trick to evaluate $\langle \mathcal{M}_1 \mathcal{M}_2^* \rangle$.

PROBLEM 3 (25%)

Consider the following two possible decays of K^-:

$$K^- \to e^- + \bar{\nu}_e \tag{2}$$
$$K^- \to \mu^- + \bar{\nu}_\mu. \tag{3}$$

The Feynman diagrams may be represented as in Fig. 1, where l is the lepton species. The Feynman amplitude will have the form:

$$\mathcal{M} = \frac{g_W^2}{8(M_W c)^2}[\bar{u}(3)\gamma_\mu(1-\gamma^5)v(2)]F^\mu, \tag{4}$$

where $F^\mu = f_K p^\mu$. Here, f_K is a real constant and p^μ is the momentum of the kaon. The kaon has zero spin.

(a) Calculate the spin-averaged quantity $\langle|\mathcal{M}|^2\rangle$ for both of these processes in order to find an explicit expression for the decay rate in each case. Calculate the ratio of the two decay rates (insert the values of the masses et.c. in the final answer so that you get a numerical value for this ratio). You can use as a known fact the decay rate is in general given by:

$$\Gamma = \frac{|\mathbf{p}_{\text{out}}|}{8\pi\hbar m_K^2 c}\langle|\mathcal{M}|^2\rangle \tag{5}$$

where $|\mathbf{p}_{\text{out}}| = (c/2m_K)(m_K^2 - m_l^2)$.

(b) The experimentally measured lifetime of K^- turns out to be 1.2×10^{-8} sec and 64% of all K^- particles decay via the $\mu^- + \bar{\nu}_\mu$ route. Compute the kaon decay constant f_K based on this information.

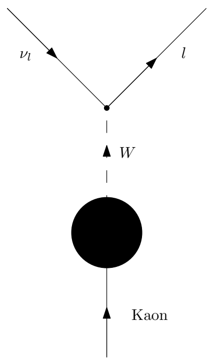

FIG. 1: Feynman diagrams for the two processes above. The black blob represents the external kaon line and the $K \to W$ vertex.

Supplementary Information

The regime of validity and the meaning of the symbols below are assumed to be known by the reader.

Mass of W: $m_W \simeq 82$ GeV. Mass of K^-: 493 MeV$/c^2$. Mass of electron and muon: $m_e = 0.51$ MeV$/c^2$ and $m_\mu = 105$ MeV$/c^2$.

$\hbar = 6.58 \times 10^{-16}$ eV$\cdot$s. Weak coupling constant: $g_W = 0.66$.

Photon propagator: $-\mathrm{i}g_{\mu\nu}/q^2$. QED vertex factor: $\mathrm{i}g_e\gamma^\mu$.

W propagator: $\frac{-\mathrm{i}(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$. Weak vertex factor: $-\frac{\mathrm{i}g_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$.

Completeness relations ($\bar{u} = u^\dagger\gamma^0$):

$$\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + mc), \ \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - mc). \tag{6}$$

We also have that $\gamma^0(\gamma^\nu)^\dagger\gamma^0 = \gamma^\nu$ whereas $\gamma^5 = (\gamma^5)^\dagger$ and γ^5 anticommutes with γ^μ.

Trace theorems: (below I use the notation $a' \equiv a^\mu\gamma_\mu$)

$$Tr(A+B) = Tr(A) + Tr(B),\ Tr(\alpha A) = \alpha Tr(A),\ Tr(ABC) = Tr(CAB) = Tr(BCA). \tag{7}$$

$$g_{\mu\nu}g^{\mu\nu} = 4,\ \{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu},\ a'b' + b'a' = 2ab. \tag{8}$$

$$\gamma_\mu\gamma^\mu = 4,\ \gamma_\mu\gamma^\nu\gamma^\mu = -2\gamma^\nu,\ \gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\mu = 4g^{\nu\lambda}. \tag{9}$$

$$\gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\gamma^\mu = -2\gamma^\sigma\gamma^\lambda\gamma^\nu,\ \gamma_\mu a'\gamma^\mu = -2a'. \tag{10}$$

$$\gamma_\mu a'b'\gamma^\mu = 4ab,\ \gamma_\mu a'b'c'\gamma^\mu = 2 - c'b'a'. \tag{11}$$

$$Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu},\ Tr(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}). \tag{12}$$

$$Tr(a'b') = 4ab,\ Tr(a'b'c'd') = 4[(ab)(cd) - (ac)(bd) + (ad)(bc)], \gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3. \tag{13}$$

$$Tr(\gamma^5) = 0,\ Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0,\ Tr(\gamma^5\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4\mathrm{i}\varepsilon^{\mu\nu\lambda\sigma}. \tag{14}$$

$$Tr(\gamma^5 a'b') = 0,\ Tr(\gamma^5 a'b'c'd') = 4\mathrm{i}\varepsilon^{\mu\nu\lambda\sigma}a_\mu b_\nu c_\lambda d_\sigma, \tag{15}$$

where $\varepsilon^{\mu\nu\lambda\sigma}$ is -1 if $\mu\nu\lambda\sigma$ is an even permutation of 0123, +1 if it is an odd permutation, 0 if any two indices are the same. Finally, the trace over an odd number of γ matrices is zero.